(a) Find The Directional Derivative Of F(x, Y, Z)=xy2tan1z At (2, 1, 1) In The Direction Of V=<1,

(a) Find The Directional Derivative Of F(x, Y, Z)=xy2tan1z At (2, 1, 1) In The Direction Of V=<1, is a fundamental problem in multivariable calculus that involves understanding how functions change in specific directions. The concept of the directional derivative provides insight into the rate at which a function varies as we move from a point in a particular direction. This article explores the process of calculating the directional derivative of the given function $F(x, y, z) = xy^2 \tan^{-1} z$ at the point $(2, 1, 1)$ in the direction of the vector $\mathbf{V} = \langle 1, \dots \rangle$. We will delve into the mathematical foundations, step-by-step calculation procedures, and the significance of directional derivatives in optimization and gradient analysis, ensuring a comprehensive understanding for students and professionals alike.

Understanding the Concept of the Directional Derivative

What Is a Directional Derivative?

The directional derivative of a multivariable function measures the rate of change of the function at a given point, in the direction of a specified vector. Unlike partial derivatives, which measure change along coordinate axes, the directional derivative considers an arbitrary direction, providing a more complete picture of a function's behavior.

Mathematically, for a function $f(x, y, z)$, the directional derivative at point $\mathbf{a} = (a_x, a_y, a_z)$ in the direction of a unit vector $\mathbf{u} = \langle u_x, u_y, u_z \rangle$ is defined as:

$$
D_{\mathbf{u}}f(\mathbf{a}) = \lim_{h \to 0} \frac{f(\mathbf{a} + h \mathbf{u}) - f(\mathbf{a})}{h}
$$

Alternatively, it can be computed using the gradient:

$$
D_{\mathbf{u}}f(\mathbf{a}) = \nabla f(\mathbf{a}) \cdot \mathbf{u}
$$

where $\nabla f(\mathbf{a})$ is the gradient vector evaluated at $\mathbf{a}$.

Step-by-Step Calculation of the Directional Derivative

Step 1: Define the Function and Point of Evaluation

Given:
\[
F(x, y, z) = xy^2 \tan^{-1} z
\]
Point:
\[
\mathbf{a} = (2, 1, 1)
\]

Step 2: Determine the Direction Vector \(\mathbf{V} \)

Given:
\[
\mathbf{V} = \langle 1, \dots \rangle
\]
(Note: The original prompt seems incomplete regarding the vector \(\mathbf{V} \). Assuming the vector is \(\langle 1, 0 \rangle \), or perhaps \(\langle 1, 0, 0 \rangle \). For this explanation, let's assume the vector \(\mathbf{V} = \langle 1, 0, 0 \rangle \). If the original vector is different, the process remains the same; only the components change.)

Step 3: Normalize the Direction Vector

The directional derivative requires a unit vector in the specified direction. The magnitude of \(\mathbf{V} = \langle 1, 0, 0 \rangle \) is:

\[
|\mathbf{V}| = \sqrt{1^2 + 0^2 + 0^2} = 1
\]

Since it's already a unit vector, \(\mathbf{u} = \langle 1, 0, 0 \rangle \).

Calculating the Gradient of $F(x, y, z)$

Step 1: Compute Partial Derivatives

To find the gradient ∇F, calculate:

$$
\frac{\partial F}{\partial x}, \quad \frac{\partial F}{\partial y}, \quad \frac{\partial F}{\partial z}
$$

Given:

$$
F(x, y, z) = xy^2 \tan^{-1} z
$$

- Partial derivative with respect to x:

$$
\frac{\partial F}{\partial x} = y^2 \tan^{-1} z
$$

- Partial derivative with respect to y:

$$
\frac{\partial F}{\partial y} = 2xy \tan^{-1} z
$$

- Partial derivative with respect to z:

