

A Confidence Interval For A Population Mean Has A Margin Of Error Of 3.9. A) Determine The Length Of

A Confidence Interval For A Population Mean Has A Margin Of Error Of 3.9. A) Determine The Length Of

Understanding confidence intervals is fundamental in statistics, especially when estimating population parameters such as the mean. When a confidence interval (CI) for a population mean has a margin of error (MOE) of 3.9, it provides valuable information about the precision of the estimate. This article aims to explore what this margin of error signifies, how to determine the length of the confidence interval, and associated concepts such as confidence levels, standard deviation, and sample size. Whether you're a student, researcher, or data analyst, grasping these concepts will enhance your statistical literacy and ability to interpret data accurately.

What Is a Confidence Interval for a Population Mean?

A confidence interval (CI) is a range of values, derived from sample data, that is likely to contain the true population parameter — in this case, the population mean (μ). It is expressed with a specified confidence level, such as 95%, indicating the probability that the interval contains the true mean if the same sampling process is repeated numerous times.

Key components of a confidence interval:

- Sample mean ($\bar{x}$): The average from the sample data.
- Margin of Error (MOE): The maximum expected difference between the sample mean and the true population mean.
- Confidence Level: The probability that the interval contains the true mean.

The general form of a confidence interval for a population mean when the population standard deviation (σ) is known is:

$$\text{CI} = \left(\bar{x} - z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}, \quad \bar{x} + z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}} \right)$$

When σ is unknown and the sample size is small, the t-distribution replaces the z-distribution.

Understanding Margin of Error (MOE)

The margin of error quantifies the uncertainty in the estimate of the population mean based on the sample data. It reflects how close the sample mean is expected to be to the actual population mean.

Definition:

- The margin of error is the amount added and subtracted from the sample mean to create the confidence interval.
- It depends on the variability in the data, the sample size, and the chosen confidence level.

Mathematically:

$$\text{MOE} = z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$

for known σ , or

$$\text{MOE} = t_{\alpha/2, df} \times \frac{s}{\sqrt{n}}$$

for unknown σ , where:

- $z_{\alpha/2}$ is the critical value from the standard normal distribution.
- $t_{\alpha/2, df}$ is the critical value from the t-distribution.
- s is the sample standard deviation.
- n is the sample size.

In this context, the MOE is given as 3.9, indicating the maximum expected deviation of the sample mean from the true population mean with the specified confidence level.

Determining the Length of the Confidence Interval

The length of the confidence interval is the total width from the lower bound to the upper bound. It provides a measure of the estimate's precision — narrower intervals imply greater precision.

Formula for the length of the confidence interval:

$$\text{Length} = 2 \times \text{MOE}$$

Given the problem statement:

- $\text{MOE} = 3.9$

Thus:

```
\boxed{
\text{Interval Length} = 2 \times 3.9 = 7.8
}
\]
```

This means the confidence interval spans a total of 7.8 units, centered around the sample mean. The interpretation is that, based on the sample data, the true population mean is estimated to lie within a 7.8-unit range.

Implications of the Confidence Level

While the problem specifies the margin of error, it does not explicitly mention the confidence level (such as 90%, 95%, or 99%). The confidence level influences the critical value used (either $z_{\alpha/2}$ or $t_{\alpha/2, df}$) and, consequently, the margin of error.

Common confidence levels:

- 90% confidence: $z_{0.05} \approx 1.645$
- 95% confidence: $z_{0.025} \approx 1.96$
- 99% confidence: $z_{0.005} \approx 2.576$

The higher the confidence level, the larger the critical value and the wider the confidence interval, leading to a higher margin of error.

Calculating Sample Size Based on Margin of Error

If the goal is to determine how large a sample size is needed to achieve a specific margin of error given a known or estimated population standard deviation, the formula is:

$$n = \left(\frac{z_{\alpha/2} \times \sigma}{\text{MOE}} \right)^2$$

Steps to determine sample size:

1. Decide on the desired margin of error (e.g., 3.9 in this problem).
2. Choose the confidence level and find the corresponding $z_{\alpha/2}$.
3. Estimate the population standard deviation (σ) either from prior data or a pilot study.
4. Plug into the formula to compute n .

Example:

Suppose $\sigma = 10$, confidence level = 95% ($z_{0.025} \approx 1.96$):

$$n = \left(\frac{1.96 \times 10}{3.9} \right)^2 \approx \left(\frac{19.6}{3.9} \right)^2 \approx (5.03)^2 \approx 25.3$$

Rounding up, a sample size of 26 would be needed to achieve a margin of error of 3.9 with 95% confidence, assuming $(\sigma = 10)$.

Practical Applications and Examples

Understanding how to determine the confidence interval length and sample size has numerous real-world applications:

- Market Research: Estimating average customer satisfaction scores within a margin of error.
- Quality Control: Measuring the average weight of products with a specified precision.
- Healthcare Studies: Estimating average blood pressure in a population with a certain confidence level.
- Public Policy: Determining average income levels in demographic surveys.

