

A Finite Line Of Charge With Linear Charge Density $\lambda = 3.35 \times 10^6 \text{ C/m}$ And Length $L = 0.588 \text{ m}$ Is Located

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Understanding electrostatics and electric fields is fundamental in physics, especially when analyzing the behavior of charge distributions. Among various charge configurations, the finite line of charge is a classic problem that provides insights into how electric fields behave in realistic situations where charges are confined to a limited region rather than an infinite extent. This article explores the detailed calculation and physical interpretation of the electric field generated by a finite line of charge with a specified linear charge density and length, delving into the mathematical formulations, practical applications, and important considerations in such scenarios.

Introduction to Finite Line Charges and Their Significance

Electrostatic phenomena involving line charges are not just theoretical curiosities—they are applicable in designing electrical components, understanding charge distributions in conductors, and analyzing fields in various engineering contexts.

A finite line of charge is a segment of a charged wire or filament with uniform linear charge density, which is a measure of how much charge is distributed along its length. Such configurations are essential because they bridge the gap between idealized infinite line charges (which simplify calculations and are useful for theoretical understanding) and realistic, finite systems encountered in practical scenarios.

In this context, the problem involves a line charge of length $(L = 0.588 \text{ m})$ meters with a linear charge density $(\lambda = 3.35 \times 10^6 \text{ C/m})$. These parameters suggest a substantial amount of charge concentrated in a relatively short segment, leading to significant electric fields that can influence nearby charges or objects.

Fundamental Concepts and Parameters

Linear Charge Density (λ)

- Definition: The amount of charge per unit length along a line, expressed as $\lambda = \frac{dq}{dl}$.
- Given value: $\lambda = 3.35 \times 10^6 \text{ C/m}$.
- Significance: A high linear charge density indicates a dense charge distribution, resulting in strong electric fields near the line.

Length of the Charge Segment (L)

- Given length: $L = 0.588 \text{ m}$.
- Implication: The finite length affects the electric field pattern, especially near the endpoints, contrasting with the simpler infinite line charge case.

Charge Total (Q)

- Total charge: $Q = \lambda \times L$.
- Calculation:
$$Q = 3.35 \times 10^6 \text{ C/m} \times 0.588 \text{ m} \approx 1.9698 \times 10^6 \text{ C}$$
- Note: This is an enormous amount of charge, illustrating the intense electric fields involved.

Mathematical Analysis of the Electric Field

Calculating the electric field (E) at a point in space due to a finite line charge involves integrating Coulomb's law over the charge distribution. The standard approach considers the position of the observation point relative to the charge segment.

Setup of the Coordinate System

- Assume the line of charge extends from $x=0$ to $x=L$ along the x-axis.
- Observation point (P) located at a distance (r) from the line, typically along a perpendicular axis.

Expression for Electric Field at a Point

For a finite line charge, the electric field at a point located at a perpendicular distance (R) from

the midpoint of the charge segment (assuming the point lies along the perpendicular bisector) can be derived as:

$$E = \frac{1}{4\pi\epsilon_0} \times \frac{\lambda}{R} \times \sin\theta$$

where:

- ϵ_0 = vacuum permittivity ($8.854 \times 10^{-12} \text{ F/m}$),
- θ = half-angle subtended by the charge segment at the observation point, given by:

$$\theta = \arctan\left(\frac{L/2}{R}\right)$$

Note: For points on the axis of the charge segment, the calculation involves integrating Coulomb's law directly, leading to:

$$E = \frac{1}{4\pi\epsilon_0} \times \frac{\lambda}{r} \left(\sin\alpha_2 + \sin\alpha_1 \right)$$

where α_1 and α_2 are the angles subtended at the observation point by the ends of the segment.

Calculating Electric Field at a Specific Point

Suppose the observation point is located at a perpendicular distance R from the center of the segment. The steps to calculate the electric field include:

1. Determining the angles α_1 and α_2 :

$$\alpha_1 = \arctan\left(\frac{L/2}{R}\right)$$

$$\alpha_2 = -\arctan\left(\frac{L/2}{R}\right)$$

2. Computing the sum of sines:

$$\sin\alpha_1 + \sin\alpha_2 = 2\sin\alpha_1$$

3. Plugging into the formula:

$$E = \frac{1}{4 \pi \epsilon_0} \times \frac{\lambda}{R} \times 2 \sin \alpha_1$$

4. Simplifying:

$$E = \frac{1}{2 \pi \epsilon_0} \times \frac{\lambda}{R} \times \sin \alpha_1$$

This demonstrates how the electric field depends on the geometry and the charge distribution.

Practical Applications and Implications

Understanding the electric field generated by finite line charges has numerous practical applications across various fields:

1. Design of Charge Storage Devices

- High linear charge densities are relevant in designing capacitors and charge storage systems where precise control of electric fields is essential.

2. Electrostatic Shielding and Safety

- Knowledge of field distributions helps in designing shielding to prevent high-voltage discharges or electric shocks.

