

A Hollow Spherical Conductor Of Radius 12 Cm Is charged To 6×10^{-6} C. Find The Electric Field Strength

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Field Strength

Understanding the electric field produced by conductors is fundamental in electrostatics, a branch of physics that examines electric charges at rest. When dealing with a hollow spherical conductor of radius 12 cm charged to 6×10^{-6} C, it is essential to analyze how the electric field behaves both inside and outside the sphere. This article provides a comprehensive explanation of how to determine the electric field strength in this scenario, exploring the principles of electrostatics, the properties of conductors, and the mathematical calculations involved.

Fundamentals of Electric Fields and Conductors

Before diving into the specific problem, let's review key concepts related to electric fields and conductors:

What Is an Electric Field?

- An electric field is a vector field around electric charges that represents the force exerted on other charges placed within the field.
- The electric field strength (E) at a point is defined as the force (F) experienced by a unit positive charge placed at that point, mathematically expressed as $E = F/q$.

Properties of Conductors in Electrostatics

- Conductors are materials that allow free movement of electric charges (usually electrons).
- When a conductor is charged, excess charge resides on its surface.
- The electric field inside a conducting material in electrostatic equilibrium is zero.
- The surface charge distribution on a conductor adjusts itself to maintain this equilibrium, resulting in unique electric field patterns.

Analyzing the Hollow Spherical Conductor

Given a hollow spherical conductor with a radius of 12 cm (0.12 meters) charged to a total charge of 6×10^{-6} Coulombs, we aim to find the electric field strength at various points:

Key Characteristics to Consider

- The total charge (Q) = 6×10^{-6} C.
- Radius (r) = 12 cm = 0.12 meters.
- The conductor is hollow, meaning it has no charge inside the shell, only on its surface.
- The behavior of the electric field depends on the position relative to the sphere's surface.

Calculating Electric Field Strength at Different Regions

The electric field around a hollow spherical conductor can be understood by considering three regions:

1. Inside the Hollow Sphere ($r < 0.12$ m)

- According to Gauss's law and the properties of conductors, the electric field inside a hollow conducting shell is zero.

- This is because charges reside on the surface, and the enclosed charge within the cavity is zero.
- Therefore, for any point inside the shell, $E = 0$.

2. On the Surface of the Sphere ($r = 0.12 \text{ m}$)

- At the surface, the electric field can be calculated using Coulomb's law or Gauss's law.
- The electric field just outside the surface of a charged conductor is given by:

$$E = \sigma / \epsilon_0$$

where:

- σ is the surface charge density.
- ϵ_0 is the permittivity of free space, approximately $8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$.

Alternatively, for a spherical charge, the electric field at the surface is:

$$E = (1 / (4\pi\epsilon_0)) (Q / r^2)$$

This formula provides the magnitude of the electric field at the surface.

3. Outside the Sphere ($r > 0.12 \text{ m}$)

- The electric field behaves like that of a point charge located at the center of the sphere.
- The electric field at a distance r from the center (outside the sphere) is:

$$E = (1 / (4\pi\epsilon_0)) (Q / r^2)$$

which diminishes with the square of the distance from the center.

Step-by-Step Calculation of Electric Field Strength

Let's now proceed with the actual calculations:

Constants Needed

- Permittivity of free space, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$
- Charge, $Q = 6 \times 10^{-6} \text{ C}$
- Radius, $r = 0.12 \text{ m}$

Calculating Electric Field at the Surface ($r = 0.12 \text{ m}$)

Using the formula:

$$E = (1 / (4\pi\epsilon_0)) (Q / r^2)$$

- First, compute the Coulomb constant, k :

$$k = 1 / (4\pi\epsilon_0) \approx 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$$

- Substitute known values:

$$E = 8.99 \times 10^9 (6 \times 10^{-6}) / (0.12)^2$$

- Calculate the denominator:

$$(0.12)^2 = 0.0144 \text{ m}^2$$

- Now compute E:

$$E = 8.99 \times 10^9 \cdot 6 \times 10^{-6} / 0.0144$$

- Simplify numerator:

$$8.99 \times 10^9 \cdot 6 \times 10^{-6} = (8.99 \times 6) \times 10^{9-6} = 53.94 \times 10^3 = 53,940$$

- Final calculation:

$$E = 53,940 / 0.0144 \approx 3,744,167 \text{ N/C}$$

Therefore, the electric field at the surface of the sphere is approximately $3.74 \times 10^6 \text{ N/C}$.

