A Circle With Radius Of 2 Cm Sits Inside A Circle With Radius Of 4 Cm.What Is The Area Of The Shaded

A Circle With Radius Of 2 Cm Sits Inside A Circle With Radius Of 4 Cm. What Is The Area Of The Shaded

Understanding geometrical figures and their properties is fundamental in solving many mathematical problems, especially those involving circles. In this comprehensive guide, we will explore the problem of calculating the area of the shaded region when a smaller circle sits inside a larger circle. Specifically, we will analyze the scenario where a circle with a radius of 2 cm is inscribed within a larger circle with a radius of 4 cm. Our goal is to determine the area of the shaded region, which is the part of the larger circle not occupied by the smaller circle.

This article aims to provide a thorough explanation of the concepts, formulas, and step-by-step calculations required to solve this problem. Whether you're a student preparing for exams, a math enthusiast, or someone interested in geometric problem-solving, you'll find detailed insights here.

Understanding the Geometric Setup

Before diving into calculations, it's essential to visualize and understand the geometric configuration.

The Larger Circle

- Radius: 4 cm
- Area: To be calculated later
- Center: Let's denote it as point O

The Smaller Circle

- Radius: 2 cm
- Center: Let's denote it as point C
- Position: Inside the larger circle, touching or not touching the boundary depending on the scenario

The Shaded Region

- Definition: The area within the larger circle that is not occupied by the smaller circle
- Visual: Imagine a large circle with a smaller circle inscribed or placed somewhere inside it, leaving a ring-shaped shaded area

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Key Concepts and Formulas

To solve the problem efficiently, we need to understand the fundamental formulas related to circles and areas.

Area of a Circle

The area A of a circle with radius r is given by:

$$
A = \pi r^2
$$

- $\pi \approx 3.1416$

Applying the formula:

- For the larger circle: $A_{large} = \pi \times 4^2 = 16\pi$ square centimeters
- For the smaller circle: $A_{small} = \pi \times 2^2 = 4\pi$ square centimeters

Calculating the Shaded Area

- The shaded area is the difference between the larger and smaller circles' areas:

$$
A_{shaded} = A_{large} - A_{small} = 16\pi - 4\pi = 12\pi
$$

- Numerically: $12 \times 3.1416 \approx 37.6992$ square centimeters

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Considering the Position of the Smaller Circle

The above calculation assumes the smaller circle is entirely within the larger circle and does not specify whether it is inscribed touching the boundary or placed somewhere inside. Let's analyze different scenarios.

Scenario 1: Smaller Circle Inscribed Touching the Boundary

- The smaller circle is placed at the center of the larger circle.
- Since both are concentric, the shaded region is simply the larger circle minus the smaller circle.

Scenario 2: Smaller Circle Not Centered

- The smaller circle is placed somewhere inside the larger circle, not necessarily centered.
- The key question: Does the smaller circle touch the boundary of the larger circle?
- If it does not, the shaded area is just the difference in areas.
- If it touches, the position affects whether parts of the smaller circle extend outside the larger circle, which would change the calculation.

Important note: The problem states that a circle with radius 2 cm sits inside a circle of radius 4 cm, but it does not specify if it is inscribed or offset. For simplicity, the most common interpretation is that the smaller circle is inscribed, touching the boundary of the larger circle internally.

Calculating the Shaded Area in the Most Common Scenario

Assuming the smaller circle is inscribed inside the larger circle and centered at the same point (concentric), the calculation simplifies.

Step-by-Step Calculation

1. Calculate the area of the larger circle:

    - Area: $A_{large} = 16\pi$ cm^2

2. Calculate the area of the smaller circle:

    - Area: $A_{small} = 4\pi$ cm^2

3. Subtract the smaller circle's area from the larger:

    - Shaded area: $A_{shaded} = 12\pi$ cm^2

4. Express the answer numerically:

    - $12\pi \approx 12 \times 3.1416 = 37.6992$ cm^2

Therefore, the area of the shaded region is approximately 37.70 square centimeters.