3. Particle Accelerators and Beam Control

- Finite charged filaments are used to steer and focus charged particle beams, requiring accurate electric field calculations.

4. Microelectromechanical Systems (MEMS)

- Small-scale devices often involve intricate charge distributions, where understanding finite line charge fields is crucial for device performance.

5. Electric Field Mapping in Conductors

- Analyzing how charges distribute along conductors and their resulting electric fields aids in predicting breakdown points or discharge paths.

Important Considerations and Limitations

While theoretical calculations provide valuable insights, practical scenarios involve complexities such as:

- Charge Distribution Variability: Real-world charge densities may not be perfectly uniform.
- Edge Effects: Near the ends of the finite segment, the electric field exhibits complex behavior.
- Environmental Factors: Presence of other charges, dielectric materials, and external fields can alter the field.
- Material Properties: Conductors and insulators influence charge mobility and distribution.

Summary and Conclusion

Analyzing a finite line of charge with a specified linear charge density of $(3.35 \times 10^6 \text{ C/m})$ and length (0.588 m) involves understanding the fundamental principles of electrostatics, precise mathematical modeling, and practical implications. The total charge stored in such a segment is approximately $(1.97 \times 10^6 \text{ C})$, which underscores the intensity of the resulting electric fields.

Calculating the electric field at various points involves integrating Coulomb's law over the charge distribution, considering geometric factors such as the observation point's position relative to the segment. These calculations are essential in designing devices and understanding phenomena where high charge densities and finite charge distributions are involved.

This thorough understanding aids engineers, physicists, and designers in predicting electric field behavior, ensuring safety, optimizing device performance, and innovating new applications in electronics, aerospace, and energy systems.

Keywords: finite line charge, linear charge density, electric field, Coulomb's law, electrostatics, charge distribution, electric field calculation, high charge density, engineering applications

Frequently Asked Questions

What is the electric field produced by a finite line of charge with linear charge density $\lambda = 3.35 \times 10^6 \text{ C/m}$ and length $L = 0.588 \text{ m}$?

The electric field at a point along the axis of the finite line of charge can be calculated using the formula $E = (k\lambda / r) [\sin \theta_2 + \sin \theta_1]$, where θ_1 and θ_2 are the angles subtended at the point by the ends of the line. Substituting the given values allows precise calculation of the field strength.

How does the length of the charge distribution ($L = 0.588 \text{ m}$) affect the electric field compared to an infinite line of charge?

Since the line is finite ($L = 0.588 \text{ m}$), the electric field is weaker at points far from the ends compared to an infinite line of charge, where the field is uniform and stronger. Finite length causes the field to diminish with distance, especially near the ends.

What is the significance of the linear charge density $\lambda = 3.35 \times 10^6 \text{ C/m}$ in electrostatics?

The linear charge density λ indicates a very high charge per unit length, which results in a strong electric field around the charge distribution. It is crucial for calculating the magnitude of the field and potential around the line of charge.

How can one calculate the potential at a point near the finite line of charge?

The potential V at a point along the axis of the line of charge can be obtained by integrating the contributions of infinitesimal charge elements along the length, using $V = (k \lambda) \int (dz / r)$, where r is the distance from each element to the point of interest.

What are the practical applications of understanding the electric field from a finite line of charge?

Understanding the electric field from a finite line of charge is essential in designing charged particle beams, understanding the behavior of electric fields in cables and wires, and in electrostatic shielding and sensor development.

How does the position of the charge distribution influence the electric field calculation?

The position determines the angles θ_1 and θ_2 and the distance r from the charge to the point where the field is calculated. Accurate position data allows precise computation of the electric field and potential at that point.

Additional Resources

A Finite Line Of Charge With Linear Charge Density $(\lambda = 3.35 \times 10^6) \text{ C/m}$ And Length $(L = 0.588) \text{ m}$ Is Located

In the realm of electrostatics, understanding the behavior of charge distributions is fundamental to a wide array of scientific and engineering applications. From designing capacitors to modeling charge interactions in nanoscale devices, the detailed analysis of charge configurations offers crucial insights. Today, we delve into a specific but illustrative scenario: a finite line of charge characterized by a linear charge density $(\lambda = 3.35 \times 10^6) \text{ C/m}$ and a length $(L = 0.588) \text{ meters}$, exploring its properties, the resulting electric field, and the implications for both theoretical understanding and practical applications.

Understanding the Physical Setup

What Is a Finite Line of Charge?

A finite line of charge refers to a segment of a charged conductor or insulator that carries a uniform distribution of electric charge along its length. Unlike an infinitely long charged wire, which simplifies calculations through symmetry, a finite line introduces additional complexity due to edge effects and the non-uniformity of the electric field at the extremities.

In our scenario, the line of charge has the following parameters:

- Linear charge density, (λ) : $(3.35 \times 10^6) \text{ C/m}$
- Length, (L) : 0.588 meters

This configuration can be visualized as a straight, uniformly charged rod or wire, with the charge evenly spread along its length.