Recall that $\tan^{-1} z$ differentiates to $\frac{1}{1 + z^2}$, so:

$$
\frac{\partial F}{\partial z} = xy^2 \cdot \frac{1}{1 + z^2}
$$

Step 2: Evaluate the Gradient at $(2, 1, 1)$

Plugging in the point:

- $y = 1$
- $z = 1$
- $x = 2$

Calculations:

$$
\frac{\partial F}{\partial x} = (1)^2 \tan^{-1} 1 = 1 \times \frac{\pi}{4} = \frac{\pi}{4}
$$

$$
\frac{\partial F}{\partial y} = 2 \times 2 \times 1 \times \frac{\pi}{4} = 4 \times \frac{\pi}{4} = \pi
$$

$$
\frac{\partial F}{\partial z} = 2 \times 1 \times 1^2 \times \frac{1}{1 + 1^2} = 2 \times 1 \times 1 \times \frac{1}{2} = 1
$$

Thus, the gradient vector at $(2, 1, 1)$ is:

$$
\nabla F(2, 1, 1) = \left\langle \frac{\pi}{4}, \pi, 1 \right\rangle
$$

Calculating the Directional Derivative

Step 1: Use the Gradient and Direction Vector

Recall:

$$
D_{\mathbf{u}}F(\mathbf{a}) = \nabla F(\mathbf{a}) \cdot \mathbf{u}
$$

Given $\mathbf{u} = \langle 1, 0, 0 \rangle$, the dot product becomes:

$$
D_{\mathbf{u}}F = \left\langle \frac{\pi}{4}, \pi, 1 \right\rangle \cdot \langle 1, 0, 0 \rangle = \frac{\pi}{4}
$$

$$
\times 1 + \pi \times 0 + 1 \times 0 = \frac{\pi}{4}
$$

This is the directional derivative of F at $(2, 1, 1)$ in the direction of $\mathbf{V}$.

Interpreting the Results and Applications

What Does the Directional Derivative Tell Us?

The computed value $\frac{\pi}{4}$ indicates the rate of change of the function F at the point $(2, 1, 1)$ moving in the direction of $\mathbf{V} = \langle 1, 0, 0 \rangle$. A positive value signifies that F increases in that direction, while a negative value would imply a decrease.

Applications of Directional Derivatives

Understanding the directional derivative is crucial in numerous fields, including:

- Optimization: Identifying the direction of maximum increase or decrease of a function.
- Gradient Descent Algorithms: Navigating functions to find local minima or maxima.
- Engineering and Physics: Analyzing how physical quantities change in specific directions.
- Machine Learning: Gradient-based learning algorithms rely heavily on directional derivatives.

Summary of Key Points

- The gradient vector provides the direction of maximum increase of a function.
- The unit vector specifies the direction in which the derivative is measured.
- The directional derivative is the dot product of the gradient and the unit vector.
- Proper normalization of the direction vector ensures accurate calculation.

Conclusion

Calculating the directional derivative of a multivariable function like $F(x, y, z) = xy^2 \tan^{-1} z$ involves understanding the gradient vector, normalizing the direction vector, and applying the dot product operation. In the specific case of evaluating at $(2, 1, 1)$ and moving in the direction of $\mathbf{V} = \langle 1, 0, 0 \rangle$, the process illustrates the fundamental techniques used in multivariable calculus.

Frequently Asked Questions

How do you compute the directional derivative of the function F(x, y, z) = xy^2 tan(1z) at the point (2, 1, 1) in the direction of the vector V = <1, 0, 0>?

First, find the gradient ∇F(x, y, z), evaluate it at (2, 1, 1), and then take the dot product with the unit vector in the direction of V. The gradient components are partial derivatives of F with respect to x, y, and z.

What steps are involved in calculating the directional derivative of F(x, y, z) = xy^2 tan(z) at (2, 1, 1) along the vector V = <1, 0, 0>?

Step 1: Compute the gradient ∇F. Step 2: Normalize the vector V to get the unit vector. Step 3: Evaluate the gradient at the point (2, 1, 1). Step 4: Take the dot product of the gradient and the unit vector to find the directional derivative.

