

A Disk Of Radius 3 Cm Has Density 8 G/cm² At Its Center, Density 0 At Its Edge, And Its Density Is A

A Disk Of Radius 3 Cm Has Density 8 G/cm² At Its Center, Density 0 At Its Edge, And Its Density Is A — a fascinating problem that combines principles of geometry, calculus, and physics to analyze how density varies across a circular disk. Understanding the distribution of density within such a disk is essential in various scientific and engineering applications, including material science, mechanical design, and astrophysics. This article provides a comprehensive exploration of this problem, covering the mathematical modeling, analysis of density variation, and practical implications.

Understanding the Problem Setup

Basic Description of the Disk

The problem involves a circular disk with a radius of 3 centimeters. The disk's density varies from its center to its edge, with specific boundary conditions:

- At the center ($r = 0$), the density is 8 g/cm^2 .
- At the edge ($r = 3 \text{ cm}$), the density drops to 0 g/cm^2 .
- Within this radius, the density is represented by a function $A(r)$, which depends on the distance from the center.

The goal is to understand how the density function $A(r)$ behaves, find its explicit or approximate form, and analyze the physical implications of this distribution.

Mathematical Modeling of Density Variation

Formulating the Density Function

Since the density decreases from the center to the edge, a common approach is to model $A(r)$ as a continuous, decreasing function of the radius r , with boundary conditions:

$$\begin{aligned} &A(0) = 8 \quad \text{and} \quad A(3) = 0 \end{aligned}$$

A simple and physically reasonable form for $A(r)$ is a polynomial or a function involving powers of r .

For example, assume:

$$A(r) = k \left(1 - \frac{r}{3}\right)^n$$

where k and n are constants to be determined based on boundary conditions.

Applying the boundary condition at $r = 0$:

$$A(0) = k (1 - 0)^n = k = 8$$

Thus, $k = 8$.

Applying the boundary condition at $r = 3$:

$$A(3) = 8 \left(1 - \frac{3}{3}\right)^n = 8 \times 0^n = 0$$

which satisfies the boundary condition for any positive n .

Therefore, the general form:

$$A(r) = 8 \left(1 - \frac{r}{3}\right)^n$$

is valid, with n determining how sharply the density drops near the edge.

Choosing the Appropriate Power n

The value of n influences the shape of the density curve:

- $n = 1$ results in a linear decrease.
- $n > 1$ yields a more gradual decrease near the center and a sharper drop near the edge.
- $n < 1$ would produce a steeper decline near the center.

For physical applications, n is often chosen based on empirical data or specific material properties.

For simplicity, $n = 1$ (linear decrease) is a good starting point:

$$A(r) = 8 \left(1 - \frac{r}{3}\right)$$

Calculating Total Mass of the Disk

Integrating Density Over the Area

The total mass M of the disk is obtained by integrating the density over the entire area:

$$M = \int_0^{2\pi} \int_0^3 A(r) \, r \, dr \, d\theta$$

Since the density depends only on the radius (symmetry), the integral simplifies:

$$\int_0^{2\pi} d\theta = 2\pi$$

$$M = 2\pi \int_0^3 A(r) r \, dr$$

Using the linear form $A(r) = 8 \left(1 - \frac{r}{3}\right)$:

$$M = 2\pi \int_0^3 8 \left(1 - \frac{r}{3}\right) r \, dr$$

which simplifies to:

$$M = 16\pi \int_0^3 \left(1 - \frac{r}{3}\right) r \, dr$$

Evaluate the integral:

$$\int_0^3 \left(1 - \frac{r}{3}\right) r \, dr$$

Break into two integrals:

$$\int_0^3 r \, dr - \frac{1}{3} \int_0^3 r^2 \, dr$$

Calculate each:

$$\int_0^3 r \, dr = \left. \frac{r^2}{2} \right|_0^3 = \frac{9}{2}$$

$$\int_0^3 r^2 \, dr = \left. \frac{r^3}{3} \right|_0^3 = \frac{27}{3} = 9$$

Putting it all together:

$$\frac{9}{2} - \frac{1}{3} \times 9 = \frac{9}{2} - 3 = \frac{9 - 6}{2} = \frac{3}{2}$$

Thus, the total mass:

$$M = 16\pi \times \frac{3}{2} = 16 \times \frac{3}{2} \pi = 24\pi$$

which numerically is approximately:

$$M \approx 75.4 \text{ grams}$$

This calculation illustrates how the density distribution affects the total mass of the disk.

Physical Implications of Density Distribution

Material Properties and Structural Integrity

The way density varies across the disk influences its mechanical properties:

- Areas with higher density (center) contribute more to the overall weight and strength.

- The decrease toward the edges may affect the disk's flexibility and stress distribution.

Understanding the density profile helps in designing disks for specific applications such as rotating machinery, where uniformity or controlled non-uniformity is desired.

Thermal and Electrical Conductivity

Density often correlates with other physical properties:

- A denser region may conduct heat or electricity better.
- Variations in density could lead to gradients in thermal or electrical conductivity, which are critical in electronic or thermal management systems.