Summary of Electric Field Strengths

- Inside the sphere ($r < 0.12 \text{ m}$): $E = 0 \text{ N/C}$
- On the surface ($r = 0.12 \text{ m}$): $E \approx 3.74 \times 10^6 \text{ N/C}$
- Outside the sphere ($r > 0.12 \text{ m}$): E decreases with $1/r^2$, calculated with the same formula for respective r values

Additional Considerations and Practical Implications

Understanding the electric field distribution around a hollow spherical conductor has several practical applications:

Electrostatic Shielding

- Hollow conducting shells can be used to shield sensitive electronic equipment from external electric fields.
- The zero electric field inside the shell ensures no interference occurs within the enclosed space.

Designing Capacitors

- Spherical conductors are often used in capacitor design, where knowing the electric field helps optimize the device's performance and safety.

Safety Precautions in High-Voltage Environments

- Knowing the electric field strength allows engineers to assess potential hazards and implement safety measures.

Summary and Final Remarks

In conclusion, analyzing the electric field around a hollow spherical conductor involves understanding the fundamental principles of electrostatics, such as Coulomb's law and Gauss's law. For a sphere with a radius of 12 cm charged to 6×10^{-6} C, the electric field:

- Is zero inside the sphere, maintaining electrostatic equilibrium.

- Reaches approximately $3.74 \times 10^6 \text{ N/C}$ at the surface.
- Diminishes as $1/r^2$ outside the sphere, following the inverse-square law.

These calculations are crucial in various scientific and engineering applications, including designing shielding devices, capacitors, and understanding the behavior of charged conductors.

By mastering these concepts and calculations, students and professionals can better understand the behavior of electric fields around conductors, a foundational aspect of electromagnetism.

Frequently Asked Questions

What is the electric field strength outside a hollow spherical conductor of radius 12 cm charged to $6 \times 10^{-6} \text{ C}$?

The electric field outside the hollow spherical conductor at a distance $r > 12 \text{ cm}$ is given by $E = kQ/r^2$. For example, at 12 cm (the surface), $E = (9 \times 10^9 \text{ Nm}^2/\text{C}^2) \times (6 \times 10^{-6} \text{ C}) / (0.12 \text{ m})^2 = 37,500 \text{ N/C}$.

What is the electric field inside the hollow spherical conductor at any point within the cavity?

The electric field inside the hollow spherical conductor (for $r < 12 \text{ cm}$) is zero because the conductor shields the interior from external electric fields.

How does the radius of the spherical conductor affect the electric field strength at its surface?

The electric field strength at the surface is directly proportional to the charge and inversely proportional to the square of the radius, given by $E = kQ/r^2$. Increasing the radius decreases the surface electric field strength.

What is the significance of the conductor being hollow in calculating the electric field?

Since the conductor is hollow and charged, the electric field inside the conducting material is zero, and all charge resides on the surface, influencing the external electric field but not creating any field inside the cavity.

How is the electric field strength different at the center of the spherical shell compared to the surface?

The electric field at the center of the spherical shell is zero, while at the surface (radius 12 cm), it is approximately 37,500 N/C, as calculated by $E = kQ/r^2$.

Can the electric field strength be calculated at any point outside the spherical conductor? If so, how?

Yes, for points outside the sphere ($r > 12$ cm), the electric field strength can be calculated using $E = kQ/r^2$, treating the charge as concentrated at the center of the sphere.

Additional Resources

A Hollow Spherical Conductor Of Radius 12 Cm Is Charged To 6×10^{-6} C. Find The Electric Field Strength — this classic problem in electrostatics offers a rich opportunity to explore the principles of electric fields, conductors, and charge distribution. Whether you're a student preparing for exams or a physics enthusiast keen to deepen your understanding, breaking down this problem step by step reveals the elegant simplicity of electrostatics principles and their practical applications. In this guide, we will walk through the concepts involved, the fundamental formulas, and the detailed computations required to determine the electric field strength around a hollow spherical conductor.

Introduction to Conductors and Electric Fields

Before tackling the problem directly, it's crucial to understand the foundational concepts:

- Conductors: Materials like copper, aluminum, and silver that allow free movement of electrons.
- Electrostatic Equilibrium: When charges are at rest, they distribute themselves on the surface of a conductor in such a way that the electric field inside the conductor is zero.
- Hollow Spherical Conductors: Conductors with a cavity inside. The charge resides on the outer surface in electrostatic equilibrium, and the electric field inside the cavity is influenced by the charge distribution on the surface.

Core Concepts and Principles

1. Charge Distribution on Conductors

In electrostatics, charges placed on a conductor move freely until they reach an equilibrium where:

- All excess charges reside on the outer surface of the conductor.
- The electric field inside the conducting material is zero.
- The electric field outside the conductor behaves as if all the charge were concentrated at the center (for spherical conductors).

2. Electric Field Inside and Outside a Hollow Sphere

- Inside the cavity (region within the hollow sphere): The electric field depends on the charge distribution on the outer surface. If the sphere is isolated and only has surface charge, the field inside the cavity is zero unless there's a charge placed within the cavity.
- On the surface: The electric field just outside a charged conductor surface is given by:

$$E = \frac{\sigma}{\epsilon_0}$$

where σ is the surface charge density.