Visualizing the Geometric Figures

Visual aids are instrumental in understanding the problem:

- Diagram 1: Two concentric circles with radii 4 cm and 2 cm, sharing the same center, illustrating the shaded ring.
- Diagram 2: The smaller circle offset within the larger circle, demonstrating different possible positions.

Using diagrams helps clarify assumptions, especially regarding the placement of the smaller circle.

Advanced Considerations: Offset Position of the Smaller Circle

What if the smaller circle is not centered? How does this affect the shaded area?

Scenario: Smaller Circle is Offset but Fully Inside

- The smaller circle is placed somewhere inside the larger circle without crossing the boundary.
- The maximum offset occurs when the smaller circle touches the boundary of the larger circle at a single point.

Determining the Position

- The maximum distance from the center of the larger circle to the center of the smaller circle is:

$$
d = R_{large} - r_{small} = 4\,cm - 2\,cm = 2\,cm
$$

- The smaller circle's center can be anywhere within this distance from the larger circle's center, provided it remains entirely inside.

Implication for Area Calculation

- Regardless of the position, if the entire smaller circle is within the larger circle, the shaded area remains the same because the areas are based on the full circles' sizes, not their positions.

Summary of Key Points

- The area of a circle is calculated as πr^2.
- When a smaller circle is entirely inside a larger circle, the shaded area is the difference of their areas.
- In our specific problem, the shaded region's area is 12π square centimeters, approximately 37.70 cm^2.
- The position of the smaller circle (centered or offset) does not change the total shaded area, as long as it remains fully within the larger circle.

Practical Applications of This Geometric Problem

Understanding how to compute shaded areas involving circles has numerous real-world applications:

- **Design and Engineering:** Calculating material usage when designing circular components with inner cutouts.
- **Architecture:** Planning circular layouts with inner courtyards or holes.
- **Manufacturing:** Estimating waste or material removal in circular objects.
- **Educational Purposes:** Teaching concepts of area, radius, and geometric reasoning.

Additional Tips for Solving Similar Problems

- Always visualize the problem with diagrams.
- Clarify the positions of the circles — concentric or offset.
- Use consistent notation for centers and radii.
- Recall basic formulas and be cautious with assumptions.
- When in doubt, consider the maximum and minimum possible positions.

Conclusion

In conclusion, when a circle with radius 2 cm sits inside a larger circle with radius 4 cm, and the smaller circle is fully contained within the larger one, the shaded area—representing the ring-shaped region—is obtained by subtracting the smaller circle's area from the larger circle's area. The calculation yields:

$$
\boxed{
\text{Shaded Area} = 12\pi \text{ cm}^2 \approx 37.70 \text{ cm}^2
}
$$

This problem exemplifies fundamental principles of circle geometry and area calculations and highlights the importance of visualization and assumptions in geometric problem-solving.

If you want to explore more, consider investigating how the shaded area changes if the smaller circle is partially outside the larger circle or how the area varies with different radii. Happy learning!

Frequently Asked Questions

What is the area of the larger circle with radius 4 cm?

The area of the larger circle is $\pi \times 4^2 = 16\pi$ cm^2.

What is the area of the smaller circle with radius 2 cm?

The area of the smaller circle is $\pi \times 2^2 = 4\pi$ cm^2.

How can we find the area of the shaded region between the two circles?

The shaded area is the difference between the areas of the larger and smaller circles: $16\pi - 4\pi = 12\pi$ cm^2.

Is the smaller circle completely inside the larger circle?

Yes, since the smaller circle with radius 2 cm sits entirely inside the larger one with radius 4 cm.

What is the significance of the radius lengths in calculating the shaded area?

The radius lengths determine each circle's area, enabling us to subtract the smaller from the larger to find the shaded region.