Example scenario:

Suppose a researcher conducts a study to estimate the average height of adult men in a city. They find a sample mean height and a margin of error of 3.9 inches at a 95% confidence level. The total confidence interval length is 7.8 inches, providing a range within which the true average height likely falls.

Factors Influencing Margin of Error and Interval Length

Several factors affect the margin of error and, consequently, the length of the confidence interval:

- Sample Size (n): Larger samples reduce the margin of error, leading to narrower intervals.
- Variability in Data (σ) or (s) : Higher variability increases the margin of error.
- Confidence Level: Higher confidence levels increase the critical value, enlarging the margin of error.
- Measurement Precision: More precise measurements reduce variability and interval length.

Summary and Key Takeaways

- The length of a confidence interval is twice the margin of error.
- Given a margin of error of 3.9, the interval length is 7.8.
- The margin of error depends on the standard deviation, sample size, and confidence level.
- To reduce the interval length, increase the sample size or improve measurement accuracy.
- Understanding these concepts allows researchers to design better studies and interpret statistical results effectively.

Conclusion

The margin of error is a crucial concept in statistical inference, representing the maximum expected deviation of a sample estimate from the true population parameter. In the context of a confidence interval for a population mean with a margin of error of 3.9, the total length of the interval is 7.8. Recognizing how this interval is constructed, what influences its width, and how to manipulate sample size and confidence levels empowers statisticians and data professionals to make informed decisions. Whether estimating population characteristics or planning data collection, grasping these principles ensures more accurate and reliable results.

Keywords for SEO Optimization:

- Confidence interval for population mean
- Margin of error
- Interval length calculation
- Sample size determination
- Statistical inference
- Confidence level
- Population estimate
- Data analysis
- Margin of error formula
- Confidence interval width

Frequently Asked Questions

What is the length of the confidence interval if the margin of error is 3.9?

The length of the confidence interval is twice the margin of error, so it is $2 \times 3.9 = 7.8$.

How is the margin of error related to the length of the confidence interval?

The margin of error is half the length of the confidence interval; thus, the interval length is twice the margin of error.

If the margin of error increases, what happens to the length of the confidence interval?

The length of the confidence interval increases proportionally, as it is twice the margin of error.

Given a margin of error of 3.9, what is the formula to find the length of the confidence interval?

$\text{Length} = 2 \times \text{Margin of Error} = 2 \times 3.9 = 7.8.$

Why is the length of the confidence interval important in statistical analysis?

It indicates the range within which the true population mean is likely to lie, with a certain level of confidence.

Can the length of the confidence interval be less than the margin of error?

No, the length of the confidence interval is always twice the margin of error, so it cannot be less.

If the margin of error were reduced to 2, what would be the new length of the confidence interval?

The new length would be $2 \times 2 = 4.$

How does sample size affect the margin of error and the confidence interval length?

A larger sample size generally reduces the margin of error, leading to a narrower confidence interval.

What assumptions are necessary to interpret the confidence interval and its margin of error?

Assumptions include a random sample, normally distributed data (or large sample size), and known confidence level.

In the context of the question, what does the margin of error of 3.9 specifically tell us?

It indicates the maximum expected difference between the sample mean and the true population mean at the specified confidence level, with the confidence interval length being 7.8.

Additional Resources

Understanding Confidence Intervals and Margin of Error: A Comprehensive Analysis

When conducting statistical analysis, especially in inferential statistics, one of the fundamental concepts is estimating population parameters based on sample data. Among these parameters, the population mean (μ) is frequently estimated using a confidence interval. A confidence interval provides a range within which we expect the true population mean to lie with a certain level of confidence. A critical component of this interval is the margin of error (MoE), which quantifies the uncertainty associated with the estimate.

In this detailed review, we will explore the scenario where a confidence interval for a population mean has a margin of error of 3.9, and we aim to determine the length of this interval. This exploration will encompass the fundamental concepts, formulas, assumptions, and implications involved in calculating the length of a confidence interval, especially in relation to the margin of error.

Fundamentals of Confidence Intervals for a Population Mean

What is a Confidence Interval?

A confidence interval (CI) for a population mean is a range of values calculated from sample data, within which the true population mean (μ) is believed to lie with a specified level of confidence (e.g., 95%). It is expressed as:

$$[\text{CI}] = \bar{x} \pm \text{MoE}$$

where:

- $\bar{x}$ is the sample mean,
- MoE is the margin of error.

Key Point: The confidence level (e.g., 95%) indicates the long-term proportion of such intervals that would contain the true mean if the process were repeated multiple times.

Components of the Confidence Interval

1. Sample Mean ($\bar{x}$): The average value from the sample data.
2. Margin of Error (MoE): The maximum expected difference between the sample estimate and the actual population parameter.
3. Confidence Level: The probability that the interval contains the true parameter.

Understanding the Margin of Error (MoE)

Definition and Significance

The margin of error reflects the precision of the estimate. A smaller MoE indicates a more precise estimate, while a larger MoE suggests more uncertainty.