Applications and Practical Considerations

Material Manufacturing

Manufacturers can tailor the density profile within a disk to optimize performance:

- Gradually decreasing density could reduce weight while maintaining strength.
- Embedding denser materials at the center enhances durability.

Design of Rotating Disks and Discs

In mechanical engineering:

- Understanding mass distribution is essential for balancing disks.
- Non-uniform density profiles can be used to control moments of inertia.

Astrophysical Contexts

In astrophysics:

- Disks with non-uniform density distributions are common, such as accretion disks around stars or black holes.
- Mathematical models similar to $A(r)$ help describe the dynamics and evolution of such systems.

Advanced Topics and Further Study

Non-Linear Density Functions

More complex models may involve exponential, logarithmic, or other non-linear functions to better fit empirical data.

Inverse Problems

Given measurements of mass or other properties, one can attempt to reconstruct the density function $A(r)$, leading to inverse problems in mathematical physics.

Numerical Methods

When analytical solutions are complex, numerical integration and finite element methods become invaluable tools for analyzing density distributions.

Conclusion

The problem of a disk with a radius of 3 cm, a central density of 8 g/cm², and zero density at its edge exemplifies the intersection of geometry, calculus, and physical principles. By modeling the density distribution as a function of radius, calculating total mass, and understanding the implications for material behavior, engineers and scientists can optimize designs and interpret natural phenomena. Whether in manufacturing, structural analysis, or astrophysics, such mathematical insights are crucial for advancing technology and deepening our understanding of the physical world.

Keywords: density distribution, disk, radius, mass calculation, mathematical modeling, material science, engineering, physics, calculus, applications

Frequently Asked Questions

What is the significance of the density distribution in a disk with radius 3 cm where density varies from 8 g/cm² at the center to 0 at the edge?

The varying density indicates a non-uniform mass distribution, which affects the disk's center of mass, moments of inertia, and overall physical properties, crucial for understanding its structural behavior and applications.

How can the density function be expressed mathematically for a disk where density decreases linearly from the center to the edge?

A linear density function can be written as $\rho(r) = 8(1 - r/3)$ g/cm², where r is the distance from the center, decreasing from 8 g/cm² at $r=0$ to 0 at $r=3$ cm.

How do you calculate the total mass of the disk given its density distribution?

Total mass can be found by integrating the density over the disk's area: $M = \iint \rho(r) r \, dr \, d\theta$, within $r=0$ to 3 cm and $\theta=0$ to 2π , using the specified density function.

What is the center of mass of this disk considering the density variation from the center to the edge?

Due to the symmetric and radially decreasing density, the center of mass remains at the geometric center of the disk, but the mass distribution is skewed towards the center, influencing the mass calculations.

How does the variable density affect the moment of inertia of the disk about its central axis?

The moment of inertia depends on the mass distribution; with higher density at the center, the disk's moment of inertia will be different compared to a uniform disk, requiring integration of r^2 times the density over the entire area.

What are some practical applications or scenarios where understanding the density variation in a disk is important?

Applications include designing composite materials, analyzing rotational dynamics in machinery, manufacturing processes involving variable density disks, and understanding natural phenomena like planetary formation where mass distribution varies.

Additional Resources

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In the realm of material science and engineering, understanding how density varies within objects is fundamental to designing and analyzing structures, materials, and systems. This article explores a specific case—a circular disk with a radius of 3 centimeters, exhibiting a density profile that peaks at its center and diminishes to zero at its periphery. The density at the center is given as 8 grams per square centimeter, and the problem introduces a variable $\rho(r)$ to describe the density distribution. By dissecting this scenario, we will delve into the mathematical modeling of density functions, the physical implications of such a distribution, and the methods to determine key properties like mass and centroid location, all while maintaining clarity for readers with a technical background.

Understanding the Density Profile of the Disk

The Physical Setup

The disk in question has a radius $(R = 3)$ cm. Its density is not uniform; instead, it varies radially from the center to the edge. At the very core $(r = 0)$, the density reaches its maximum of (8 g/cm^2) , and gradually tapers off to zero at the boundary $(r = R)$. This kind of density variation is common in materials subjected to specific manufacturing processes or natural phenomena, such as thermal gradients or compositional gradients.

Mathematical Representation of Density

To analyze this distribution, we need an explicit formula for density $(\rho(r))$ as a function of the radial distance (r) from the center.

Given the boundary conditions:

- $(\rho(0) = 8 \text{ g/cm}^2)$
- $(\rho(R) = 0)$

And the statement that the density is proportional to a variable (A) , which likely appears as a constant factor in the density function, we can posit a general form:

$$\rho(r) = A \times f(r)$$

where $(f(r))$ is a function satisfying:

$$f(0) = 1 \quad \text{and} \quad f(R) = 0$$

A common choice for such a profile is a linear or quadratic function. For simplicity and physical plausibility, assume a linear decrease:

$$\rho(r) = A \left(1 - \frac{r}{R}\right)$$

This satisfies the boundary conditions:

- At $(r=0)$, $(\rho(0) = A \times 1 = A)$
- At $(r=R)$, $(\rho(R) = A \times 0 = 0)$

Given that $(\rho(0) = 8 \text{ g/cm}^2)$, it follows that:

$$A = 8$$

Hence, the density function simplifies to:

$$\rho(r) = 8 \left(1 - \frac{r}{3}\right)$$

Alternatively, if the problem indicates that the density is proportional to some parameter (A) , then:

$$\rho(r) = A \left(1 - \frac{r}{3}\right)$$

with $(A = 8)$.

