A Boy Bought Some Pencils For N60.00. If He Had Paid N1 Less For Each Pencil, He Could Have Bought 5

A Boy Bought Some Pencils For N60.00. If He Had Paid N1 Less For Each Pencil, He Could Have Bought 5

Understanding the problem involving the boy's purchase of pencils provides an excellent opportunity to explore basic algebra, word problem strategies, and financial reasoning. This article delves into the details of the problem, breaking down the mathematical concepts involved, and offering insights on how to approach similar problems effectively.

Introduction to the Problem

At first glance, the problem presents a straightforward scenario:

- A boy buys a certain number of pencils for a total of N60.00.
- If he had paid N1 less per pencil, he could have bought 5 more pencils than he actually did.

This combination of facts allows us to derive an equation and solve for the unknowns, specifically the number of pencils purchased and the price per pencil.

Breaking Down the Problem

Let’s identify the key variables:

- Let x be the number of pencils the boy originally bought.
- Let p be the price per pencil in Naira.

From the problem, two main pieces of information are available:

1. The total amount spent is N60.00:

$$
x \times p = 60
$$

2. If he paid N1 less per pencil, he could have bought 5 more pencils:

\[

(x + 5) \text{ pencils} \text{ at } (p - 1) \text{ Naira each}
\]

The total cost in this case must also equal N60.00, since the total money remains the same:

\[
(x + 5) \times (p - 1) = 60
\]

These two equations form the basis for solving the problem.

Formulating the Equations

Based on the above, the equations are:

Equation 1:

\[
x \times p = 60
\]

Equation 2:

\[
(x + 5)(p - 1) = 60
\]

Our goal is to find the values of x and p that satisfy both equations.

Solving the Equations

To solve the problem, we can express p from Equation 1:

\[
p = \frac{60}{x}
\]

Substitute p into Equation 2:

\[
(x + 5) \left(\frac{60}{x} - 1 \right) = 60
\]

Simplify the expression inside the parentheses:

\[
(x + 5) \left(\frac{60 - x}{x} \right) = 60
\]

Multiply both sides by x to clear the denominator:

\[
(x + 5)(60 - x) = 60x
\]

Now, expand the left side:

\[
(x)(60 - x) + 5(60 - x) = 60x
\]

\[
60x - x^2 + 300 - 5x = 60x
\]

Combine like terms:

\[
(60x - 5x) - x^2 + 300 = 60x
\]

\[
55x - x^2 + 300 = 60x
\]

Bring all terms to one side:

\[
- x^2 + 55x + 300 - 60x = 0
\]

Simplify:

\[
- x^2 - 5x + 300 = 0
\]

Multiply through by -1 to make the quadratic more manageable:

\[
x^2 + 5x - 300 = 0
\]

This quadratic equation can be solved using the quadratic formula:

\[

x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

where \(a = 1 \), \(b = 5 \), and \(c = -300 \).

Calculate the discriminant:

\[
\Delta = b^2 - 4ac = 25 - 4(1)(-300) = 25 + 1200 = 1225
\]

Since 1225 is a perfect square (35^2), the solutions are:

\[
x = \frac{-5 \pm 35}{2}
\]

Calculate both possibilities:

1. \[
x = \frac{-5 + 35}{2} = \frac{30}{2} = 15
\]
2. \[
x = \frac{-5 - 35}{2} = \frac{-40}{2} = -20
\]

Since the number of pencils cannot be negative, we discard x = -20.

Thus, x = 15.

Now, find p:

\[
p = \frac{60}{x} = \frac{60}{15} = 4
\]

Therefore, the boy bought 15 pencils, each priced at N4.00.

Verification of the Solution

To ensure the solution’s accuracy, verify the second condition:

- If he paid N1 less per pencil, i.e., N3 per pencil:

Number of pencils he could buy with the same N60:

\[
\frac{60}{p - 1} = \frac{60}{4 - 1} = \frac{60}{3} = 20
\]

Number of pencils he could buy in this case:

$$
20
$$

Compare with the original number:

$$
15 + 5 = 20
$$

Since the numbers match, the solution is consistent.

Key Takeaways and Lessons

This problem exemplifies several important concepts in algebra and problem-solving:

- **Formulating equations from word problems:** Identifying variables and translating the given information into mathematical expressions.
- **Using substitution:** Expressing one variable in terms of another to reduce the number of unknowns.
- **Solving quadratic equations:** Applying the quadratic formula to find real solutions.
- **Verifying solutions:** Checking the solutions against the original problem to ensure correctness.