Mathematically, for a mean, the MoE is typically calculated as:

$$\text{MoE} = z^* \times \frac{s}{\sqrt{n}}$$

for a known standard deviation, or

$$\text{MoE} = t^* \times \frac{s}{\sqrt{n}}$$

when the standard deviation is unknown and the sample size is small, where:

- z^* is the critical value from the standard normal distribution,
- t^* is the critical value from the Student's t-distribution,
- s is the sample standard deviation,
- n is the sample size.

Note: The critical value depends on the confidence level and the distribution (normal or t-distribution).

Given the Margin of Error: 3.9

In our scenario, the MoE is given as 3.9. This value signifies the maximum expected deviation between the sample mean and the true population mean at the specified confidence level.

Determining the Length of the Confidence Interval

What Is the Length of a Confidence Interval?

The length (or width) of a confidence interval is the total span from the lower bound to the upper bound. It is calculated as:

$$\text{Length} = \text{Upper Bound} - \text{Lower Bound}$$

Since the confidence interval is symmetric around the sample mean ($\bar{x}$), the length is twice the margin of error:

$$\boxed{\text{Length} = 2 \times \text{MoE}}$$

Therefore:

$$\text{Length} = 2 \times 3.9 = 7.8$$

In other words, if the margin of error is known to be 3.9, the total length of the confidence interval is 7.8 units.

Implications of the Interval Length

- The length provides a measure of the estimate's precision: shorter intervals imply more precise estimates.
- Larger margins of error or sample variability lead to longer intervals.
- The length depends on the confidence level, sample size, and population variability.

Deeper Dive: How Margin of Error Relates to Confidence Level and Sample Size

The Role of Critical Values (z or t)

The critical value is determined based on:

- The desired confidence level (e.g., 95%, 99%)
- The sample size (which influences whether to use z or t distribution)

For example:

- At 95% confidence, $(z \approx 1.96)$ for large samples.
- For smaller samples, the t-distribution is used, and (t) varies with degrees of freedom.

Sample Standard Deviation and Variability

The standard deviation (or sample standard deviation) influences the margin of error:

- Larger variability increases MoE.
- Smaller variability reduces MoE.

Sample Size Impact

- Increasing the sample size (n) decreases the margin of error, as the standard error $(s / \sqrt{n})$ diminishes.
- To achieve a specific MoE, a larger sample size may be required if variability is high.

Applying the Concept: How to Calculate the Interval Length from MoE

Given:

- Margin of error $(\text{MoE} = 3.9)$

The total length of the confidence interval is straightforward:

$$\boxed{\text{Interval Length} = 2 \times 3.9 = 7.8}$$

This calculation assumes the confidence interval is symmetric, which is standard in most cases when estimating a mean.

Practical Considerations and Real-World Applications

Interpreting the Interval Length

A length of 7.8 units suggests that, at the specified confidence level, the true population mean is estimated to lie within a range 7.8 units wide. The narrower the interval, the more precise the estimate, which is often desired in decision-making processes in various fields

such as healthcare, economics, manufacturing, and social sciences.

Impacts of Confidence Level on Interval Length

- Higher confidence levels (e.g., 99%) lead to larger critical values (e.g., $z^* \approx 2.576$), thus increasing the margin of error and the interval length.
- Conversely, lower confidence levels (e.g., 90%) produce narrower intervals.

Balancing Precision and Confidence

Researchers must balance the desire for precision (smaller interval length) with the need for confidence (higher confidence levels). Adjustments in sample size or variability can help optimize this balance.

Conclusion: Key Takeaways and Summary

- The margin of error directly influences the size of a confidence interval. Specifically, the length of the interval is twice the margin of error.
- Given a margin of error of 3.9, the total length of the confidence interval is 7.8.
- The length of the confidence interval encapsulates the precision of the estimate; shorter intervals imply more precise estimates.
- Factors influencing the margin of error and, by extension, the interval length include the confidence level, sample size, and population variability.
- Understanding these relationships helps statisticians and researchers design studies that meet their accuracy and confidence requirements.

In summary, when the margin of error for a population mean is specified as 3.9, the total length of the confidence interval is simply twice that value, resulting in an interval length of 7.8. This fundamental relationship is vital in interpreting the results of statistical estimates and in planning data collection strategies to achieve desired levels of precision.

Final Note: Remember that the actual values for sample size, standard deviation, and critical values are essential for precise calculations in practice. This analysis assumes the standard symmetric confidence interval framework, which is standard in most inferential statistical procedures.

A Confidence Interval For A Population Mean Has A Margin Of Error Of 3.9 A Determine The Length Of

A Confidence Interval For A Population Mean Has A Margin Of Error Of 3.9 A Determine The Length

Of

Related Articles

- [5Need 200% Perfect Answer In Typing. Please Solve In Less Than 20minutes.And Give Answer In Typing.i](#)
- [\(a\) The Icosahedron Has 20 Triangular Faces, With 5 Triangles At Each Vertex. Using These Facts, How](#)
- [A Line AB, 2000 M Long, Lying At An Elevation Of 500 M Measures 8.65 Cm On A Vertical Photograph For](#)

[Back to Home](#)