A Line Passes Through The Point (-8,5) And Has A Slope Of -3/4 .Write An Equation In Slope-intercept

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Understanding how to derive the equation of a line given a point and a slope is a fundamental skill in algebra and coordinate geometry. Whether you're a student preparing for exams or someone interested in graphing lines for practical applications, mastering this concept is essential. In this article, we will explore the process of finding the slope-intercept form of a line when given a specific point and slope, using the example of a line passing through (-8, 5) with a slope of -3/4. We will break down each step, discuss relevant concepts, and provide tips for solving similar problems efficiently.

Understanding the Slope-Intercept Form of a Line

What Is the Slope-Intercept Form?

The slope-intercept form of a straight line's equation is given by:

\[y = mx + b \]

where:
- m is the slope of the line,
- b is the y-intercept, the point where the line crosses the y-axis.

This form is particularly useful because it directly provides information about the slope and the y-intercept, allowing easy graphing and analysis of the line.

Why Is It Important?

Knowing the slope-intercept form enables you to:
- Quickly graph the line by plotting the y-intercept and using the slope.
- Understand the behavior of the line, such as whether it is increasing or decreasing.
- Write equations of lines when you have the slope and a point on the line.

Given Data and What It Means

Point (-8, 5)

This point indicates that when \(x = -8 \), then \(y = 5 \). It lies somewhere on the line we are trying to find.

Slope -3/4

The slope tells us how steep the line is and in which direction it goes:
- A negative slope indicates the line is decreasing; as \(x \) increases, \(y \) decreases.
- The magnitude \(\frac{3}{4} \) indicates that for every 4 units increase in \(x \), \(y \) decreases by 3 units.

Deriving the Equation Step-by-Step

Step 1: Recall the Point-Slope Formula

The point-slope form of a line's equation is:

\[y - y_1 = m(x - x_1) \]

where:
- \((x_1, y_1) \) is a known point on the line,
- \(m \) is the slope.

In our case:
- \((x_1, y_1) = (-8, 5) \),
- \(m = -\frac{3}{4} \).

Step 2: Substitute Known Values into the Point-Slope Formula

Plugging in, we get:

\[y - 5 = -\frac{3}{4}(x + 8) \]

Note: Since the point is \((-8, 5)\), the formula uses \((x - (-8)) = x + 8 \).

Step 3: Simplify and Convert to Slope-Intercept Form

Distribute the slope:

$$ y - 5 = -\frac{3}{4}x - \frac{3}{4} \times 8 $$

Calculate:

$$ -\frac{3}{4} \times 8 = -\frac{3}{4} \times 8 = -\frac{3 \times 8}{4} = -\frac{24}{4} = -6 $$

Now, write:

$$ y - 5 = -\frac{3}{4}x - 6 $$

Add 5 to both sides to solve for y:

$$ y = -\frac{3}{4}x - 6 + 5 $$

$$ y = -\frac{3}{4}x - 1 $$

This is the slope-intercept form of the line.

Final Equation in Slope-Intercept Form

The equation of the line passing through (-8, 5) with a slope of -3/4 is:

$$ y = -\frac{3}{4}x - 1 $$

This equation succinctly describes the line's behavior and can be used for graphing or further analysis.

Additional Tips and Practice

Tips for Writing Equations of Lines

- Always identify the given point and slope.
- Use the point-slope form as an intermediate step if needed.
- Convert to slope-intercept form by isolating y.
- Simplify fractions and constants carefully to avoid errors.

Practice Problems

1. Find the equation of a line passing through (2, -3) with a slope of 4.
2. Write the equation of a line passing through the point (0, 7) with a slope

of -2/3.
3. Given the point (-5, 4) and a slope of 1/2, find the slope-intercept form of the line.

