A 75 Kg Skier Accelerates From Rest To A Speed Of 12 M/s In 3.0 Seconds On A Frictionless Hill. What

A 75 Kg Skier Accelerates From Rest To A Speed Of 12 M/s In 3.0 Seconds On A Frictionless Hill. What does this scenario reveal about the fundamental principles of physics, specifically kinematics and dynamics? Understanding this situation provides insight into how forces, mass, and motion interrelate, and allows us to explore the concepts of acceleration, work-energy, and the conservation of energy in idealized conditions. Whether you are a physics student, a skiing enthusiast interested in the science behind the sport, or simply curious about how force and motion work, analyzing this case can deepen your comprehension of mechanical principles.

Understanding the Scenario: Basic Concepts and Definitions

Before delving into calculations, it's essential to clarify the key concepts involved in this scenario.

What is Kinematics?

Kinematics is the branch of mechanics that describes the motion of objects without considering the forces that cause the motion. In this case, we focus on the skier's change in velocity over time.

What is Dynamics?

Dynamics examines the forces acting on objects and how these forces influence motion. Here, we analyze the force responsible for accelerating the skier.

Key Quantities Involved

- Mass (m): 75 kg
- Initial velocity (u): 0 m/s (rest)
- Final velocity (v): 12 m/s
- Time taken (t): 3.0 seconds
- Acceleration (a): To be calculated
- Force (F): To be calculated
- Work done (W): To be determined

Calculating the Acceleration of the Skier

The first step is to determine how quickly the skier accelerates during the descent.

Using Kinematic Equations

The basic equation relating initial velocity, final velocity, acceleration, and time is:

$$ v = u + a t $$

Since the skier starts from rest:

$$ 12\, \mathrm{m/s} = 0 + a \times 3.0\, \mathrm{s} $$

Solving for acceleration:

$$ a = \frac{12\, \mathrm{m/s}}{3.0\, \mathrm{s}} = 4\, \mathrm{m/s^2} $$

Result: The skier accelerates at a rate of 4 meters per second squared.

Determining the Force Acting on the Skier

According to Newton's Second Law, the net force acting on an object is proportional to its mass and acceleration:

$$ F = m a $$

Plugging in the known values:

$$ F = 75\, \mathrm{kg} \times 4\, \mathrm{m/s^2} = 300\, \mathrm{N} $$

Interpretation: A force of 300 newtons is responsible for accelerating the skier up to 12 m/s in 3 seconds.

Work-Energy Perspective: How Is Energy Transferred?

The acceleration on a frictionless hill implies that the skier's potential energy is converted into kinetic energy.

Calculating the Kinetic Energy

Kinetic energy (KE) at the final speed:

$$ KE = \frac{1}{2} m v^2 $$

$$ KE = \frac{1}{2} \times 75\, \mathrm{kg} \times (12\, \mathrm{m/s})^2 $$

$$ KE = 37.5 \times 144 = 5400\, \mathrm{J} $$

Result: The skier gains 5400 joules of kinetic energy during the acceleration.

Work Done by the Force

Since the hill is frictionless, all the work done by the force results in kinetic energy:

$$ W = F \times d $$

where d is the distance traveled during the acceleration.

To find d, use the kinematic equation:

$$ v^2 = u^2 + 2 a d $$

$$ (12)^2 = 0 + 2 \times 4 \times d $$

$$ 144 = 8 d $$

$$ d = \frac{144}{8} = 18\, \mathrm{m} $$

Now, verify work:

$$ W = F \times d = 300\, \mathrm{N} \times 18\, \mathrm{m} = 5400\, \mathrm{J} $$

which matches the kinetic energy, confirming energy conservation.

Implications of the Physics in Real-World Skiing

While this idealized scenario assumes a frictionless hill, real-world conditions involve various forces and factors.

Friction and Air Resistance

- Friction between skis and snow opposes motion, reducing acceleration.
- Air resistance increases with speed, also affecting acceleration and maximum speed.

Energy Losses and Safety Considerations

- Energy is often dissipated as heat due to friction.
- Understanding forces and energy transfer helps in designing safer ski slopes and equipment.

Optimizing Skiing Performance

- Skiers aim to maximize their initial acceleration while maintaining control.
- Knowledge of physics can inform training and equipment choices.

Advanced Analysis: Calculating the Power and Force Distribution

Beyond basic force and energy calculations, we can explore how power varies during acceleration.

Instantaneous Power

Power at any moment is:

$$ P = F \times v $$

At the final velocity:

$$ P = 300\, \mathrm{N} \times 12\, \mathrm{m/s} = 3600\, \mathrm{W} $$

This indicates that at the end of acceleration, the skier's muscles or the applied force are exerting a power of approximately 3.6 kilowatts.

