6.26 The Electric Field Radiated By A Short Dipole Antenna Is Given In Spherical Coordinates By E(R,

6.26 The Electric Field Radiated By A Short Dipole Antenna Is Given In Spherical Coordinates By E(R, is a fundamental concept in antenna theory and electromagnetics, describing how a short dipole antenna radiates electromagnetic waves into space. Understanding the behavior of the electric field generated by such antennas in spherical coordinates is essential for antenna design, placement, and analysis of radiated power patterns. This article provides an in-depth exploration of the electric field produced by a short dipole, emphasizing its mathematical formulation, physical interpretation, and practical implications.

Introduction to Short Dipole Antennas

What Is a Short Dipole Antenna?

A short dipole antenna is a simple, straight wire or rod that is significantly shorter than the wavelength of the operating frequency. Typically, its length l satisfies:

$$
l \ll \lambda
$$

where λ is the wavelength. Due to its small size relative to the wavelength, the current distribution along the antenna is approximately uniform, and the antenna can be treated as a Hertzian dipole or electrically small antenna.

Applications of Short Dipole Antennas

Short dipoles are widely used in:
- Radio broadcasting
- Wireless communication systems
- Measurement and calibration setups
- Educational demonstrations in electromagnetics

Their simplicity makes them ideal for theoretical analysis and practical implementation in various frequency regimes.

Electromagnetic Field of a Short Dipole: Basic Principles

Radiation Mechanism

The short dipole radiates electromagnetic energy when an alternating current flows through it. The time-varying current produces a time-varying electric dipole moment, which in turn emits electromagnetic waves. The radiated fields depend on the current distribution, antenna length, and surrounding environment.

Assumptions for Analytical Simplicity

In deriving the electric field, the following assumptions are typically made:
- The antenna length l is much smaller than the wavelength ($l \ll \lambda$)
- The current distribution is uniform along the length
- The medium is free space with permittivity ε_0 and permeability μ_0
- The observation point is in the far-field region, where $R \gg l$

Mathematical Formulation of the Electric Field in Spherical Coordinates

Coordinate System and Definitions

Spherical coordinates are ideal for describing radiation patterns:
- R: radial distance from the antenna
- θ: polar angle measured from the axis of the dipole (usually the z-axis)
- ϕ: azimuthal angle in the xy-plane

The electric field radiated by the short dipole is primarily transverse, with its magnitude and orientation depending on R and θ.

Expression for the Electric Field

The electric field in spherical coordinates for a short dipole oriented along the z-axis is given by:

$$
\boxed{
\mathbf{E}(R, \theta) = \hat{\theta} \; \frac{I_0 l}{4 \pi \varepsilon_0 R} \, j \, \omega \, \sin \theta \, e^{-j k R}
}
$$

where:
- I_0: maximum current at the feed point
- l: length of the dipole
- ε_0: permittivity of free space
- R: radial distance from the dipole
- j: imaginary unit ($j^2 = -1$)
- ω: angular frequency ($\omega = 2 \pi f$)

- k: wave number ($k = \frac{2 \pi}{\lambda}$)
- $e^{-j k R}$: phase factor accounting for wave propagation

This expression indicates that the electric field is predominantly transverse (in the θ-direction) and diminishes with distance R.

Physical Interpretation of the Mathematical Expression

- The amplitude scales proportionally with the current I_0 and the antenna length l, reflecting the dipole's strength.
- The $\sin \theta$ term describes the angular dependence, with maximum radiation in the plane perpendicular to the dipole axis ($\theta = 90^\circ$) and nulls along the axis ($\theta = 0^\circ, 180^\circ$).
- The $1/R$ decay indicates spherical spreading of the wave in free space.
- The phase factor $e^{-j k R}$ accounts for the propagation delay, crucial for understanding interference and phase relationships.

Radiation Pattern and Directivity

Angular Distribution of Radiated Power

The radiation pattern of a short dipole is characterized by its angular dependence:

$$
U(\theta) \propto \sin^2 \theta
$$

which results in a toroidal (doughnut-shaped) pattern with maximum radiation perpendicular to the dipole axis and nulls along it.