Solutions:

1. $y - (-3) = 4(x - 2) \Rightarrow y + 3 = 4x - 8 \Rightarrow y = 4x - 11$
2. Since the point is on the y-axis (x=0), the y-intercept is 7, and because the slope is -2/3, the equation is:

$$y = -\frac{2}{3}x + 7$$

3. $y - 4 = \frac{1}{2}(x + 5) \Rightarrow y - 4 = \frac{1}{2}x + \frac{5}{2} \Rightarrow y = \frac{1}{2}x + \frac{5}{2} + 4 \Rightarrow y = \frac{1}{2}x + \frac{5}{2} + \frac{8}{2} = \frac{1}{2}x + \frac{13}{2}$

Conclusion

Understanding how to find the equation of a line given a point and a slope is a vital skill in algebra. By following the steps outlined—using the point-slope form and converting to slope-intercept form—you can accurately derive the equation of any line with known data. The specific example of a line passing through (-8, 5) with a slope of -3/4 illustrates this process clearly, resulting in the equation:

$$y = -\frac{3}{4}x - 1$$

Mastering this technique not only enhances your mathematical problem-solving skills but also prepares you for more complex topics in geometry, calculus, and real-world applications such as engineering and data analysis. Keep practicing with different points and slopes to become more comfortable and confident in deriving line equations efficiently.

Frequently Asked Questions

What is the slope of the line passing through the point (-8, 5)?

The slope of the line is -3/4, as given in the problem.

How do you write the equation of a line in slope-intercept form?

The slope-intercept form is y = mx + b, where m is the slope and b is the y-intercept.

How do you find the y-intercept (b) using the point (-8, 5) and slope -3/4?

Substitute the point and slope into $y = mx + b$: $5 = (-3/4)(-8) + b$. Simplify to find b.

What is the value of b in the equation?

Calculate: $5 = (-3/4)(-8) + b \rightarrow 5 = 6 + b \rightarrow b = 5 - 6 = -1$.

What is the equation of the line in slope-intercept form?

The equation is $y = -3/4\, x - 1$.

Can you verify that the point (-8, 5) lies on the line y = -3/4 x - 1?

Yes, substitute $x = -8$: $y = -3/4\, (-8) - 1 = 6 - 1 = 5$, which matches the y-coordinate.

Why is the slope-intercept form useful for this problem?

It clearly shows the slope and y-intercept, making it easy to write the equation once the slope and point are known.

Additional Resources

A Line Passes Through The Point (-8, 5) And Has A Slope Of -3/4: An In-Depth Exploration of Slope-Intercept Form and Its Applications

Understanding the fundamental principles of linear equations forms a cornerstone of algebra and analytic geometry. Among these principles, the slope-intercept form of a line offers a straightforward and intuitive way to describe a straight line's position and steepness. In this article, we delve into a specific problem: determining the equation of a line that passes through a given point, (-8, 5), with a specified slope of -3/4. We will explore the underlying concepts, the step-by-step derivation of the equation, and the broader significance of such calculations in mathematics and real-world contexts.

Foundations of the Slope-Intercept Form

What Is the Slope-Intercept Form?

The slope-intercept form is one of the most common ways to represent a straight line algebraically. It is expressed as:

$$ y = mx + b $$

where:

- y represents the dependent variable (the vertical coordinate).

- x represents the independent variable (the horizontal coordinate).

- m is the slope of the line, indicating its steepness and direction.

- b is the y-intercept, the point where the line crosses the y-axis.

This form is favored for its simplicity and the immediate visual interpretation it provides: knowing the slope and y-intercept allows plotting the line quickly and understanding its behavior.

The Significance of Slope and Y-Intercept

- Slope (m): Measures the rate of change of y with respect to x. A negative slope, such as $-3/4$, indicates the line descends as it moves from left to right, reflecting an inverse relationship between x and y.

- Y-Intercept (b): Indicates the value of y when x is zero. It anchors the line on the y-axis, serving as a starting point for graphing.

In our problem, the slope is given as $-3/4$, and the line passes through a specific point, $(-8, 5)$. Using these pieces of information, we can determine the specific equation that describes this line.

