A Firm Has The Production Function Q=F(K,L)=K1/2L1/4, Where K>0 And L>0. It Chooses Its Inputs

Introduction

A firm has the production function $Q = F(K, L) = K^{1/2} L^{1/4}$, where $K > 0$ and $L > 0$. It chooses its inputs, capital (K) and labor (L), to maximize output or profit based on market conditions. Understanding how this firm makes input decisions involves exploring the properties of its production function, the concept of marginal products, and the optimization strategies employed in the context of cost minimization and profit maximization. This article delves into the characteristics of this specific production function, examines the firm's input choice behavior, and discusses the implications for efficiency, costs, and output.

Properties of the Production Function

1. Cobb-Douglas Form and Homogeneity

The given production function is a form of Cobb-Douglas, characterized by its multiplicative structure with constant exponents:

- $Q = K^{1/2} L^{1/4}$

This form exhibits several important properties:

1. **Constant Returns to Scale:** Because the exponents sum to 3/4 (< 1), the production function is diminishing returns to scale. When both inputs are scaled by a factor $t > 0$, output scales by $t^{3/4}$:

2. $Q(tK, tL) = (tK)^{1/2} (tL)^{1/4} = t^{1/2} K^{1/2} t^{1/4} L^{1/4} = t^{3/4} Q(K, L)$

3. This indicates that doubling inputs results in less than double the output, characteristic of diminishing returns.

2. Marginal Products of Capital and Labor

The marginal product (MP) of an input measures the additional output produced by an extra unit of that input holding other inputs constant:

- **Marginal Product of Capital (MP_K):**

Calculated by taking the partial derivative of Q with respect to K:

$$MP_K = \partial Q/\partial K = (1/2) K^{-1/2} L^{1/4}$$

- **Marginal Product of Labor (MP_L):**

Calculated by taking the partial derivative of Q with respect to L:

$$MP_L = \partial Q/\partial L = (1/4) K^{1/2} L^{-3/4}$$

Both MP_K and MP_L are positive for K, L > 0, but diminish as inputs increase due to the negative exponents in the derivatives.

Input Choice and Optimization

1. Cost Minimization Framework

The firm's primary goal in input selection is often to minimize total cost for a given level of output or to maximize profit subject to cost constraints. Assuming input prices w (for labor) and r (for capital), the firm's problem can be formulated as:

$$\text{Minimize } C = rK + wL$$
$$\text{Subject to } Q = K^{1/2} L^{1/4} = Q_0$$

Here, Q_0 is the desired output level. The solution involves choosing K and L to satisfy this constraint while minimizing costs.

2. Using the Lagrangian Method

To solve the cost minimization problem, the Lagrangian function is constructed:

$$L = rK + wL + \lambda (Q_0 - K^{1/2} L^{1/4})$$

Setting partial derivatives to zero yields the first-order conditions:

- $\partial L/\partial K = r - (1/2) \lambda K^{-1/2} L^{1/4} = 0$
- $\partial L/\partial L = w - (1/4) \lambda K^{1/2} L^{-3/4} = 0$
- $\partial L/\partial \lambda = Q_0 - K^{1/2} L^{1/4} = 0$

These conditions help determine the optimal K and L in terms of input prices and the desired output level.

3. Deriving the Input Ratio (Isoquant) and Marginal Rate of Technical Substitution (MRTS)

The MRTS indicates the rate at which the firm can substitute labor for capital without changing output:

$$MRTS_{L,K} = - (MP_L) / (MP_K) = - [(1/4) K^{1/2} L^{-3/4}] / [(1/2) K^{-1/2} L^{1/4}]$$

Calculating this yields:

$$MRTS_{L,K} = - (1/4) / (1/2) \ K^{1/2 + 1/2} \ L^{-3/4 - 1/4} = - (1/2) \ K^{1} L^{-1}$$

Thus, the MRTS simplifies to:

$$MRTS_{L,K} = - (1/2) \ (K / L)$$

This indicates that the rate at which the firm can substitute between capital and labor depends on their input ratio, scaled by a factor of 1/2.

Optimal Input Choice and Isocosts

1. Equating the MRTS to the Input Price Ratio

In equilibrium, the firm chooses K and L such that the MRTS equals the ratio of input prices:

$$MRTS_{L,K} = - w / r$$

Substituting the MRTS:

$$- (1/2) (K / L) = - w / r$$

Which simplifies to:

$$K / L = 2 (w / r)$$

This ratio guides the firm's choice of input proportions, balancing marginal productivity and input costs.

2. Solving for K and L

Using the input ratio, the firm can express one input in terms of the other:

- $L = K / [2(w/r)]$

Plugging this into the production constraint $Q_0 = K^{1/2} L^{1/4}$:

$$Q_0 = K^{1/2} (K / [2(w/r)])^{1/4}$$

Or:

$$Q_0 = K^{1/2} K^{1/4} [2(w/r)]^{-1/4} = K^{3/4} [2(w/r)]^{-1/4}$$

Solving for K:

$$K^{3/4} = Q_0 [2(w/r)]^{1/4}$$

$$K = [Q_0 (2(w/r))^{1/4}]^{4/3}$$

Similarly, L can be derived from the ratio:

$$L = K / [2(w/r)]$$

which completes the optimal input determination process.

