A Circle Has A Centre At (1,2). One Point On Its Circumference Is (-3,-1). What Is The Radius Of The

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Understanding the properties and calculations related to circles are fundamental in geometry. When given a circle's center and a point on its circumference, determining the radius becomes a straightforward task involving the distance formula. This article provides a comprehensive guide to solving such problems, with detailed explanations, step-by-step calculations, and tips to enhance your understanding of circle geometry.

Understanding the Components of a Circle

Before delving into calculations, it’s essential to grasp the fundamental components of a circle:

- Centre (h, k): The fixed point equidistant from all points on the circle.
- Radius (r): The distance from the centre to any point on the circle.
- Circumference: The boundary of the circle, passing through all points at distance r from the centre.
- Point on the circumference: Any point lying exactly on the circle's boundary.

In the problem at hand, the circle's centre is given as (1, 2), and a point on its circumference is (-3, -1). The goal is to determine the circle's radius, which is the distance from the centre to this point.

Mathematical Approach to Find the Radius

The key to solving this problem lies in the distance formula, which calculates the distance between two points in a coordinate plane.

The Distance Formula

Given two points, (x_1, y_1) and (x_2, y_2), the distance d between them is:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

This formula is derived from the Pythagorean theorem, applied to the right-angled triangle formed between the points.

Applying the Distance Formula to Our Problem

Given:

- Center $(x_1, y_1) = (1, 2)$
- Point on circumference $(x_2, y_2) = (-3, -1)$

The radius r is the distance between these two points:

$$
r = \sqrt{(-3 - 1)^2 + (-1 - 2)^2}
$$

Step-by-Step Calculation of the Radius

Let's perform the calculation step-by-step:

Step 1: Calculate the differences in x and y

$$
\Delta x = -3 - 1 = -4
$$
$$
\Delta y = -1 - 2 = -3
$$

Step 2: Square the differences

$$
(\Delta x)^2 = (-4)^2 = 16
$$
$$
(\Delta y)^2 = (-3)^2 = 9
$$

Step 3: Sum the squared differences

$$
16 + 9 = 25
$$

Step 4: Take the square root to find the radius

$$
r = \sqrt{25} = 5
$$

Therefore, the radius of the circle is 5 units.

Summary of the Calculation Process

Step	Action	Result
1	Find the differences in x and y coordinates	$\Delta x = -4$, $\Delta y = -3$
2	Square the differences	16 and 9
3	Sum the squares	25
4	Take the square root	5

Hence, the circle's radius is 5 units.

Additional Tips and Considerations

- Coordinate Subtraction Significance: Remember that subtraction order affects the sign but not the magnitude when squaring, so the absolute difference is what matters.
- Square Root Calculation: Always verify your square root result, especially in more complex problems.
- Unit Consistency: Ensure all coordinates are in the same units to maintain accuracy.
- Visual Representation: Drawing the circle and points on graph paper can help visualize the problem and understand the distances better.

Related Concepts and Applications

Understanding how to find the radius of a circle from given points has several applications:

- Geometry Problems: Calculating distances between points and understanding circle properties.
- Coordinate Geometry: Fundamental in graphing circles, ellipses, and other conic sections.
- Real-world Applications: GPS technology, navigation, and engineering often involve calculating distances between points.
- Design and Engineering: Creating precise circular components based on given dimensions.

Practice Problems to Master the Concept

To strengthen your understanding, try solving these similar problems:

1. Find the radius of a circle with center at (3, -4) and a point on the circumference at (7, 2).
2. Determine the radius of a circle with center at (-2, 5) passing through the point (1, 1).
3. A circle has a radius of 10 units, and its center is at (4, -3). Find the coordinates of a point on the circumference that is directly above the center.

Conclusion

Calculating the radius of a circle when given the center and a point on its circumference is a fundamental skill in coordinate geometry. By applying the distance formula carefully, you can determine the radius accurately and efficiently. Remember to follow each step diligently, visualize the points when possible, and practice with various problems to become confident in your geometric problem-solving skills.

In our specific problem, the radius of the circle with center at (1, 2) and a point (-3, -1) on its circumference is 5 units. This straightforward calculation exemplifies the power of the distance formula in solving real-world and academic geometry problems.

Keywords: circle geometry, radius calculation, distance formula, coordinate geometry, center and point on circle, circle radius formula, geometry problems, math practice, coordinate plane, circle properties

Frequently Asked Questions

How do you find the radius of a circle given its center and a point on its circumference?

The radius is the distance between the center and the point on the circumference, calculated using the distance formula: $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$.

What are the coordinates of the circle's center in this problem?

The center of the circle is at the point (1, 2).

Which point lies on the circumference of the circle?

The point (-3, -1) is on the circumference of the circle.

How do you compute the radius from the given points?

Calculate the distance between the center (1, 2) and the point on the circumference (-3, -1) using the distance formula.

What is the formula for the distance between two points (x_1, y_1) and (x_2, y_2)?

Distance = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$.

What is the calculated radius of the circle in this problem?

The radius is $\sqrt{[(-3 - 1)^2 + (-1 - 2)^2]} = \sqrt{[(-4)^2 + (-3)^2]} = \sqrt{[16 + 9]} = \sqrt{25} = 5$.

Why is the distance formula used here?

Because the radius is the straight-line distance between the circle's center and a point on its circumference.

Can you express the radius as a simplified radical?

Yes, the radius is $\sqrt{25}$, which simplifies to 5.