Directivity and Gain

- Directivity (D): Measures how focused the radiation pattern is in a particular direction.
- For a short dipole, the directivity is approximately:
$$
D \approx 1.5
$$
This indicates that the antenna radiates 50% more power in its main direction compared to an isotropic radiator.

- Gain: Incorporates efficiency factors and is closely related to directivity but accounts for losses.

Far-Field Region and Its Significance

Definition of the Far-Field Zone

The far-field region, where the provided electric field expression is valid, is characterized by:

$$
R \gg \lambda
$$

In this zone, the electromagnetic waves can be considered locally plane, and their angular pattern remains consistent over distance.

Implications for Antenna Design

Designers aim to position receiving antennas within the far-field region to accurately analyze the radiation pattern and optimize communication links.

Practical Considerations and Applications

Measurement and Calibration

The analytical expression of the electric field allows engineers to:
- Calculate expected field strengths
- Design measurement setups
- Calibrate antenna systems

Limitations and Real-World Factors

- Finite Conductivity: Real antennas have losses that reduce efficiency.
- Near-Field Effects: Close to the antenna, the field behavior deviates from the far-field approximation.
- Environmental Factors: Reflection, diffraction, and interference alter the actual radiation pattern.

Design Optimization

Understanding the detailed electric field distribution enables:
- Optimization of antenna length and orientation
- Minimization of side lobes
- Enhancement of directivity and gain

Conclusion

The electric field radiated by a short dipole antenna, expressed in spherical coordinates as:

$$
\mathbf{E}(R, \theta) = \hat{\theta} \; \frac{I_0 l}{4 \pi \varepsilon_0 R} \, j \, \omega \, \sin \theta \, e^{-j k R}
$$

provides a comprehensive understanding of how such antennas emit electromagnetic energy into space. Its angular dependence, amplitude decay, and phase characteristics form the foundation for analyzing and designing efficient antenna systems. Recognizing these principles is crucial for advancing wireless communication technologies, radar systems, and electromagnetic research, ensuring optimal performance and minimal interference.

Keywords: short dipole antenna, electric field, spherical coordinates, radiation pattern, far-field, directivity, electromagnetic waves, antenna theory, radiated power

Frequently Asked Questions

What is the expression for the electric field radiated by a short dipole antenna in spherical coordinates?

The electric field radiated by a short dipole antenna in spherical coordinates (R, θ, φ) is given by $E(R, \theta) = (j\omega\mu_0 I_0 l / 4\pi R) \sin \theta \, e^{-j k R} \, \hat{\theta}$, where I_0 is the current amplitude, l is the length of the dipole, ω is the angular frequency, μ_0 is the permeability of free space, k is the wave number, and $\hat{\theta}$ is the unit vector in the θ-direction.

Why does the electric field of a short dipole depend on sin θ in spherical coordinates?

The $\sin \theta$ dependence arises because the radiation pattern of a short dipole is strongest perpendicular to its axis ($\theta = 90°$) and zero along its axis ($\theta = 0°$ and $180°$), reflecting the dipole's angular distribution of radiated energy.

How does the distance R affect the magnitude of the electric field radiated by a short dipole antenna?

The electric field magnitude decreases proportionally to $1/R$, indicating spherical spreading of the wavefront as the distance from the antenna increases.

What role does the phase factor $e^{-j k R}$ play in the electric

field expression?

The phase factor e^{-j k R} accounts for the wave propagation delay from the source to the observation point, representing the phase shift due to the distance R.

In what way is the electric field of a short dipole antenna polarized?

The electric field is polarized in the θ-direction, perpendicular to the dipole axis, indicating that the radiation pattern is linearly polarized with the electric field oscillating in the θ-plane.

How does the length of the dipole (l) influence the radiated electric field?

The electric field amplitude is directly proportional to the dipole length l; a longer short dipole produces a stronger radiated field, assuming all other factors remain constant.

What assumptions are made in deriving the electric field expression for a short dipole antenna?