Average Power

Over the entire acceleration:

$$ P_{avg} = \frac{W}{t} = \frac{5400\, \mathrm{J}}{3\, \mathrm{s}} = 1800\, \mathrm{W} $$

This average power reflects the energy transfer rate during the acceleration phase.

Conclusion: The Interplay of Force, Energy, and Motion

This scenario exemplifies fundamental physics principles in action. The skier's acceleration from rest to 12 m/s in 3 seconds on a frictionless hill demonstrates the direct relationship between force, mass, and acceleration, as described by Newton's second law. Additionally, converting potential energy (from height) into kinetic energy showcases the conservation of energy, emphasizing how work done by forces results in energetic changes.

Understanding these principles is not only academically interesting but also practically valuable for athletes, engineers, and safety professionals involved in winter sports. Recognizing how forces and energy transfer influence motion can lead to optimized equipment, safer skiing environments, and improved performance.

Whether analyzing a simple physics problem or considering complex real-world scenarios, the foundational concepts of mechanics remain central to understanding movement and energy transformations in our physical world.

Frequently Asked Questions

What is the acceleration of the skier during the descent?

The acceleration can be calculated using the formula a = (final velocity - initial velocity) / time = (12 m/s - 0) / 3.0 s = 4 m/s^2.

What is the net force acting on the skier during the acceleration?

Using Newton's second law, F = m a = 75 kg 4 m/s^2 = 300 N.

How much kinetic energy does the skier gain after reaching 12 m/s?

Kinetic energy = 0.5 m v^2 = 0.5 75 kg $(12 m/s)^2$ = 0.5 75 144 = 5400 Joules.

What is the work done on the skier during the acceleration?

Since the hill is frictionless, the work done equals the change in kinetic energy, which is 5400 Joules.

If the skier's mass was doubled, how would that affect the acceleration?

On a frictionless hill, the acceleration depends only on gravity and the slope angle, not mass. So, doubling the mass would not change the acceleration.

What assumptions are made in calculating the acceleration and force in this scenario?

Assumptions include a frictionless hill, constant acceleration during the entire 3 seconds, and no external forces like air resistance affecting the motion.

Additional Resources

A 75 Kg Skier Accelerates From Rest To A Speed Of 12 M/s In 3.0 Seconds On A Frictionless Hill. What Does This Tell Us About Physics, Motion, and Energy Transfer?

Introduction

In the realm of physics, the motion of objects under various forces offers profound insights into fundamental principles governing the universe. The scenario of a skier, starting from rest and reaching a speed of 12 meters per second (m/s) in a mere 3.0 seconds on a frictionless hill, exemplifies these principles vividly. This situation serves as an excellent case study to explore concepts such as acceleration, force, work-energy relationships, and conservation of energy.

At first glance, it appears straightforward: a skier accelerates downhill without friction, gaining kinetic energy. However, dissecting this scenario reveals a wealth of physical insights, connecting theoretical physics with real-world applications. This article aims to analyze the problem comprehensively, addressing the underlying physics, calculations, and implications.

Section 1: Understanding the Scenario – Key Parameters and Setup

1.1 The Physical Context

A skier with a mass of 75 kg begins at rest atop a frictionless hill, accelerating uniformly to a final speed of 12 m/s over 3 seconds. The absence of friction implies that the primary force influencing the skier's acceleration originates from gravity and the geometric configuration of the hill.

1.2 Assumptions and Simplifications

- Frictionless Surface: No resistive forces oppose the skier's motion.
- Uniform Acceleration: The skier's acceleration remains constant throughout the 3 seconds.
- Negligible Air Resistance: The effect of air drag is ignored for simplicity.
- Initial and Final Conditions:
- Initial velocity, $u = 0 \, \text{m/s}$.
- Final velocity, $v = 12 \, \text{m/s}$.
- Time taken, $t = 3.0 \, \text{s}$.

1.3 Relevance of the Scenario

This setup offers a textbook example to analyze kinematic and dynamic quantities such as acceleration, force, work, and energy transfer. It also provides an opportunity to interpret real-world skiing dynamics within a physics framework.

Section 2: Calculating Acceleration and Force

2.1 Determining the Acceleration

Given the initial and final velocities over a specific time frame, the average acceleration a can be calculated using the kinematic equation:

$$
a = \frac{v - u}{t}
$$

Substituting the known values:

$$
a = \frac{12\,\text{m/s} - 0\,\text{m/s}}{3\,\text{s}} = 4\,\text{m/s}^2
$$

This indicates the skier's speed increases uniformly at 4 m/s^2 during the 3 seconds.

2.2 Calculating the Force Acting on the Skier

Newton's second law relates force, mass, and acceleration:

$$
F = m \times a
$$

Given $m = 75\, \mathrm{kg}$:

$$
F = 75\, \mathrm{kg} \times 4\, \mathrm{m/s}^2 = 300\, \mathrm{N}
$$

Thus, an average net force of 300 N acts on the skier, propelling them downhill.