Can the shaded area be zero if the circles overlap?

No, the shaded area would be zero only if the circles coincided exactly, but here, with the smaller inside the larger, the shaded area is positive.

What formula do we use to find the area of a circle?

The formula is Area = $\pi \times \text{radius}^2$.

How does the position of the smaller circle inside the larger impact the shaded area?

If the smaller circle is completely inside, the shaded area is simply the difference of their areas; partial overlap would require more complex calculations.

What is the numerical value of the shaded area in square centimeters?

The shaded area is 12π cm^2, which is approximately 37.7 cm^2 when π is approximated as 3.14.

Additional Resources

Geometry: Exploring the Overlapping Areas of Circles with Different Radii

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Introduction

Imagine a perfectly round circle with a radius of 4 centimeters (cm), within which another smaller circle of radius 2 cm is nestled. This configuration is a classic problem in geometry that combines the elegance of mathematical visualization with practical applications in fields like engineering, design, and even art. The question at hand is: What is the area of the shaded region—the part of the larger circle that is not overlapped by the smaller circle?

Understanding this problem requires a comprehensive exploration of circle geometry, including concepts of area, chord lengths, and segment calculations. This article provides an in-depth analysis, breaking down each step to clarify the geometric principles involved, and ultimately arriving at a precise answer for the shaded area.

Setting the Scene: The Circles and Their Relationships

Let's start by clearly defining our circles:

- Large circle: Radius $R = 4\,cm$
- Small circle: Radius $r = 2\,cm$

The smaller circle is inside the larger one, and the critical question is: Where are these circles positioned?

Key Assumption:
For simplicity and to create a meaningful intersection, we assume that the smaller circle is completely

inside the larger circle, and that their centers are separated by some distance d. The problem does not specify the position explicitly, but for maximum overlap (and thus to analyze the shaded area), the most relevant scenario is when the smaller circle is entirely inside and possibly tangent or overlapping with the larger circle.

Visualizing the Configuration

Before diving into calculations, it's essential to visualize the setup:

- The larger circle is centered at point O.
- The smaller circle is centered at point O', located somewhere within the larger circle such that the entire smaller circle lies inside the larger circle.
- The distance between the centers $d = |OO'|$ determines how much the circles overlap.

Possible scenarios include:

1. Complete containment without overlap: The smaller circle is entirely inside the larger circle with no part outside—no shaded region in this case.
2. Partial overlap: The smaller circle intersects the larger circle, creating a lens-shaped intersection.
3. Full containment: The smaller circle is tangent to the larger circle internally, with the centers aligned such that the entire small circle fits inside the big one.

Given the nature of the question ("A circle with radius 2 cm sits inside a circle with radius 4 cm"), the most intriguing case is when the smaller circle partially overlaps with the larger circle, creating a shaded region—the part of the larger circle not overlapped by the smaller circle.

However, since the problem does not specify the position of the smaller circle, a common interpretation is that the smaller circle is entirely inside the larger circle, and the shaded area refers to the area of the larger circle minus the area of the smaller circle.

To cover all bases, we'll analyze two typical cases:

- Case 1: The smaller circle is concentric with the larger circle.
- Case 2: The smaller circle is offset, overlapping with the larger circle, and the overlapping region (lens shape) is considered.

Case 1: Concentric Circles

The Simplest Scenario

Suppose both circles share the same center O. This is the simplest case, often used as a starting point.

- Area of the large circle:

$$
A_{\text{large}} = \pi R^2 = \pi \times 4^2 = 16\pi\,cm^2
$$

- Area of the small circle:

$$
A_{\text{small}} = \pi r^2 = \pi \times 2^2 = 4\pi\,cm^2
$$

- Shaded area (the part of the larger circle not overlapped):

$$
A_{\text{shaded}} = A_{\text{large}} - A_{\text{small}} = 16\pi - 4\pi = 12\pi\,cm^2
$$

Result:

$$
\boxed{
A_{\text{shaded}} = 12\pi\,cm^2 \approx 37.699\,cm^2
}
$$

This is straightforward and applies if the circles are perfectly aligned concentrically.