What is the significance of knowing the radius in a circle problem?

Knowing the radius helps in understanding the size of the circle and can be used to find its area, circumference, or other properties.

Additional Resources

A Circle Has A Centre At (1,2). One Point On Its Circumference Is (-3,-1). What Is The Radius Of The?

Understanding the fundamental properties of circles is a cornerstone of geometry, a branch of mathematics that explores shapes, sizes, and the spatial relationships between objects. The question posed—determining the radius of a circle given its center and a point on its circumference—serves as a classic example of applying coordinate geometry principles. This article aims to dissect the problem comprehensively, exploring the underlying concepts, mathematical formulations, and practical applications associated with such geometric calculations.

Introduction to Circles in Coordinate Geometry

What Is a Circle?

A circle is a set of all points in a plane that are equidistant from a fixed point called the center. The fixed distance from the center to any point on the circle is known as the radius. The simplicity and symmetry of circles make them a fundamental object of study in geometry, with applications spanning engineering, physics, computer graphics, and more.

Representation of a Circle on the Coordinate Plane

In coordinate geometry, a circle can be represented algebraically using the standard form:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

where:
- (h, k) are the coordinates of the center,
- r is the radius.

This form clearly indicates that knowing the center and a point on the circle allows us to compute the radius directly.

Understanding the Given Data

Coordinates of the Center

The problem states that the circle has a center at $(1, 2)$. This means that:
- The x-coordinate of the center is 1.

- The y-coordinate of the center is 2.
This point acts as the fixed reference point from which all points on the circle are equidistant.

Point on the Circumference

One point on the circle's circumference is $(-3, -1)$. This point, by definition, satisfies the circle's equation, and its distance from the center is exactly the radius.

Implication of the Data

The data provided allows us to:
- Confirm the circle's radius by calculating the distance between the center and the given point.
- Use the distance formula from coordinate geometry, which is a pivotal tool in such problems.

Mathematical Foundations: The Distance Formula

The Distance Formula

Given two points (x_1, y_1) and (x_2, y_2), the distance d between them is:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

This formula stems from the Pythagorean theorem, which relates the lengths of the sides of a right triangle.

Application to the Circle Problem

In this scenario:
- $(x_1, y_1) = (1, 2)$ (the center)
- $(x_2, y_2) = (-3, -1)$ (a point on the circumference)

The distance between these two points will be the radius r:

$$
r = \sqrt{(-3 - 1)^2 + (-1 - 2)^2}
$$

Calculating the Radius

Step-by-Step Calculation

Let's compute the radius systematically:

1. Calculate the differences in x and y:
- $\Delta x = -3 - 1 = -4$
- $\Delta y = -1 - 2 = -3$

2. Square these differences:
- $(\Delta x)^2 = (-4)^2 = 16$
- $(\Delta y)^2 = (-3)^2 = 9$

3. Sum the squared differences:
- $16 + 9 = 25$

4. Take the square root to find the radius:
- $r = \sqrt{25} = 5$

Result: The radius of the circle is 5 units.

The Significance of the Radius in Geometric Context

Understanding the Role of the Radius

The radius is a critical measure, defining the size of the circle. It influences various properties such as:
- The circumference: $C = 2\pi r$
- The area: $A = \pi r^2$
- The extent of the circle in the plane

Knowing the radius allows for precise calculations of these and other related measures, essential in engineering designs, navigation, and computer graphics.

Applications of Radius Calculations

Accurately determining the radius from given points is fundamental in:
- Designing circular components in machinery
- Mapping regions in geographical information systems
- Creating graphical representations in computer-aided design (CAD)
- Analyzing orbits in astrophysics

This fundamental calculation illustrates how abstract geometric principles have tangible real-world applications.

Further Analytical Perspectives

Verifying the Result

One way to validate the computed radius is to substitute the radius back into the circle's equation:

$$
(x - 1)^2 + (y - 2)^2 = r^2
$$

Using the point $(-3, -1)$:

$$
(-3 - 1)^2 + (-1 - 2)^2 = 16 + 9 = 25
$$

which equals r^2 when $r = 5$. This confirms the correctness of the calculation.

Alternative Methods and Their Validity

Although the distance formula is straightforward, alternative methods like geometric constructions or coordinate transformations can also be employed, especially in more complex scenarios involving rotated or translated circles.

Concluding Remarks and Broader Implications

The problem of determining a circle's radius from its center and a point on its circumference exemplifies core principles of coordinate geometry. It highlights how algebraic formulas like the distance formula serve as powerful tools for solving geometric problems with precision and clarity.

Broader Significance:
- Such calculations underpin advanced mathematical modeling in multiple fields.
- They foster a deeper understanding of the relationship between algebra and geometry.
- They enable practical problem-solving in technology, science, and engineering.

Educational Value:
Mastering these fundamental concepts equips students and practitioners to approach more complex geometric challenges, fostering analytical thinking and problem-solving skills.

Final Thought:
The simplicity of the calculation belies its importance; understanding how to derive the radius from basic data reinforces the interconnectedness of geometric principles and their applications in the real world. The radius of this particular circle, calculated as 5 units, is not just a numerical answer but a gateway to exploring broader mathematical and practical concepts.

In summary, given the center at $(1, 2)$ and a point $(-3, -1)$ on the circumference, the radius of the circle is 5 units. This result is obtained through straightforward application of the distance formula, emphasizing the elegance and utility of coordinate geometry in solving real-world geometric problems.

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