A Batch Of 50 Items Contains 10 Defective Items. Suppose 10 Items Are Selected At Random And Tested.

A Batch Of 50 Items Contains 10 Defective Items. Suppose 10 Items Are Selected At Random And Tested. This scenario is a classic example encountered in quality control and statistical sampling. Understanding the probability and implications of such sampling is crucial for manufacturers, quality inspectors, and statisticians alike. When dealing with batches of products, determining the likelihood that a sample contains defective items helps in assessing overall quality and making informed decisions about acceptance or rejection of the entire batch.

In this article, we will explore the statistical concepts underlying this problem, including the hypergeometric distribution, probability calculations, and real-world applications. We aim to provide a comprehensive understanding of how to analyze such sampling scenarios, interpret results, and apply these insights in quality assurance processes.

Understanding the Scenario: Key Concepts

Before diving into calculations, it is essential to clarify the scenario's elements and relevant statistical principles.

The Batch and Its Composition

- Total items in the batch: 50
- Number of defective items: 10
- Number of non-defective items: 40

The Sampling Process

- Items selected at random: 10
- Items tested for defects
- Sampling is assumed to be without replacement (once an item is selected, it is not returned to the batch)

Goals of the Analysis

- Determine the probability that a certain number of defective items are found in the sample
- Assess the likelihood of detecting defective items during sampling
- Understand how the sample reflects the overall batch quality

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Statistical Foundations: The Hypergeometric Distribution

The problem at hand is a classic example of the hypergeometric distribution, which models the probability of drawing a specific number of successes (defective items) in a fixed number of draws from a finite population without replacement.

Definition of the Hypergeometric Distribution

The hypergeometric distribution gives the probability of k successes (defective items) in n draws from a population of size N containing K successes, without replacement. Its probability mass function (PMF) is:

$$
P(X = k) = \frac{\binom{K}{k} \binom{N - K}{n - k}}{\binom{N}{n}}
$$

where:
- N = total number of items in the population (50)
- K = total number of defective items (10)
- n = number of items sampled (10)
- k = number of defective items found in the sample (0, 1, 2, ..., up to min(K, n))

Why the Hypergeometric Distribution?

Since sampling is without replacement, the probability of each draw depends on previous draws, making the hypergeometric distribution the appropriate model rather than the binomial distribution, which assumes independent trials with replacement.

Calculating Probabilities: Examples and Insights

Let's explore specific probability calculations to understand the likelihood of various outcomes when selecting 10 items from the batch.

Probability of Finding Exactly 0 Defective Items

This corresponds to testing all selected items as non-defective.

$$
P(X=0) = \frac{\binom{10}{0} \binom{40}{10}}{\binom{50}{10}}
$$

Calculating:

- $\binom{10}{0} = 1$
- $\binom{40}{10}$: number of ways to choose all non-defective items in the sample
- $\binom{50}{10}$: total ways to select any 10 items

Using combinatorial calculations:

$$
P(X=0) = \frac{1 \times \binom{40}{10}}{\binom{50}{10}}
$$

Similarly, we can compute the probability for $k=1, 2, ..., 10$, but in practice, it’s often more useful to understand the expected number of defective items in the sample.

Expected Number of Defective Items in the Sample

The expected value $E[X]$ for the hypergeometric distribution is:

$$
E[X] = n \times \frac{K}{N}
$$

Plugging in the known values:

$$
E[X] = 10 \times \frac{10}{50} = 10 \times 0.2 = 2
$$

Interpretation: On average, testing 10 items from the batch will reveal about 2 defective items. This expectation guides quality control decisions, suggesting that inspecting a subset gives a reasonable estimate of the defect rate in the entire batch.

Probability of Detecting at Least One Defective Item

One practical question is: What is the probability that the sample contains at least one defective item? This is often more relevant than the exact distribution of defective items.

Using the complement rule:

$$
P(\text{at least one defective}) = 1 - P(\text{no defective items})
$$

We already have $P(X=0)$ from earlier:

$$
P(\text{no defective}) = \frac{\binom{40}{10}}{\binom{50}{10}}
$$

Calculating or approximating this value gives insights into the effectiveness of sampling in detecting defects.

Implications in Quality Control

Understanding these probabilities has significant implications:

Batch Acceptance Criteria

- If the probability of detecting at least one defect is high, sampling provides a reliable method for batch acceptance.
- Conversely, if this probability is low, more extensive testing may be warranted.

Estimating the True Defect Rate

- Sampling outcomes inform estimations about the overall defect rate.
- For example, finding 2 defective items in a sample suggests roughly a 20% defect rate, aligning with the known 20% in the batch.

Cost-Benefit Analysis

- Sampling fewer items reduces inspection costs but may increase the risk of passing defective batches.
- Testing more items improves accuracy but incurs higher costs.

Practical Applications and Strategies

Applying these statistical insights can improve quality control processes:

Designing Effective Sampling Plans

- Determine sample sizes based on desired confidence levels.
- Use hypergeometric calculations to estimate detection probabilities.