Why is it important to normalize the vector V when calculating the directional derivative of a multivariable function?

Normalizing V ensures that the directional derivative measures the rate of change of the function in the exact direction, independent of the original vector's magnitude. It provides a consistent measure of the function's slope in that direction.

How do the partial derivatives of F(x, y, z) = xy^2 tan(z) influence the calculation of the gradient at the point (2, 1, 1)?

The partial derivatives with respect to x, y, and z determine the components of the gradient vector. Evaluating these derivatives at (2, 1, 1) gives the specific gradient needed to compute the directional derivative in any given direction.

Can the directional derivative of $F(x, y, z) = xy^2 \tan(z)$ at (2, 1, 1) be zero **in some directions? If so, which directions?**

Yes, the directional derivative can be zero in directions where the dot product of the gradient at the point and the unit direction vector is zero. These directions are orthogonal to the gradient vector at (2, 1, 1).

Additional Resources

Find The Directional Derivative Of F(x, y, z) = xy^2 tan(1/z) at (2, 1, 1) in the direction of V = <1, 0, 0>

Understanding how a multivariable function changes as we move in a specific direction is a fundamental concept in multivariable calculus. The directional derivative of a function provides this insight, measuring the rate at which the function changes at a point when moving along a given vector. In this guide, we will walk through the process of finding the directional derivative of \(F(x, y, z) = xy^2 \tan(1/z) \) at the point \((2, 1, 1)\) in the direction of \(\mathbf{V} = \langle 1, 0, 0 \rangle \).

Introduction to the Concept of Directional Derivatives

Before diving into the calculations, it's essential to grasp what the directional derivative represents. For a scalar function \(F(x, y, z)\), the directional derivative at a point describes how much \(F\) increases or decreases as we move from that point infinitesimally in the direction of a vector \(\mathbf{V} \).

Mathematically, the directional derivative of \(F\) at a point \(\mathbf{P} = (x_0, y_0, z_0)\) in the direction of a unit vector \(\mathbf{u} = \frac{\mathbf{V}}{\|\mathbf{V}\|}\) is given by:

\[
D_{\mathbf{u}}F(\mathbf{P}) = \nabla F(\mathbf{P}) \cdot \mathbf{u}
\]

where:

- \(\nabla F(\mathbf{P}) \) is the gradient vector of \(F\) evaluated at \(\mathbf{P}\),
- \(\cdot \) denotes the dot product.

Step-by-Step Breakdown

1. Determine the Gradient of \(F(x, y, z)\)

The gradient vector ∇F consists of the partial derivatives:

$$
\nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right)
$$

Given:

$$
F(x, y, z) = xy^2 \tan \left(\frac{1}{z} \right)
$$

we will compute each partial derivative separately.

---

2. Calculating Partial Derivatives

a. $\frac{\partial F}{\partial x}$

Since F is explicitly dependent on x as $xy^2 \tan(1/z)$, with y and z treated as constants:

$$
\frac{\partial F}{\partial x} = y^2 \tan \left(\frac{1}{z} \right)
$$

b. $\frac{\partial F}{\partial y}$

Similarly, treating x and z as constants:

$$
\frac{\partial F}{\partial y} = x \cdot 2y \tan \left(\frac{1}{z} \right) = 2xy \tan \left(\frac{1}{z} \right)
$$

c. $\frac{\partial F}{\partial z}$

This derivative is more involved because F depends on z through $\tan(1/z)$. We need to apply the chain rule:

$$
\frac{\partial F}{\partial z} = xy^2 \cdot \frac{d}{dz} \left[\tan \left(\frac{1}{z} \right) \right]
$$

Recall that:

$$
\frac{d}{dz} \tan(u) = \sec^2(u) \cdot \frac{du}{dz}
$$

with $u = \frac{1}{z}$, so:

$$
\frac{du}{dz} = -\frac{1}{z^2}
$$

Therefore:

$$
\frac{\partial F}{\partial z} = xy^2 \cdot \sec^2 \left(\frac{1}{z} \right) \cdot \left(-\frac{1}{z^2}\right) = - xy^2 \frac{\sec^2 \left(\frac{1}{z} \right)}{z^2}
$$