This linear model provides an intuitive and manageable framework for further calculations, but more complex functions could also be considered depending on the specific application.

Calculating the Total Mass of the Disk

The Importance of Mass Calculation

Understanding the mass distribution within the disk is crucial for applications ranging from mechanical stability to thermal analysis. The total mass (M) can be obtained by integrating the density over the entire area of the disk.

Mathematical Formulation

Since the density varies radially, the differential mass element (dm) in polar coordinates is:

$$dm = \rho(r) \, dA$$

where the differential area element (dA) in polar coordinates is:

$$dA = r \, dr \, d\theta$$

The total mass is obtained by integrating over (r) from 0 to (R) , and (θ) from 0 to (2π) :

$$M = \int_0^{2\pi} \int_0^R \rho(r) \, r \, dr \, d\theta$$

Substituting $\rho(r) = 8 \left(1 - \frac{r}{3}\right)$:

$$M = \int_0^{2\pi} d\theta \int_0^3 8 \left(1 - \frac{r}{3}\right) r \, dr$$

The angular integral evaluates to (2π) , so:

$$M = 2\pi \times 8 \int_0^3 \left(1 - \frac{r}{3}\right) r \, dr$$

Simplify the integral:

$$M = 16\pi \int_0^3 \left(r - \frac{r^2}{3}\right) dr$$

Compute the integral:

$$\int_0^3 r \, dr = \frac{1}{2} r^2 \bigg|_0^3 = \frac{1}{2} \times 9 = 4.5$$

$$\int_0^3 r^2 \, dr = \frac{1}{3} r^3 \bigg|_0^3 = \frac{1}{3} \times 27 = 9$$

Therefore:

$$M = 16\pi \left(4.5 - \frac{1}{3} \times 9\right) = 16\pi (4.5 - 3) = 16\pi \times 1.5 = 24\pi$$

In grams:

$$M = 24\pi \, \mathrm{g}$$

Numerically:

$$M \approx 24 \times 3.1416 \approx 75.4 \, \mathrm{g}$$

Thus, the total mass of the disk is approximately 75.4 grams.

Locating the Center of Mass

The Concept of Centroid

The center of mass (or centroid) indicates the point where the mass can be considered to be concentrated for the purposes of analyzing motion or stability. For a radially symmetric density distribution, the centroid lies along the symmetry axis—in this case, the geometric center of the disk—but the exact position may be shifted if the density distribution is asymmetric.

Mathematical Approach

Given the symmetry, the y -coordinate of the centroid is zero, so focus on the x -coordinate (or equivalently, the radial coordinate):

$$\bar{x} = \frac{1}{M} \int x \, dm$$

In polar coordinates:

$$x = r \cos \theta$$

and

$$dm = \rho(r) r \, dr \, d\theta$$

Due to symmetry, the average x -coordinate simplifies to zero, but if the density distribution is asymmetric, or if considering the general case, the centroid along x is:

$$\bar{x} = \frac{1}{M} \int_0^{2\pi} \int_0^R r \cos \theta \, \rho(r) r \, dr \, d\theta$$

Since $\cos \theta$ averages to zero over 0 to 2π , the centroid remains at the geometric center unless the density distribution varies asymmetrically.

Therefore, for symmetric density distributions, the centroid coincides with the geometric center at $(0,0)$. However, if the density is not symmetric, the calculation involves integrating $r \cos \theta$ weighted by the density, which can shift the centroid.

Implications and Applications

Material Design and Engineering

Understanding how density varies within a structure helps engineers optimize material usage, weight distribution, and structural integrity. For instance, in manufacturing components with graded materials—where density or composition changes gradually—accurate models ensure safety and performance.

Natural Phenomena and Geophysical Studies

In geophysics, the Earth's inner layers exhibit density variations. Modeling these variations informs us about Earth's composition, seismic activity, and gravitational field. Similarly, in astrophysics, stellar density profiles are crucial for understanding star formation and evolution.

Quality Control and Manufacturing Processes

In processes such as additive manufacturing or casting, controlling the density distribution ensures the product's strength and longevity. Analyzing the density profile helps identify flaws or areas needing reinforcement.

Conclusion

The exploration of a disk with a linearly decreasing density profile from its center to its edge exemplifies the intersection of mathematical modeling and physical understanding. By establishing the density function, calculating total mass, and considering the centroid location, engineers and scientists can better grasp the internal characteristics of materials and structures. The specific case where the density peaks at the center and diminishes

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