Case 2: Offset Circles with Partial Overlap

Exploring the Overlap: The Lens-Shaped Region

In many practical problems, the smaller circle isn't perfectly centered. Instead, it is offset, creating an intersection with the larger circle. The area of the shaded region then becomes the part of the larger circle not overlapped by the smaller one.

Suppose:

- The small circle is inside the larger circle.
- The centers are separated by a distance d, with $0 \le d \le R - r$.

The overlap region (lens shape) is bounded by two circular segments—one from each circle.

Key parameters:

- Distance between centers: d
- Overlap area: Calculated via the formula for the intersection of two circles.

Calculating the Overlap Area for Partially Intersecting Circles

The area of intersection between two circles of radii R and r, separated by distance d, is given by:

$$
A_{\text{intersect}} = r^2 \cos^{-1}\left(\frac{d^2 + r^2 - R^2}{2 d r} \right) + R^2 \cos^{-1}\left(\frac{d^2 + R^2 - r^2}{2 d R} \right) - \frac{1}{2} \sqrt{(-d + r + R)(d + r - R)(d - r + R)(d + r + R)}
$$

This formula accounts for the area of the overlapping lens-shaped region.

Note:
- The square root term is known as the intersecting chord area adjustment.
- The angles are in radians.

Applying the formula in a practical context

Suppose the centers are separated by $d = 2\,cm$. Then:

- $R = 4\,cm$
- $r = 2\,cm$
- $d = 2\,cm$

Calculate the two angles:

$$
\theta_1 = \cos^{-1} \left(\frac{d^2 + r^2 - R^2}{2 d r} \right) = \cos^{-1} \left(\frac{(2)^2 + (2)^2 - (4)^2}{2 \times 2 \times 2} \right) = \cos^{-1} \left(\frac{4 + 4 - 16}{8} \right) = \cos^{-1} \left(\frac{-8}{8} \right) = \cos^{-1}(-1) = \pi
$$

Similarly,

$$
\theta_2 = \cos^{-1} \left(\frac{d^2 + R^2 - r^2}{2 d R} \right) = \cos^{-1} \left(\frac{4 + 16 - 4}{2 \times 2 \times 4} \right) = \cos^{-1} \left(\frac{16}{16} \right) = \cos^{-1}(1) = 0
$$

Now, the intersection area simplifies:

$$
A_{\text{intersect}} = r^2 \times \pi + R^2 \times 0 - \frac{1}{2} \times \sqrt{(-d + r + R)(d + r - R)(d - r + R)(d + r + R)}
$$

Calculations:

- $r^2 \pi = 4 \pi$
- Square root term:

$$
\sqrt{(-2 + 2 + 4)(2 + 2 - 4)(2 - 2 + 4)(2 + 2 + 4)} = \sqrt{(4)(0)(4)(8)} = \sqrt{0} = 0
$$

So,

$$
A_{\text{intersect}} = 4 \pi - 0 = 4 \pi \approx 12.566\,cm^2
$$

Interpretation:
The overlap region has an area of approximately 12.566 cm^2.

Determining the Shaded Area

If the smaller circle is offset and partially overlaps with the larger circle, the shaded area—the part of the larger circle not overlapped—is:

$$
A_{\text{shaded}} = A_{\text{large}} - A_{\text{intersect}} = 16 \pi - 4 \pi = 12 \pi \approx 37.699\,cm^2
$$

This calculation indicates that, in this specific configuration, the shaded region is the part of the larger circle outside the overlapping lens, approximately 37.699 cm^2.

Practical Implications and Applications

Understanding the area of the shaded region has real-world significance:

- Design and Engineering: Calculating material usage or coverage areas in circular components.
- Optics

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