Applications of Similar Problems

Problems similar to this one are common in everyday life, especially in shopping, budgeting, and financial planning. For example, understanding how discounts or price reductions affect the quantity of items one can purchase is crucial for making informed purchasing decisions.

Some practical applications include:

- Calculating how discounts influence the quantity of goods bought.
- Budgeting expenses when prices change.
- Analyzing the impact of price variations on purchasing power.

Additional Practice Problems

To reinforce understanding, consider attempting similar problems:

1. A customer bought 20 apples for N100. If each apple had been N2 cheaper, he could have bought 15 apples with the same amount. Find the original price per apple.

2. A shopkeeper sold a dozen pens for N120. If he had offered a N5 discount on each pen, he would have made the same total sale. What was the original price per pen?

3. Jane bought 8 books for N160. If she paid N2 less for each book, she could have bought 2 more books with the same money. Find the original price per book.

Conclusion

This problem about the boy and his pencils provides a clear example of how algebra can be used to solve real-world problems involving money, quantities, and prices. The key steps involve setting up equations based on the given conditions, simplifying, and solving quadratic equations. By practicing such problems, learners enhance their critical thinking, problem-solving skills, and understanding of basic algebraic concepts.

Whether you're a student preparing for exams or a curious learner, mastering these types of problems boosts your confidence in handling everyday financial decisions and mathematical challenges. Remember, the ability to translate real-world situations into mathematical language is a vital skill that transcends classroom learning and applies broadly in life.

Frequently Asked Questions

What is the total amount the boy paid for the pencils?

N60.00

How many pencils did the boy buy initially?

The exact number is not given, but it can be calculated based on the information provided.

If the boy had paid N1 less for each pencil, how many pencils could he have bought?

He could have bought 5 pencils.

How can we determine the original number of pencils bought?

By setting up equations based on the total cost and the reduced price per pencil, then solving for the number of pencils.

What is the price per pencil in the original scenario?

It can be calculated once the number of pencils is known, using the total cost of N60.00.

What is the price per pencil if he paid N1 less and could buy 5 pencils?

It is the total amount he would pay divided by 5 pencils, i.e., (original total cost minus N1 times the number of pencils) divided by 5.

How does reducing the price by N1 per pencil affect the number of pencils he can buy?

Reducing the price per pencil allows him to purchase more pencils with the same total amount.

Can we find the original number of pencils bought using the given data?

Yes, by setting up the equations based on total cost and price per pencil, we can solve for the original number.

What is the mathematical approach to solve this problem?

Use algebraic equations: let n be the original number of pencils, then set up equations for total cost and the reduced price scenario, then solve for n.

Additional Resources

A Boy Bought Some Pencils For N60.00. If He Had Paid N1 Less For Each Pencil, He Could Have Bought 5 pencils is an intriguing problem that combines basic algebra with problem-solving skills. It challenges readers to think carefully about the relationships between the number of items purchased, their individual prices, and the total amount spent. This type of problem is common in mathematics, especially in topics related to linear equations and word problems. Understanding how to approach such questions is essential for students and anyone interested in developing logical reasoning and quantitative skills.

Introduction
Mathematics often involves solving real-life or hypothetical problems that require analyzing given data, forming equations, and deriving solutions. The problem statement "A boy bought some pencils for N60.00. If he had paid N1 less for each pencil, he could have bought 5" is a classic example of a word problem that can be approached systematically.

Such problems are not only useful for academic purposes but also for developing critical thinking. By breaking down the problem into manageable parts, we can find the number of pencils bought, the price per pencil, and how the total cost relates to these variables.

Understanding the Problem

Before jumping into calculations, it's important to interpret the problem carefully:

- The boy bought some pencils, say n pencils.
- The total amount paid was N60.00.
- If he had paid N1 less for each pencil, he would have been able to buy 5 pencils instead.

From this, two main pieces of information are clear:

1. The total cost for n pencils at the original price.
2. The alternative scenario where each pencil costs N1 less, and the number of pencils bought would be 5.

Breaking Down the Data

Let's define variables:

- Let n = number of pencils originally bought.
- Let p = price per pencil in the original scenario.

From the problem:

- Total cost = N60.00.
- Therefore, the relationship between n and p:

$n \times p = 60$

- In the alternative scenario:

- The new price per pencil = p - 1.
- The number of pencils that could be bought at this new price = 5.

Hence:

$5 \times (p - 1)$ = total amount spent in the alternative scenario.

But since the total amount spent remains N60.00, the total cost in this scenario is also N60.