- Outside the sphere: The sphere behaves as if all the charge were concentrated at its center, and the electric field at a distance r from the center (for $r \geq R$) is:

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

Step-by-Step Solution

Step 1: Gather Data and Constants

- Radius of the spherical conductor, $(R = 12\text{ cm} = 0.12\text{ m})$
- Total charge, $(Q = 6 \times 10^{-6}\text{ C})$
- Vacuum permittivity, $(\epsilon_0 = 8.854 \times 10^{-12}\text{ F/m})$

Step 2: Identify What Is Being Asked

The problem requests the electric field strength. Typically, in such problems, the focus is on the electric field just outside the surface of the conductor, since the electric field inside the conducting material is zero and inside the cavity, if uncharged, the field is zero.

Step 3: Determine the Electric Field at the Surface

Key principle: For a spherical conductor of radius (R) carrying total charge (Q) , the electric field just outside the surface is given by:

$$E_{\text{surface}} = \frac{1}{4\pi \epsilon_0} \frac{Q}{R^2}$$

which simplifies to:

$$E_{\text{surface}} = \frac{k Q}{R^2}$$

where $(k = \frac{1}{4\pi \epsilon_0} \approx 9 \times 10^9, \text{Nm}^2/\text{C}^2)$.

Step 4: Calculation of Electric Field Strength at the Surface

Plugging in the known values:

$$E = \frac{9 \times 10^9 \times 6 \times 10^{-6}}{(0.12)^2}$$

Calculating step-by-step:

1. Square the radius:

$$\begin{aligned} & \backslash[\\ & (0.12)^2 = 0.0144\backslash, \text{m}^2 \\ & \backslash] \end{aligned}$$

2. Compute numerator:

$$\begin{aligned} & \backslash[\\ & 9 \times 10^9 \times 6 \times 10^{-6} = 9 \times 6 \times 10^{9-6} = 54 \times 10^3 = 54,000 \\ & \backslash] \end{aligned}$$

3. Compute the electric field:

$$\begin{aligned} & \backslash[\\ & E = \frac{54,000}{0.0144} \approx 3,750,000\backslash, \text{N/C} \\ & \backslash] \end{aligned}$$

or

$$\begin{aligned} & \backslash[\\ & \boxed{ \\ & E \approx 3.75 \times 10^6\backslash, \text{N/C} \\ & } \\ & \backslash] \end{aligned}$$

This is the electric field just outside the surface of the hollow spherical conductor.

Additional Considerations

Electric Field Inside the Conductor and Cavity

- Inside the conducting material: The electric field is zero.
- Inside the hollow cavity (assuming it's empty and uncharged): The electric field is zero, because no charge resides within the cavity unless specifically placed there.

Electric Field at Distances Greater Than (R)

- For points outside the sphere (at a distance $(r > R)$), the electric field behaves as if all charge were concentrated at the center:

$$E(r) = \frac{k Q}{r^2}$$

- For example, at a distance $(r = 24\text{ cm})$:

$$E = \frac{9 \times 10^9 \times 6 \times 10^{-6}}{(0.24)^2} = \frac{54,000}{0.0576} \approx 937,500\text{ N/C}$$

which is half of the field at the surface, following the inverse square law.

Practical Insights and Applications

Understanding how to compute the electric field around a charged conductor has numerous real-world applications:

- Designing shielding: Conductors are used to shield sensitive electronic components from external electric fields.

- Electrostatic precipitators: Harness electric fields to remove particulates from gases.
- Capacitors: Knowledge of electric field distribution helps optimize capacitor design.
- Electrostatic safety: Recognizing the magnitude of electric fields helps assess the risk of discharge or sparks.

Summary and Key Takeaways

- The electric field just outside a hollow spherical conductor is given by:

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2}$$

- For the given problem:

$$R = 0.12 \text{ m}, \quad Q = 6 \times 10^{-6} \text{ C}$$

the electric field strength at the surface is approximately 3.75 million N/C.

- Inside the conductor (including the cavity), the electric field is zero in electrostatic equilibrium.
- The behavior of electric fields around conductors is fundamental to understanding electrostatics and designing various electrical devices.

Final Remarks

This problem exemplifies the elegance of electrostatics: charges reside on the outer surface of conductors, and the electric field behaves predictably based on charge and geometry. Mastery of these principles enables engineers and physicists to manipulate electric fields for technological innovations, ensuring devices operate safely and efficiently.

Always remember: the distribution of charge and the resulting electric fields are governed by fundamental laws, and understanding these allows for precise control and application in real-world scenarios.

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