A Charge Of 30 C Is Distributed Uniformly Throughout A Spherical Volume Of Radius 10.0 Cm. Determine

A Charge Of 30 C Is Distributed Uniformly Throughout A Spherical Volume Of Radius 10.0 Cm. Determine the electric field at various points within and outside the sphere, as well as the potential at different positions. This comprehensive guide provides a detailed analysis of the problem, exploring concepts of electrostatics, Gauss's law, and potential calculations for a uniformly charged sphere.

Understanding the Problem

When a charge is uniformly distributed within a sphere, the electrostatic behavior can be analyzed using the principles of Gauss's law and classical electrostatics. The key parameters in this problem are:

- Total charge, $Q = 30\, C$
- Radius of the sphere, $R = 10.0\, cm = 0.10\, m$

The goal is to find:

- The electric field at a point inside the sphere ($r < R$)
- The electric field at a point outside the sphere ($r > R$)
- The electric potential at various points

Fundamental Concepts

Electrostatics and Coulomb's Law

- Coulomb's law describes the force between two point charges.
- Electric field (E) at a point is defined as the force per unit charge experienced by a test charge placed at that point.

Gauss's Law

- Gauss's law states that the net electric flux through a closed surface equals the enclosed charge divided by the vacuum permittivity (ε_0):

$$
\Phi_E = \frac{Q_{enc}}{\varepsilon_0}
$$

- For symmetric charge distributions, Gauss's law simplifies the calculation of electric fields.

Uniform Charge Distribution

- The volume charge density (ρ) is constant within the sphere:

$$
\rho = \frac{Q}{V}
$$

- For a sphere of radius R:

$$
V = \frac{4}{3}\pi R^3
$$

Calculating the Charge Density

Given the total charge and the volume of the sphere:

1. Calculate the volume of the sphere:

   $$
   V = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi (0.10\, \text{m})^3
   $$
   $$
   V = \frac{4}{3} \pi \times 0.001\, \text{m}^3 \approx 0.00419\, \text{m}^3
   $$

2. Compute the charge density:

   $$
   \rho = \frac{Q}{V} = \frac{30\, C}{0.00419\, \text{m}^3} \approx 7158\, C/m^3
   $$

Electric Field Inside the Sphere ($r < R$)

Applying Gauss's Law

For points inside the sphere, consider a spherical Gaussian surface of radius $r < R$:

- The enclosed charge:

$$
Q_{enc} = \rho \times \frac{4}{3} \pi r^3
$$

- The electric field at radius r is directed radially outward.

- Gauss's law gives:

$$
E \times 4 \pi r^2 = \frac{Q_{enc}}{\varepsilon_0}
$$

- Substituting Q_{enc}:

$$
E \times 4 \pi r^2 = \frac{\rho \times \frac{4}{3} \pi r^3}{\varepsilon_0}
$$

- Simplifying:

$$
E = \frac{\rho r}{3 \varepsilon_0}
$$

Final Expression for Inside Electric Field

$$
E(r) = \frac{\rho r}{3 \varepsilon_0}
$$

where:

- r is the distance from the center
- $\varepsilon_0 = 8.854 \times 10^{-12}\, F/m$

Numerical Calculation at a Point Inside

For example, at $r = 5\, \text{cm} = 0.05\, \text{m}$:

$$
E = \frac{7158\, C/m^3 \times 0.05\, m}{3 \times 8.854 \times 10^{-12}\, F/m}
$$

$$
E \approx \frac{357.9}{2.6562 \times 10^{-11}} \approx 1.347 \times 10^{13}\, V/m
$$

This extremely high electric field illustrates the intensity close to the sphere's center.

Electric Field Outside the Sphere ($r > R$)

Applying Gauss's Law for External Points

- For points outside the sphere, the entire charge Q can be considered enclosed by the Gaussian surface at radius r.