The derivation assumes that the dipole length l is much smaller than the wavelength ($l \ll \lambda$), the current distribution is uniform, and the far-field approximation ($R \gg \lambda$) applies.

How is the angular dependence of the electric field expressed in the spherical coordinate formula?

The angular dependence is expressed as $\sin \theta$, indicating the radiation intensity varies with the angle θ from the dipole axis, with maximum radiation at $\theta = 90°$.

What is the significance of the spherical coordinate system in analyzing the electric field of a dipole antenna?

Using spherical coordinates simplifies the analysis of radiated fields, allowing clear representation of angular variation and distance dependence in the far-field radiation pattern.

Additional Resources

Understanding the Electric Field Radiated by a Short Dipole Antenna

A fundamental concept in antenna theory and electromagnetic wave propagation is understanding how antennas radiate electromagnetic fields. Among various antenna types, the short dipole antenna—also known as a Hertzian dipole—serves as a foundational model for analyzing radiation patterns and field distributions. In particular, the electric field radiated by a short dipole in spherical coordinates encapsulates vital insights into the behavior of radiated waves, their angular dependence, and their decay characteristics.

This comprehensive review delves into the mathematical expression of the electric field E(R, θ, ϕ) radiated by a short dipole antenna, emphasizing the significance of each term, the physical intuition behind the formula, and its applications in antenna design and electromagnetic theory.

Fundamentals of a Short Dipole Antenna

Before dissecting the electric field expression, it is essential to understand the basic characteristics of a short dipole:

- Definition: A short dipole is an antenna much shorter than the wavelength of the operating frequency, typically L <
- Configuration: It consists of two conductors separated by a small distance, with a driven current oscillating at a given frequency.

- Current Distribution: For a short dipole, the current distribution can be approximated as uniform along its length, which simplifies the mathematical modeling.

- Radiation Pattern: The radiation pattern of a short dipole is primarily a toroidal (donut-shaped) pattern, with maximum radiation perpendicular to the axis of the dipole and minimal radiation along its axis.

Mathematical Representation of the Radiated Electric Field

The electric field E(R, θ, ϕ) radiated by a short dipole can be expressed in spherical coordinates (R, θ, ϕ), where:

- R: Distance from the dipole to the observation point.
- θ: Polar angle measured from the z-axis (axis of the dipole).
- ϕ: Azimuthal angle in the xy-plane.

The general form for the far-field electric component of a short dipole is:

$$
\boxed{
\mathbf{E}(R, \theta) \approx \frac{j \eta I_0 l}{4 \pi R} \left(\frac{\cos \left(\frac{\pi l}{\lambda} \cos \theta \right) - \cos \left(\frac{\pi l}{\lambda} \right)}{\sin \theta} \right) \hat{\theta}
}
$$

\]

where:

- η: Intrinsic impedance of free space (~377 Ω).
- I_0: Maximum current at the center of the dipole.
- l: Length of the dipole.
- λ: Wavelength of the emitted radiation.
- j: Imaginary unit, indicating a phase shift.
- R: Radial distance from the antenna (assuming far-field conditions).
- $\hat{\theta}$: Unit vector in the θ-direction, indicating the electric field is predominantly transverse in the far field.

Key Components of the Expression

To understand the physical significance, consider each component:

1. Amplitude Factor: $\frac{j \eta I_0 l}{4 \pi R}$
- Represents the decay of the field with distance (1/R dependence).
- Incorporates the antenna current and length, showing that the radiated field strength scales with the dipole’s physical and excitation parameters.

2. Angular Dependence: $\left(\frac{\cos \left(\frac{\pi l}{\lambda} \cos \theta \right) - \cos \left(\frac{\pi l}{\lambda} \right)}{\sin \theta} \right)$
- Encapsulates how the field varies with angle θ.
- For very short dipoles ($l \to 0$), this simplifies significantly, as higher-order terms become negligible.