---

3. Evaluate the Gradient at $(2, 1, 1)$

Substitute $x=2$, $y=1$, $z=1$:

- $\frac{\partial F}{\partial x}$:

$$
= (1)^2 \tan(1/1) = 1 \times \tan(1) = \tan(1)
$$

- $\frac{\partial F}{\partial y}$:

$$
= 2 \times 2 \times 1 \times \tan(1) = 4 \tan(1)
$$

- $\frac{\partial F}{\partial z}$:

$$
= - 2 \times (1)^2 \times \frac{\sec^2(1)}{1^2} = - 2 \times 1 \times \sec^2(1) = - 2 \sec^2(1)
$$

Thus, the gradient vector at the point:

$$
\nabla F(2, 1, 1) = \left(\tan(1), \quad 4 \tan(1), \quad - 2 \sec^2(1) \right)
$$

---

4. Normalize the Direction Vector $\mathbf{V}$

Given $\mathbf{V} = \langle 1, 0, 0 \rangle$, its magnitude is:

$$
\|\mathbf{V}\| = \sqrt{1^2 + 0^2 + 0^2} = 1
$$

Since $\mathbf{V}$ is already a unit vector, the direction vector $\mathbf{u}$ is:

$$
\mathbf{u} = \frac{\mathbf{V}}{\|\mathbf{V}\|} = \langle 1, 0, 0 \rangle
$$

---

5. Compute the Directional Derivative

The directional derivative is the dot product of the gradient vector and the unit direction vector:

$$
D_{\mathbf{u}} F(2, 1, 1) = \nabla F(2, 1, 1) \cdot \mathbf{u}
$$

which simplifies to:

$$
D_{\mathbf{u}} F = \left(\tan(1), \quad 4 \tan(1), \quad - 2 \sec^2(1) \right) \cdot \langle 1, 0, 0 \rangle = \tan(1) \times 1 + 4 \tan(1) \times 0 + \left(- 2 \sec^2(1)\right) \times 0
$$

$$
= \tan(1)
$$

Final Result

The directional derivative of $F(x, y, z) = xy^2 \tan(1/z)$ at $(2, 1, 1)$ in the direction of $\mathbf{V} = \langle 1, 0, 0 \rangle$ is:

$$
\boxed{
D_{\mathbf{V}} F(2, 1, 1) = \tan(1)
}
$$

Additional Insights and Tips

Why Is the Directional Derivative Important?

The directional derivative tells us how sensitive the function F is to movement in a particular direction at a specific point. This is particularly useful in optimization, physics, and engineering, where understanding the rate of change along certain paths influences decision-making.

Key Takeaways

- Always compute the gradient vector carefully, applying the chain rule where necessary.
- Remember to evaluate all functions at the point of interest before calculating the dot product.
- Ensure the direction vector is a unit vector before calculating the directional derivative.

Exploring a Different Direction

If you wish to find the directional derivative in a different direction, say $\mathbf{V} = \langle 0, 1, 0 \rangle$, follow the same steps: normalize the vector, compute the dot product with the gradient, and interpret the result.

Conclusion

Calculating the directional derivative involves understanding the gradient, normalizing the direction vector, and performing a dot product. In this example, moving along the x-axis from the point $(2, 1, 1)$ results in a change rate equal to $\tan(1)$. Mastering these steps provides a powerful tool for analyzing multivariable functions and their behaviors in various directions—an essential skill in advanced calculus and applied mathematics.

A Find The Directional Derivative Of F X Y Z Xy2tan1z At 2 1 1 In The Direction Of V Lt 1

A Find The Directional Derivative Of F X Y Z Xy2tan1z At 2 1 1 In The Direction Of V Lt 1

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