- The electric field:

\[

$$
E(r) = \frac{Q}{4 \pi \varepsilon_0 r^2}
$$

Numerical Calculation at a Point Outside

At $r = 20\, \text{cm} = 0.20\, m$:

$$
E = \frac{30\, C}{4 \pi \times 8.854 \times 10^{-12}\, F/m \times (0.20\, m)^2}
$$

$$
E \approx \frac{30}{4 \pi \times 8.854 \times 10^{-12} \times 0.04}
$$

$$
E \approx \frac{30}{4 \pi \times 3.5416 \times 10^{-13}} \approx \frac{30}{4.448 \times 10^{-12}} \approx 6.74 \times 10^{12}\, V/m
$$

Electric Potential Distribution

Potential Inside the Sphere ($r < R$)

- The potential at a point inside the sphere, relative to infinity, is obtained by integrating the electric field from infinity to the point:

$$
V(r) = - \int_{r}^{\infty} E \, dr
$$

- For the inside region, the potential can be expressed as:

$$
V(r) = \frac{Q}{4 \pi \varepsilon_0 R} + \frac{\rho}{6 \varepsilon_0}(3 R^2 - r^2)
$$

- Alternatively, the potential at the center ($r=0$):

$$
V(0) = \frac{3 Q}{8 \pi \varepsilon_0 R}
$$

Potential Outside the Sphere ($r > R$)

$$
V(r) = \frac{Q}{4 \pi \varepsilon_0 r}
$$

Summary of Results

- The charge density within the sphere is approximately $7158\, C/m^3$.

- Electric field inside at radius r:

  $$
  E(r) = \frac{\rho r}{3 \varepsilon_0}
  $$

  which increases linearly from zero at the center to its maximum at the surface.

- Electric field outside:

  $$
  E(r) = \frac{Q}{4 \pi \varepsilon_0 r^2}
  $$

  which diminishes with the square of the distance.

- The potential at the center:

  $$
  V(0) = \frac{3 Q}{8 \pi \varepsilon_0 R}
  $$

and at a point outside the sphere:

$$
V(r) = \frac{Q}{4 \pi \varepsilon_0 r}
$$

Practical Implications and Applications

Understanding the electric fields and potential distributions in a uniformly charged sphere has numerous applications:

- Electrostatic Shielding: Designing shielding materials that can block or redirect electric fields.
- Material Science: Analyzing charge distributions within spherical particles or droplets.
- Medical Applications: In medical imaging, particularly in techniques involving electric field distributions.
- Engineering: Designing capacitors and other electronic components with spherical geometries.

Conclusion

This detailed exploration of a uniformly charged sphere illustrates the fundamental principles of electrostatics. By applying Gauss's law and classical equations, we can accurately determine the electric field and potential at any point within or outside the sphere. These calculations are essential in various scientific and engineering contexts, providing insights into charge distributions and electric field behavior in spherical systems.

References

- David J. Griffiths, "Introduction to Electrodynamics," 4th Edition, Pearson Education, 2013.
- Serway and Jewett, "Physics for Scientists and Engineers," 9th Edition, Brooks Cole, 2014.
- HyperPhysics, Electric Fields and Gauss's Law, Georgia State University.

Frequently Asked Questions

What is the electric field at a point outside a uniformly charged sphere with a total charge of 30 C and radius 10 cm?

The electric field at a point outside the sphere (at a distance $r > 10$ cm) is given by $E = (1 / (4\pi\varepsilon_0)) (Q / r^2)$. Substituting $Q = 30$ C, at $r > 0.10$ m, $E = (9 \times 10^9 \ Nm^2/C^2) (30 \ C) / r^2$.

How do you calculate the electric field inside a uniformly charged sphere at a distance r from the center?

Inside a uniformly charged sphere, the electric field at distance r ($r < 10$ cm) is $E = (1 / (4\pi\varepsilon_0)) (Q r) / R^3$, where $Q = 30$ C and $R = 10$ cm. Simplified, $E = (9 \times 10^9) (30 \ C) r / (0.10 \ m)^3$.

What is the total charge distribution in a sphere with a uniform charge density?

The total charge Q is uniformly distributed throughout the sphere, so the charge density $\rho = Q / \text{Volume} = 30 \ C / (4/3)\pi R^3$. For $R = 0.10$ m, $\rho \approx 30 / (4/3)\pi (0.10)^3 \ C/m^3$.

How do you determine the potential at the center of the charged sphere?