Setting Up the Equations

Based on the above, we can establish the following equations:

1. Original purchase:

$$
n \times p = 60
$$

2. Alternative scenario:

$$
5 \times (p - 1) = 60
$$

From the second, we can solve for p:

$$
5(p - 1) = 60 \Rightarrow p - 1 = \frac{60}{5} \Rightarrow p - 1 = 12
$$

$$
p = 13
$$

Now, knowing p = 13, substitute into the first equation:

$$
n \times 13 = 60 \Rightarrow n = \frac{60}{13} \approx 4.615\ldots
$$

This result is not an integer, which suggests a need to re-express the problem carefully.

Re-evaluating the Approach

The inconsistency indicates that the total amount spent in the alternative scenario (buying 5 pencils at N1 less per pencil) isn't necessarily the same as the original total N60. The problem states:

> If he had paid N1 less for each pencil, he could have bought 5.

This implies that the total amount he would have spent in the alternative scenario is:

$$
\text{Number of pencils} \times \text{new price per pencil} = \text{Total amount he would have paid}
$$

But it does not state that this total equals N60. Instead, it implies that the total amount he would have paid in the alternative scenario is:

$$
\text{Total amount in alternative scenario} = (n) \times (p - 1)
$$

And this total amount would be enough to buy 5 pencils at that new price:

$$
5 \times (p - 1) = \text{Total amount paid in alternative scenario}
$$

But since the total amount paid originally is N60, and the only change is the price per pencil, the total paid remains N60 in the alternative scenario as well, which suggests:

$$
n \times (p - 1) = 60
$$

Now, we have two equations:

1. $n \times p = 60$
2. $n \times (p - 1) = 60$

Subtracting the second from the first:

$$
n \times p - n \times (p - 1) = 0

\]

\[
n \times p - n \times p + n = 0
\]

\[
n = 0
\]

Which is impossible, indicating this approach is flawed.

Correct Interpretation and Solution

Given the problem statement, the most logical interpretation is:

- The boy bought n pencils at a price p per pencil for a total of N60:

\[
n \times p = 60
\]

- If he had paid N1 less per pencil, then he could have bought 5 pencils with that money:

\[
5 \times (p - 1) = \text{Total money spent in the alternative scenario}
\]

- Since the total money spent remains N60, the total cost in the alternative scenario is N60:

\[
5 \times (p - 1) = 60
\]

From this, solve for p:

\[
p - 1 = \frac{60}{5} = 12
\]

\[
p = 13
\]

Now, find n:

\[

n \times p = 60 \Rightarrow n \times 13 = 60 \Rightarrow n = \frac{60}{13} \approx 4.615\ldots
\]

Again, a non-integer, which suggests the problem is designed with the assumption that n must be an integer.

Key Point: The problem is typical of algebraic word problems where the number of pencils and prices are integers. Therefore, to find integer solutions, we need to consider that the initial data might have been:

- Total cost: N60
- The number of pencils n is an integer (say, n), and the price per pencil is p.

And the condition: "If he had paid N1 less for each pencil, he could have bought 5" implies:

\[
(n) \times p = 60
\]
and
\[
5 \times (p - 1) = \text{some amount} \leq 60
\]

But since the problem states "he could have bought 5" pencils at p - 1, and the total cost would be:

\[
5 \times (p - 1)
\]

which must be less than or equal to N60, but given the structure of the problem, it is likely that N60 is the total amount he spent initially, and the alternative scenario involves paying N1 less per pencil to buy 5 pencils.

Therefore, the total amount he would spend in the alternative scenario is:

\[
5 \times (p - 1)
\]

This must be equal to or less than N60, but more precisely, since the problem states "If he had paid N1 less for each pencil, he could have bought 5", it suggests that:

\[
5 \times (p - 1) \leq 60
\]

```
and

\[
n \times p = 60
\]

Now, because paying N1 less per pencil allows him to buy 5 pencils, it
implies:

\[
5 \times (p - 1) = \text{total amount he spent in the alternative scenario}
\]

and this total must be less than or equal to N60, but since the original
total was N60, and paying less per pencil allows him to buy more pencils, the
total cost in the alternative scenario should be less than or equal to 60.

Putting it all together, the key is to find n and p such that:

- \( n \times p = 60 \)
- \( 5 \times (p - 1) \leq 60 \)
- \( n \geq 5 \) (since he bought some pencils initially, and the scenario
with 5 pencils at the reduced price is feasible)

Now, let's look for integer solutions:

Suppose \( p \) divides 60:

Possible values of \( p \):

- 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Calculate \( n = 60 / p \) for each:

| p | n = 60/p | 5(p
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