2.3 Direction and Nature of the Force

Since the hill is frictionless, the net force responsible for acceleration is primarily component of gravity along the slope's incline, or an external force applied by the skier (e.g., pushing or tucking to increase speed). For simplicity, we assume the force is an effective component of gravitational pull along the hill.

Section 3: Energy Perspective – Work, Kinetic Energy, and Conservation

3.1 Kinetic Energy Gained

The kinetic energy (KE) of the skier at the final velocity is given by:

\[

$$
KE = \frac{1}{2} m v^2
$$

Calculating:

$$
KE = \frac{1}{2} \times 75\, \text{kg} \times (12\, \text{m/s})^2 = 37.5 \times 144 = 5400\, \text{J}
$$

This energy represents the work done on the skier during the acceleration phase.

3.2 Work-Energy Theorem

The work-energy theorem states that the net work done on an object equals its change in kinetic energy:

$$
W_{net} = \Delta KE
$$

Since the initial KE is zero (at rest), the work done by the forces on the skier is 5400 Joules.

3.3 Role of Gravity

On a frictionless hill, the primary source of this energy transfer is gravitational potential energy converting into kinetic energy. The component of gravitational force acting along the hill's slope does work, accelerating the skier.

Section 4: Inferring the Hill's Geometry and Inclination

4.1 Relationship Between Acceleration and Hill Incline

The acceleration of a skier on an inclined plane without friction is governed by:

$$
a = g \sin \theta
$$

where:

- $g = 9.8\, \text{m/s}^2$ (acceleration due to gravity),
- θ is the angle of the hill.

Rearranged:

$$
\sin \theta = \frac{a}{g} = \frac{4}{9.8} \approx 0.408
$$

Thus,

$$
\theta = \arcsin(0.408) \approx 24.0^\circ
$$

4.2 Hill Length and Height

Knowing the acceleration and time, we can estimate the length of the hill, assuming the skier starts from rest and accelerates uniformly:

$$
s = ut + \frac{1}{2} a t^2
$$

Since $u = 0$:

$$
s = \frac{1}{2} \times 4\, \text{m/s}^2 \times (3\, \text{s})^2 = 0.5 \times 4 \times 9 = 18\, \text{m}
$$

The skier travels approximately 18 meters along the slope to reach 12 m/s.

The vertical height h of the hill can be found using:

$$
h = s \sin \theta = 18\, \text{m} \times 0.408 \approx 7.34\, \text{m}
$$

This suggests the hill has a vertical drop of approximately 7.34 meters over an 18-meter slope.

4.3 Energy Consistency Check

The initial potential energy at the top:

$$
PE = m g h = 75\, \text{kg} \times 9.8\, \text{m/s}^2 \times 7.34\, \text{m} \approx 75 \times 9.8 \times 7.34 \approx 5398\, \text{J}
$$

This closely matches the calculated kinetic energy of 5400 J, confirming energy conservation in this frictionless model.

Section 5: Broader Implications and Real-World Considerations

5.1 Practical Limitations and Real-World Dynamics

While the idealized physics scenario provides clarity, real-world skiing involves additional factors:

- Friction: Both between skis and snow, and air resistance, which dissipate energy.
- Variable Terrain: Hills are rarely perfectly uniform; inclines change.
- Human Factors: Skiers exert variable forces, and their motion isn't perfectly uniform.
- Safety and Control: Accelerating at high rates may be unsafe or unfeasible in practice.

Despite these complexities, the physics principles outlined here help in designing safer, more efficient ski slopes and understanding the energy dynamics at play.

5.2 Engineering and Design Applications

Understanding the physics of acceleration and energy transfer informs:

- Slope Design: Ensuring appropriate inclines for desired speeds.
- Safety Measures: Calculating maximum safe speeds and accelerations.
- Equipment Development: Creating skis and gear optimized for energy transfer and control.

5.3 Educational and Analytical Value

This scenario serves as a rich teaching example for physics students, illustrating the interplay between forces, energy, and motion. It emphasizes how theoretical calculations align with real-world phenomena, fostering a deeper understanding of classical mechanics.

Conclusion

The analysis of a 75 kg skier accelerating from rest to 12 m/s over 3 seconds on a frictionless hill encapsulates core physics principles. The calculated acceleration of 4 m/s^2, the force of 300 N, and the energy transfer of approximately 5400 Joules collectively demonstrate the fundamental relationships between force, motion, and energy conservation.

From the hill's inclination to the energy transformations, each aspect reveals the elegance of physics in explaining everyday phenomena. While simplified models omit real-world complexities like friction and air resistance, they provide vital foundational insights crucial for advancing both scientific understanding and practical applications in sports engineering, safety, and education.

This scenario underscores the power of physics to decode motion, informing how we design, analyze, and appreciate the dynamic world around us.

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