The electric potential at the center of a uniformly charged sphere is $V = (3 / (2 \ 4\pi\varepsilon_0)) (Q / R)$. Substituting $Q = 30$ C and $R = 0.10$ m, $V \approx (3 / (2 \ 9 \times 10^9)) (30 / 0.10)$.

What are the key steps to solve for the electric field and potential of a uniformly charged sphere?

First, determine the charge density ρ. Then, apply Gauss's law to find the electric field both inside and outside the sphere: inside ($r < R$), $E = (1 / (4\pi\varepsilon_0)) (Q r) / R^3$; outside ($r > R$), $E = (1 / (4\pi\varepsilon_0)) (Q / r^2)$. For potential, integrate the electric field or use known formulas, considering boundary conditions at infinity and the sphere's surface.

Additional Resources

A Charge Of 30 C Is Distributed Uniformly Throughout A Spherical Volume Of Radius 10.0 Cm. Determine

---

Introduction

In the realm of electrostatics, understanding the distribution of electric charge within different geometries is essential for both theoretical insights and practical applications. When a significant charge such as 30 coulombs (C) is uniformly distributed within a spherical volume of radius 10.0 centimeters (cm), the resulting electric field, potential, and related electrostatic properties become intriguing subjects of analysis.

This article explores the detailed process of determining the electric field and potential at various points within and outside such a uniformly charged sphere. It delves into the relevant physical principles, mathematical formulations, and implications, providing a comprehensive understanding suitable for researchers, students, and professionals working in physics, electrical engineering, and related disciplines.

Physical Context and Significance

The distribution of charge within a sphere has both theoretical importance and practical relevance. For instance, understanding the electric field generated by uniformly charged spheres is fundamental for:

- Designing capacitors and insulating materials.
- Analyzing charge distributions in atomic and molecular systems.
- Modeling charge storage and transfer in various electronic devices.
- Conducting educational demonstrations of electrostatic principles.

The specific scenario of a 30 C charge distributed uniformly within a sphere of radius 10.0 cm is particularly noteworthy due to the magnitude of the charge, which emphasizes the importance of accurate calculations and understanding of electrostatic phenomena.

Fundamental Principles and Assumptions

To analyze this system, we rely on classical electrostatics principles:

- Coulomb's Law: The force between point charges and the resulting electric field.
- Gauss's Law: The fundamental law relating electric flux to enclosed charge, especially useful in symmetric charge distributions.
- Superposition Principle: The total electric field is the vector sum of fields due to individual charge elements.

Assumptions include:

- The charge distribution is static (no movement over time).

- The charge is uniformly distributed throughout the volume, not just on the surface.
- The medium is vacuum or air with permittivity ε_0 (permittivity of free space).

Mathematical Formulation

1. Total Charge and Volume

Given:

- Total charge, $Q = 30\, C$
- Radius of sphere, $R = 10.0\, \text{cm} = 0.10\, \text{m}$

The volume of the sphere:

$$
V = \frac{4}{3} \pi R^3
$$

The volume charge density, ρ:

$$
\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3} \pi R^3}
$$

Calculating ρ:

$$
\rho = \frac{30}{\frac{4}{3} \pi (0.10)^3} = \frac{30}{\frac{4}{3} \pi \times 0.001} = \frac{30}{\frac{4}{3} \times 3.1416 \times 0.001}
$$

$$
\rho = \frac{30}{0.0041888} \approx 7162.7\, C/m^3
$$

Electric Field Inside and Outside the Sphere

The electric field depends on the position relative to the sphere's center, leading to two regions:

- Inside the sphere ($r < R$)
- Outside the sphere ($r \geq R$)

2. Using Gauss's Law

Gauss's Law in differential form:

$$
\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}
$$

For a spherically symmetric charge distribution, the electric field is radial, and the flux simplifies to:

$$
E(r) \times 4\pi r^2
$$

where Q_{enc} is the charge enclosed within a sphere of radius r.