Implications for Cost and Production

1. Total Cost Function

With the optimal inputs K and L, the total cost is:

$$C = rK + wL$$

Substituting K and L from above allows the firm to calculate minimal costs for producing a given output level Q_0.

2. Economies of Scale and Diminishing Returns

Since the production function exhibits diminishing returns to scale (exponents sum to less than 1), increasing inputs proportionally results in less than proportional increases in output. This affects the firm's decision-making regarding scale:

- Large-scale expansion may not proportionally increase output, raising average costs.

- Optimal input combinations balance the marginal gains against input costs to maximize profit.

Conclusion

The production function $Q = K^{1/2} L^{1/4}$ presents a nuanced view of how a firm can allocate its capital and labor to maximize efficiency and profitability. Its Cobb-Douglas structure provides clear insights into marginal products, input substitution, and cost minimization. The key takeaway is that the firm's input choices are driven by the interplay between marginal productivity, input prices, and the diminishing returns inherent in the production process. By understanding the properties of the production function and

Frequently Asked Questions

What is the production function given for the firm?

The production function is $Q = F(K, L) = K^{1/2} L^{1/4}$, where $K > 0$ and $L > 0$.

How does the firm decide the optimal combination of capital (K) and labor (L)?

The firm chooses inputs by maximizing profit or minimizing cost subject to producing a given level of output, often using techniques like setting marginal rate of technical substitution equal to input price ratios.

What is the marginal product of capital (MP_K) for this production function?

The marginal product of capital is $MP_K = \partial Q/\partial K = (1/2) K^{-1/2} L^{1/4}$.

What is the marginal product of labor (MP_L) for this production function?

The marginal product of labor is $MP_L = \partial Q/\partial L = (1/4) K^{1/2} L^{-3/4}$.

How can the firm determine the cost-minimizing input combination for a given output level?

The firm sets the ratio of marginal products equal to the ratio of input prices, i.e., $(MP_K / MP_L) = (w / r)$, where w is the wage rate and r is the rental rate of capital.

Is the production function exhibiting diminishing marginal returns?

Yes, as K and L increase, the marginal products decrease, indicating diminishing marginal returns.

How does the exponents in the production function relate to returns to scale?

Since the sum of exponents is $1/2 + 1/4 = 3/4 < 1$, the production function exhibits decreasing returns to scale.

What would happen to output Q if the firm doubles both inputs K and L?

If the firm doubles both inputs, output would increase by a factor of $2^{1/2 + 1/4} = 2^{3/4} \approx 1.68$, indicating less than doubling due to decreasing returns to scale.

Additional Resources

Production Function Analysis: K1/2 L1/4 and the Firm’s Input Choice

Understanding how firms decide on the optimal combination of inputs to maximize output or minimize costs is fundamental in microeconomics. The production function $Q = F(K, L) = K^{1/2} L^{1/4}$ offers a rich case to explore these concepts. In this detailed review, we will delve into the mathematical structure of this specific production function, interpret its properties, analyze the firm's input decisions, and discuss the economic implications of its form.

Introduction to the Production Function

A production function describes the relationship between inputs used by a firm—namely capital (K) and labor (L)—and the resulting output (Q). The given function:

$$Q = K^{1/2} L^{1/4}$$

is a Cobb-Douglas type, characterized by:

- Constant Returns to Scale (CRS), if the sum of exponents equals 1.
- Diminishing Marginal Returns in each input individually.
- Input Substitutability, allowing the firm to trade-off between capital and labor while maintaining output levels.

This function suggests that both inputs are productive but with diminishing returns, and the exponents reflect their relative importance.

Mathematical Properties of the Production Function

Exponents and Returns to Scale

- The exponents are 1/2 for K and 1/4 for L.
- Sum of exponents: $1/2 + 1/4 = 3/4$.
- Since $3/4 < 1$, the function exhibits decreasing returns to scale overall. When all inputs are scaled by a common factor $t > 1$, output increases by $t^{3/4}$, which is less than t.

Implication:
- Doubling inputs results in less than double output, indicating the firm experiences diminishing returns to scale.

Marginal Products (MP)

The marginal product of each input measures the additional output from an incremental increase in that input, holding the other constant.

- Marginal Product of Capital (MP_K):

$$
MP_K = \frac{\partial Q}{\partial K} = \frac{1}{2} K^{-1/2} L^{1/4}
$$

- Marginal Product of Labour (MP_L):

$$
MP_L = \frac{\partial Q}{\partial L} = \frac{1}{4} K^{1/2} L^{-3/4}
$$

Analysis:

- Both MP_K and MP_L are positive when $K, L > 0$.
- MP_K diminishes as K increases ($K^{-1/2}$ term).
- MP_L diminishes as L increases ($L^{-3/4}$ term).
- The marginal products highlight diminishing marginal returns, a typical feature of real-world production.