Decision-Making Frameworks

- Establish thresholds for acceptable defect detection probabilities.
- Decide on batch acceptance or rejection based on sampling outcomes.

Continuous Improvement

- Use sampling data to identify defect trends.
- Adjust manufacturing processes to reduce defect rates.

Additional Considerations and Limitations

While the hypergeometric distribution provides a solid foundation, real-world scenarios may introduce complexities:

- **Sampling Bias:** Ensuring truly random sampling is crucial; biases can distort probabilities.
- **Batch Homogeneity:** Assumes uniform defect distribution; clustering defects may affect outcomes.
- **Cost and Time Constraints:** May limit the size of samples, affecting the accuracy of defect rate estimates.

Understanding these factors helps refine sampling strategies and interpret results more accurately.

Conclusion

Analyzing the scenario where a batch of 50 items contains 10 defective units, and 10 are randomly tested, involves fundamental statistical concepts rooted in the hypergeometric distribution. By calculating probabilities such as the likelihood of detecting defective items or the expected number of defects in the sample, quality control professionals can make informed decisions about batch acceptance and process improvements.

This approach underscores the importance of proper sampling plans and statistical literacy in maintaining product quality. Whether for manufacturing, inspection, or research, applying these principles ensures that sampling strategies are both efficient and effective, ultimately contributing to better quality assurance and customer satisfaction.

Keywords: quality control, hypergeometric distribution, defect detection, sampling plan, probability, batch testing, statistical analysis, quality assurance

Frequently Asked Questions

What is the probability that all 10 randomly selected items are non-defective?

The probability that all 10 items are non-defective is given by the hypergeometric distribution:
P = C(40,10) / C(50,10), where C(n, k) is the combination of n items taken k at a time.

What is the probability of selecting exactly 2 defective items in the sample?

The probability is given by:
P = [C(10,2) C(40,8)] / C(50,10), where C(10,2) chooses the defective items and C(40,8) chooses the non-defective items.

How many ways can 10 items be selected from the batch of 50?

The total number of ways is C(50,10), which is the combination of 50 items taken 10 at a time.

What is the expected number of defective items in the selected sample?

The expected number of defective items is given by:
E = (number of defective items in batch) (sample size) / total items = 10 10 / 50 = 2.

Is selecting 10 items at random a suitable method for estimating the defect rate in the batch?

Yes, sampling 10 items at random provides an estimate of the defect rate, and the probability calculations help assess the likelihood of different defect scenarios in the sample.

How does the hypergeometric distribution apply to this problem?

The hypergeometric distribution models the probability of drawing a specific number of defective items in a sample without replacement, which is exactly applicable here where items are selected randomly from the batch.

Additional Resources

A Batch Of 50 Items Contains 10 Defective Items. Suppose 10 Items Are Selected At Random And Tested.

Introduction

In the realm of quality control and manufacturing, understanding the likelihood of defective items within a batch is crucial for maintaining standards, reducing costs, and ensuring customer satisfaction. The problem statement—A batch of 50 items contains 10 defective items. Suppose 10 items are selected at random and tested—serves as a fundamental example of probability analysis in quality assurance processes. This scenario offers an excellent foundation for exploring hypergeometric probability distributions, sampling techniques, and the implications of random testing in real-world manufacturing environments.

This article aims to thoroughly examine this problem, analyze the probabilistic outcomes, and discuss its relevance in industrial quality control. We will delve into the mathematical underpinnings, practical applications, and potential extensions of this problem, providing a comprehensive resource for researchers, quality managers, and students alike.

Background and Context

Quality Control in Manufacturing

Manufacturing industries routinely produce large batches of items that must meet certain quality standards. Defective items—those that do not conform to specifications—can lead to customer dissatisfaction, returns, recalls, and reputation damage. To mitigate these risks, sampling methods are employed to estimate the overall quality of a batch without inspecting every item.

The Importance of Random Sampling

Random sampling ensures an unbiased representation of the batch's quality. When selecting a subset of items at random, the probability of finding defective items within that subset informs decisions about batch acceptance, process adjustments, or further testing.

The Scenario: A Closer Look

Batch Composition

- Total items in batch: 50
- Defective items: 10
- Non-defective items: 40

Sampling Process

- Number of items selected: 10
- Selection method: Random and without replacement

This scenario models a classic hypergeometric distribution setting, where the probability of picking a certain number of defective items in the sample depends on the composition of the batch and the sample size.

Mathematical Framework

Hypergeometric Distribution

The hypergeometric distribution describes the probability of drawing a specific number of successes (defective items) in a fixed number of draws without replacement from a finite population containing a known number of successes and failures.