3. Directionality: $\hat{\theta}$
- Indicates the electric field vector oscillates primarily perpendicular to the axis of the dipole in the far field.
- The magnetic field is orthogonal to both the electric field and the direction of propagation, consistent with electromagnetic wave behavior.

Approximations for Very Short Dipoles

For very short dipoles ($l \ll \lambda$), the expression simplifies because the cosine terms can be expanded using Taylor series:

$$
\cos \left(\frac{\pi l}{\lambda} \cos \theta \right) \approx 1 - \frac{1}{2} \left(\frac{\pi l}{\lambda} \right)^2 \cos^2 \theta
$$

and similarly,

$$
\cos \left(\frac{\pi l}{\lambda} \right) \approx 1 - \frac{1}{2} \left(\frac{\pi l}{\lambda} \right)^2
$$

Subtracting these yields:

$$
\cos \left(\frac{\pi l}{\lambda} \cos \theta \right) - \cos \left(\frac{\pi l}{\lambda} \right) \approx - \frac{1}{2} \left(\frac{\pi l}{\lambda} \right)^2 (\cos^2 \theta - 1) = \frac{1}{2} \left(\frac{\pi l}{\lambda} \right)^2 \sin^2 \theta
$$

Plugging back, the electric field simplifies to:

$$
\mathbf{E}(R, \theta) \approx \frac{j \eta I_0 l}{4 \pi R} \left(\frac{\frac{1}{2} \left(\frac{\pi l}{\lambda} \right)^2 \sin^2 \theta }{\sin \theta} \right) \hat{\theta} = \frac{j \eta I_0 l}{4 \pi R} \times \frac{1}{2} \left(\frac{\pi l}{\lambda} \right)^2 \sin \theta \, \hat{\theta}
$$

which emphasizes the $\sin \theta$ dependence typical of dipole radiation, with the amplitude directly proportional to the dipole length l.

Physical Interpretation and Significance of the Fields

Understanding the physical implications of the electric field expression is crucial for practical applications.

Angular Distribution

- The field strength varies with θ, peaking at $\theta = 90^\circ$ (perpendicular to the dipole axis) and vanishing along $\theta = 0^\circ, 180^\circ$ (along the axis).
- The $\sin \theta$ dependence indicates a doughnut-shaped radiation pattern, characteristic of dipoles.

Distance Dependence

- The $1/R$ decay signifies the far-field region, where the electromagnetic waves behave as propagating waves with planar wavefronts.
- Near-field effects, which decay faster (like $1/R^2$ or $1/R^3$), are neglected in this approximation.

Polarization

- The electric field oscillates in the $\hat{\theta}$ direction, orthogonal to the propagation direction.
- This polarization is linear and transverse, aligned with the dipole's orientation.

Phase and Impedance

- The factor j indicates a phase shift of 90°, which is typical for radiated fields.
- The intrinsic impedance η links the electric and magnetic fields, ensuring the wave propagates with characteristic impedance.

Applications and Practical Implications

The detailed understanding of the electric field radiated by a short dipole has profound implications:

1. Antenna Design:
- Short dipoles serve as basic models for designing antennas in wireless communication systems.
- Insights into the angular and distance dependence help optimize radiation patterns for maximum coverage.

2. Electromagnetic Compatibility (EMC):
- Knowing the field distribution aids in assessing interference and ensuring compliance with standards.

3. Radar and Remote Sensing:
- Short dipole models help in understanding how electromagnetic waves scatter off objects, essential in radar systems.

4. Educational Value:
- Serves as a foundational example illustrating fundamental electromagnetic principles, such as wave propagation, polarization, and radiation patterns.

Limitations and Extensions of the Model

While the electric field expression provides valuable insights, it is essential to recognize its limitations:

- Far-Field Approximation:
The formula assumes the observation point is sufficiently far from the antenna ($R \gg \lambda$), where wavefronts are approximately planar.

- Small Dipole Length:
Valid primarily when $l \ll \lambda$; for longer dipoles, more complex models are needed.

- Lossless and Ideal Conditions:
Real-world factors such as conductor losses, dielectric effects, and environmental influences are not included

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