Elasticities of Substitution

The elasticity of substitution measures how easily the firm can substitute between K and L without changing output.

- For Cobb-Douglas functions:

$$
\sigma = 1
$$

which implies unitary elasticity of substitution; inputs can substitute for each other at a constant rate.

Cost Minimization and Input Choice

The firm's goal: Minimize total cost

$$
C = wL + rK
$$

subject to producing a given output Q_0.

Given:

$$
Q_0 = K^{1/2} L^{1/4}
$$

Method:

- Use the Lagrangian approach:

$$
\mathcal{L} = rK + wL + \lambda (Q_0 - K^{1/2} L^{1/4})
$$

- First-order conditions:

$$
\frac{\partial \mathcal{L}}{\partial K} = r - \lambda \frac{1}{2} K^{-1/2} L^{1/4} = 0
$$

$$
\frac{\partial \mathcal{L}}{\partial L} = w - \lambda \frac{1}{4} K^{1/2} L^{-3/4} = 0
$$

\]

- Solving for \(\lambda \):

\[
\lambda = 2 r K^{1/2} L^{-1/4}
\]

\[
\lambda = 4 w K^{-1/2} L^{3/4}
\]

- Equate these expressions:

\[
2 r K^{1/2} L^{-1/4} = 4 w K^{-1/2} L^{3/4}
\]

- Simplify:

\[
2 r K^{1/2} L^{-1/4} = 4 w K^{-1/2} L^{3/4}
\]

\[
\Rightarrow r K^{1/2} L^{-1/4} = 2 w K^{-1/2} L^{3/4}
\]

- Multiply both sides by \(K^{1/2} \) and \(L^{1/4} \):

\[
r K^{1/2} L^{-1/4} \times K^{1/2} L^{1/4} = 2 w K^{-1/2} L^{3/4} \times K^{1/2}
L^{1/4}
\]

which simplifies to:

\[
r K L^{0} = 2 w L^{1}
\]

\[
r K = 2 w L
\]

- Implication:

\[
\boxed{
\frac{K}{L} = \frac{2 w}{r}
}

\]

This ratio indicates the firm’s optimal capital-labor ratio depends on input prices.

Implications of Input Choice

Input Demand Functions

Using the ratio:

\[
K = \frac{2 w}{r} L
\]

and the production function:

\[
Q_0 = K^{1/2} L^{1/4}
\]

substitute \(K \):

\[
Q_0 = \left(\frac{2 w}{r} L \right)^{1/2} L^{1/4} = \left(\frac{2 w}{r} \right)^{1/2} L^{1/2} \times L^{1/4} = \left(\frac{2 w}{r} \right)^{1/2} L^{3/4}
\]

Solve for \(L \):

\[
L^{3/4} = \frac{Q_0}{\left(\frac{2 w}{r} \right)^{1/2}}
\]

\[
L = \left[\frac{Q_0}{\left(\frac{2 w}{r} \right)^{1/2}} \right]^{4/3}
\]

Similarly, \(K \):

\[
K = \frac{2 w}{r} L
\]

which provides explicit demand functions:

$$
L^ = \left(\frac{Q_0^{4/3}}{\left(\frac{2 w}{r} \right)^{2/3}} \right)
$$

$$
K^ = \frac{2 w}{r} L^
$$

Economic intuition:

- As input prices change, the optimal K/L ratio adjusts proportionally.
- Higher wage w increases the demand for capital relative to labor, and vice versa.

Cost Functions and Economies of Scale

The cost function for producing output Q_0:

$$
C(w, r, Q_0) = r K^ + w L^
$$

Substituting the demand functions:

$$
C = r \times \frac{2 w}{r} L^ + w L^ = 2 w L^ + w L^ = 3 w L^
$$

Since $L^$ is expressed in terms of Q_0, the cost function becomes:

$$
C(w, r, Q_0) = 3 w \left(\frac{Q_0}{ \left(\frac{2 w}{r} \right)^{1/2} } \right)^{4/3}
$$

which reflects the cost of producing a given level of output.

Economic Intuition and Implications

1. Diminishing Marginal Returns:

- Both inputs exhibit diminishing marginal productivity, which is typical for most production processes.
- The exponents less than 1 signify that increasing one input alone yields progressively

smaller increases in output.

2. Input Choice and Price Sensitivity:

- The optimal input ratio depends on input prices.
- When the wage \(w \) rises relative to the rental rate \(r \), the firm shifts towards more capital.
- Conversely, if \(r \) increases, the firm prefers more labor, assuming other factors constant.

3. Returns to Scale and Efficiency:

- Since

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