The probability mass function (PMF) is given by:

$$
P(X = k) = \frac{\binom{D}{k} \binom{N - D}{n - k}}{\binom{N}{n}}
$$

Where:
- N: Total population size (here, 50)
- D: Number of defective items (here, 10)
- n: Number of items sampled (here, 10)
- k: Number of defective items found in the sample

Calculating Probabilities for Different Number of Defective Items

Probabilities for 0 to 10 defective items in the sample

Using the hypergeometric formula, we can compute these probabilities for $k = 0, 1, 2, ..., 10$.

Number of Defective Items k	Probability $P(X=k)$
0	$\frac{\binom{10}{0}\binom{40}{10}}{\binom{50}{10}}$
1	$\frac{\binom{10}{1}\binom{40}{9}}{\binom{50}{10}}$
2	$\frac{\binom{10}{2}\binom{40}{8}}{\binom{50}{10}}$
3	$\frac{\binom{10}{3}\binom{40}{7}}{\binom{50}{10}}$
4	$\frac{\binom{10}{4}\binom{40}{6}}{\binom{50}{10}}$
5	$\frac{\binom{10}{5}\binom{40}{5}}{\binom{50}{10}}$
6	$\frac{\binom{10}{6}\binom{40}{4}}{\binom{50}{10}}$
7	$\frac{\binom{10}{7}\binom{40}{3}}{\binom{50}{10}}$
8	$\frac{\binom{10}{8}\binom{40}{2}}{\binom{50}{10}}$
9	$\frac{\binom{10}{9}\binom{40}{1}}{\binom{50}{10}}$
10	$\frac{\binom{10}{10}\binom{40}{0}}{\binom{50}{10}}$

Calculating these probabilities yields insights into the likelihood of encountering various levels of defectiveness in the sample.

Practical Implications

Probability of Finding No Defective Items

A critical metric for quality assurance is the probability of detecting at least one defective item in the sample.

$$
P(\text{at least one defective}) = 1 - P(0 \text{ defective})
$$

where

$$
P(0 \text{ defective}) = \frac{\binom{10}{0}\binom{40}{10}}{\binom{50}{10}}
$$

This probability informs the confidence level that the batch is free of defects if no defective items are found during testing.

Probability of Detecting Multiple Defectives

Similarly, the probability of detecting exactly k defective items can guide decision-making, such as whether to reject a batch or conduct further testing.

Extended Analysis: Expected Number of Defective Items in the Sample

The expected value $E[X]$ of defective items in the sample is given by:

$$
E[X] = n \times \frac{D}{N} = 10 \times \frac{10}{50} = 2
$$

This means, on average, testing 10 items from this batch will reveal about 2 defective items. This expectation helps in planning sampling strategies and setting quality thresholds.

Variability and Confidence Intervals

Understanding the variance of X can inform the reliability of the sampling process:

$$
\text{Var}(X) = n \times \frac{D}{N} \times \left(1 - \frac{D}{N}\right) \times \frac{N - n}{N - 1}
$$

\]

Plugging in the values:

$$
\text{Var}(X) = 10 \times \frac{10}{50} \times \frac{40}{50} \times \frac{40}{49} \approx 1.63
$$

The standard deviation $\sigma \approx 1.28$ indicates the typical fluctuation around the expected 2 defective items in samples of size 10.

Practical Applications in Quality Control

Batch Acceptance Sampling

Using this probabilistic analysis, quality managers can establish acceptance criteria. For example:

- Acceptance number: Allow up to 2 defective items in the sample.
- Rejection criteria: Reject batch if more than 2 defective items are found.

The probabilities calculated inform the consumer's risk (accepting a bad batch) and producer's risk (rejecting a good batch), which are vital in designing robust sampling plans.

Risk Management and Decision-Making

Understanding these probabilities helps balance the risks of:

- Type I error (α): Rejecting a good batch
- Type II error (β): Accepting a bad batch

Adjusting sampling sizes and acceptance thresholds based on the hypergeometric distribution ensures optimal quality control strategies.

Limitations and Assumptions

Despite its utility, this analysis hinges on several assumptions:

- Random and independent sampling without replacement.
- Known and fixed batch composition.
- No bias in the selection process.
- Defective items are equally likely to be selected.

In real-world scenarios, factors such as non-random sampling, correlated defects, or variable defect rates can complicate the analysis.

Extensions and Further Research

The presented model can be extended in several ways:

- Multiple-stage sampling for larger batches.
- Sequential testing where testing continues until certain criteria are met.
- Bayesian approaches incorporating prior information about defect rates.
- Simulation studies to assess the robustness of sampling plans under varying conditions.

Conclusion

The problem—A batch of 50 items contains 10 defective items. Suppose 10 items are selected at random and tested—serves as a fundamental example illustrating the principles of probability theory in quality control. By leveraging the hypergeometric distribution, we gain insights into the likelihood of detecting defects, planning effective sampling strategies, and managing risks in manufacturing processes.

Understanding these probabilistic frameworks empowers quality professionals to make data-driven decisions, optimize inspection procedures, and uphold high standards of product quality. As manufacturing technologies evolve and complexity increases, such rigorous analytical methods remain indispensable tools in the ongoing pursuit of excellence.

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