Significance of the Parameters

- Linear Charge Density (λ) : This parameter indicates how much charge is distributed per unit length. A high (λ) such as $(3.35 \times 10^6) \text{ C/m}$ signifies an extremely dense charge distribution, characteristic of high-voltage conductors or specialized electrostatic setups.
- Length (L) : At 0.588 meters , the line is relatively short, which accentuates the importance of considering edge effects and the non-uniformity of electric fields near the ends.

Mathematical Foundations of Electric Field Calculation

Electric Field Due to a Finite Line of Charge

Calculating the electric field $(\mathbf{E})$ generated by a finite line of charge involves integrating Coulomb's law over the length of the charge distribution. For a point in space located at a position vector $(\mathbf{r})$, the electric field due to a small element of charge $(dq =$

λdy at position y along the wire is:

$$d\mathbf{E} = \frac{1}{4\pi \epsilon_0} \frac{dq}{r'^2} \hat{\mathbf{r}}'$$

Where:

- $\epsilon_0 \approx 8.854 \times 10^{-12}$ F/m is the vacuum permittivity.
- r is the distance from the charge element to the point of observation.
- $\hat{\mathbf{r}}$ is the unit vector pointing from the charge element to the observation point.

Geometry of the Setup

Assuming the line of charge lies along the x -axis from $x=0$ to $x=L$, and the observation point is located somewhere in space, perhaps along the perpendicular bisector or at some angle, the integration becomes more manageable with coordinate geometry.

Simplifications for Symmetry

If the observation point is on the perpendicular bisector of the line segment, the problem simplifies:

- The electric field components along the axis of the line cancel due to symmetry.
- The net electric field points directly away from or toward the charged line, perpendicular to its length.

Expression for the Electric Field at a Point Along the Perpendicular Bisector

For a point located at a perpendicular distance r from the center of a finite line of charge, the electric field magnitude is given by:

$$E = \frac{1}{4\pi \epsilon_0} \frac{2\lambda}{r} \left(\sin \theta_2 + \sin \theta_1 \right)$$

Where:

- θ_1 and θ_2 are the angles between the line segment endpoints and the observation point, measured from the perpendicular bisector.

In the case where the point is directly perpendicular to the center, the angles are symmetric, simplifying the calculation further.

Implications of High Charge Density

Electric Field Magnitudes and Safety Considerations

Given the high linear charge density, the electric fields generated near the wire are expected to be extraordinarily intense. Such fields can reach magnitudes that cause ionization of surrounding air, leading to corona discharge or electrical breakdown.

- Estimating the field strength: Using the formula for the electric field at the center (or at a specified point), one can quantify the potential for corona discharge. For example, at a distance of 1 meter (a typical safety boundary), the field could be in the order of several million volts per meter, well above the dielectric strength of air ($\sim 3 \text{ MV/m}$).
- Safety implications: Structures or devices employing such high charge densities must incorporate insulation, shielding, or operate in vacuum conditions to prevent unintended discharges.

Applications and Risks

- High-voltage transmission lines: Although real-world lines are much longer, understanding the fields of finite segments helps in designing insulation and safety clearances.
- Electrostatic precipitators: Devices that utilize strong electric fields to remove particulate matter from gases.
- Scientific research: Creating controlled high-field environments for experiments in plasma physics or materials science.

Practical Considerations in Real-World Scenarios

Charge Distribution and Stability

Maintaining a uniform charge density of $(\lambda = 3.35 \times 10^6) \text{ C/m}$ over a 0.588-meter length requires sophisticated insulation and charge management. Typically, such charge densities are achieved in controlled laboratory environments using specialized electrodes or charge injection techniques.

Challenges:

- Charge leakage: High charge densities tend to leak or dissipate over time due to conduction through the surrounding medium.
- Field emission: The intense electric fields can cause electrons to be emitted from the conductor surface, altering the charge distribution and leading to potential instability.

Measuring and Verifying the Charge Distribution

Experimental validation involves:

- Using non-contact electric field sensors to map the field intensity at various points.
- Employing charge-sensitive electrometers or Faraday cages to measure the total charge accumulated.
- Ensuring the uniformity of (λ) along the length through careful electrode design.

Conclusion: The Significance of Finite Line Charge

Analysis

The study of finite lines of charge with high linear charge densities is more than a theoretical exercise; it underpins vital technological applications and safety protocols in high-voltage engineering, particle physics, and materials science. By dissecting the mathematical principles governing the electric fields generated by such charge distributions, engineers and scientists can design safer, more efficient devices, and better understand natural phenomena involving intense electrostatic fields.

In our specific scenario, a line of charge with $(\lambda = 3.35 \times 10^6) \text{ C/m}$ and length $(L = 0.588)$ meters presents a compelling case study. Its analysis reveals the profound influence of geometry and charge density on the electric field's magnitude and distribution. Whether used for experimental setups or theoretical models, understanding these principles ensures that we harness electrostatics effectively and safely, paving the way for innovations across multiple fields.

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