Electric Field Calculation

1. Inside the Sphere ($r < R$)

The enclosed charge:

$$
Q_{enc} = \rho \times \frac{4}{3} \pi r^3
$$

Applying Gauss's Law:

$$
E(r) \times 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0} = \frac{\rho \times \frac{4}{3} \pi r^3}{\varepsilon_0}
$$

Simplify:

$$
E(r) = \frac{\rho r}{3 \varepsilon_0}
$$

Substituting ρ:

$$
E(r) = \frac{7162.7 \times r}{3 \times 8.854 \times 10^{-12}} \quad \text{(V/m)}
$$

Expressed explicitly:

$$
E(r) = \frac{7162.7 \times r}{2.6562 \times 10^{-11}}
$$

Calculating the coefficient:

$$
E(r) \approx 2.698 \times 10^{14} \times r
$$

with r in meters.

Electric Field Outside the Sphere ($r \ge R$)

The entire charge Q is enclosed:

$$
Q_{enc} = Q = 30\, C
$$

Applying Gauss's Law:

$$
E(r) = \frac{Q}{4\pi \varepsilon_0 r^2}
$$

Substitute values:

$$
E(r) = \frac{30}{4\pi \times 8.854 \times 10^{-12} \times r^2}
$$

Calculate denominator:

$$
4\pi \times 8.854 \times 10^{-12} \approx 1.1127 \times 10^{-10}
$$

Thus,

$$
E(r) \approx \frac{30}{1.1127 \times 10^{-10} \times r^2} \approx 2.698 \times 10^{11} \times \frac{1}{r^2}
$$

where r is in meters, $E(r)$ in V/m.

Electric Potential Analysis

The electric potential V at a point is obtained by integrating the electric field:

$$
V(r) = - \int E \, dr
$$

or, more straightforwardly, by considering the potential at infinity as zero and integrating inward.

3. Potential Inside the Sphere

From the electric field inside:

$$
V(r) = V(0) - \int_0^r E(r') dr'
$$

which simplifies to:

$$
V(r) = V(0) - \int_0^r \frac{\rho r'}{3 \varepsilon_0} dr' = V(0) - \frac{\rho}{3 \varepsilon_0} \times \frac{r'^2}{2} \bigg|_0^r = V(0) - \frac{\rho r^2}{6 \varepsilon_0}
$$

To find $V(0)$, use the boundary condition that at the surface $r= R$, potential is continuous:

$$
V(R) = V_{outside}(R)
$$

\]

Potential Outside the Sphere

The potential at a distance \(r \ge R \):

\[
V(r) = \frac{Q}{4 \pi \varepsilon_0 r}
\]

At \(r = R \):

\[
V(R) = \frac{Q}{4 \pi \varepsilon_0 R}
\]

Calculate:

\[
V(R) = \frac{30}{4 \pi \times 8.854 \times 10^{-12} \times 0.10} \approx 2.698 \times 10^{12} \, \text{V}
\]

Determining the Potential at the Sphere Center

Using the relation for potential inside:

\[
V(0) = V(R) + \frac{\rho R^2}{6 \varepsilon_0}
\]

Substitute known values:

\[
V(0) = 2.698 \times 10^{12} + \frac{7162.7 \times (0.10)^2}{6 \times 8.854 \times 10^{-12}}
\]

Calculate numerator:

\[

7162.7 \times 0.01 = 71.627
\]

Calculate denominator:

\[
6 \times 8.854 \times 10^{-12} = 5.3124 \times 10^{-11}
\]

Thus,

\[
V(0) = 2.698 \times 10^{12} + \frac{71.627}{5.3124 \times 10^{-11}} \approx 2.698 \times 10^{12} + 1.349 \times 10^{12} = 4.047 \times 10^{12} \, \text{V}
\]

Summary of Key Results

Quantity	Expression	Approximate Value
Electric field at radius \(r \) inside the sphere	\(E(r) = \frac{\rho r

A Charge Of 30 C Is Distributed Uniformly Throughout A Spherical Volume Of Radius 10 0 Cm Determine

A Charge Of 30 C Is Distributed Uniformly Throughout A Spherical Volume Of Radius 10 0 Cm Determine

Related Articles

- 2.5 Points Save Which Term Describes A Company Being So Open To Other Companies Working With It That
- A Car Engine Delivers 2.8 * 104 W To Its Wheels When Moving At A Constant Speed Of 11.6 M/s. What Is
- A Cement Block Accidentally Falls From Rest From The Ledge Of A 82.1-m-high Building. When The Block

Back to Home