Deriving the Equation: Step-by-Step Approach

Step 1: Recall the Point-Slope Form

The point-slope form of a line is:

$$ y - y_1 = m(x - x_1) $$

where:

- (x_1, y_1) is a known point on the line.

- m is the slope.

Given the point $(-8, 5)$ and slope $-3/4$, the point-slope form becomes:

$$ y - 5 = -\frac{3}{4}(x + 8) $$

This form is advantageous because it directly incorporates a specific point and slope, laying the groundwork for conversion to slope-intercept form.

Step 2: Expand and Simplify to Slope-Intercept Form

Starting from the point-slope form:

$$ y - 5 = -\frac{3}{4}(x + 8) $$

Distribute the slope:

$$ y - 5 = -\frac{3}{4}x - \frac{3}{4} \times 8 $$

Calculate:

$$ -\frac{3}{4} \times 8 = -\frac{3 \times 8}{4} = -\frac{24}{4} = -6 $$

So, the equation becomes:

$$ y - 5 = -\frac{3}{4}x - 6 $$

Next, add 5 to both sides to solve for y:

$$ y = -\frac{3}{4}x - 6 + 5 $$

Simplify:

$$ y = -\frac{3}{4}x - 1 $$

This is the slope-intercept form of the line passing through (-8, 5) with a slope of -3/4.

Step 3: Final Equation and Interpretation

The equation in slope-intercept form is:

$$ \boxed{y = -\frac{3}{4}x - 1} $$

This equation fully characterizes the line, enabling graphing, analysis, and application.

Geometric and Analytical Insights

Graphical Interpretation

Plotting this line involves identifying the y-intercept -1 and using the slope $-3/4$:

- The y-intercept is at (0, -1). Plot this point on the coordinate plane.

- The slope $-3/4$ indicates that for every 4 units moved horizontally to the right, the line drops 3 units vertically.

- From (0, -1), moving 4 units right to $x=4$ yields:

$$ y = -\frac{3}{4} \times 4 - 1 = -3 - 1 = -4 $$

- Plotting (4, -4) confirms the slope's effect.

- Alternatively, starting from the point (-8, 5), applying the slope:

$$ \text{Rise} = -3, \quad \text{Run} = 4 $$

- Moving from (-8, 5) to (-4, 2), the line passes through these points, confirming consistency.

Analytical Significance

Knowing the full equation allows for various analyses:

- Intersection with other lines: Solving for simultaneous equations to find crossing points.

- Distance calculations: Determining perpendicular distances from points to the line.

- Transformations: Applying shifts, rotations, or reflections by manipulating the equation.

Broader Applications and Contexts

Real-World Implications

Linear equations are foundational in modeling real-world phenomena:

- Economics: Cost and revenue models often assume linear relationships.

- Physics: Motion with constant velocity can be represented by linear functions.

- Engineering: Structural analysis and signal processing rely on linear models.

In practical scenarios, the ability to derive an equation from a point and slope enables professionals to fit models to data, predict outcomes, and optimize systems.

Educational Significance

Mastering such derivations fosters:

- Problem-solving skills: Developing systematic approaches to algebraic problems.

- Conceptual understanding: Appreciating how geometric intuition aligns with algebraic expressions.

- Preparation for advanced topics: Serving as a foundation for calculus, differential equations, and analytical geometry.

Conclusion: The Power of Simple Equations

The process of deriving the equation of a line passing through a specific point with a known slope exemplifies the elegance and utility of algebra. The slope-intercept form, with its ease of use and interpretability, remains an essential tool in both academic and practical domains. Through this detailed exploration, we see how a straightforward problem encapsulates core mathematical concepts, demonstrating the interconnectedness of geometry, algebra, and real-world applications. Whether plotting a graph, analyzing data, or solving complex problems, understanding how to manipulate and interpret linear equations continues to be a valuable skill in the mathematical toolkit.

In summary:

- The line passing through (-8, 5) with a slope of -3/4 has the equation:

\[y = -\frac{3}{4}x - 1 \]

- Derived via the point-slope method and simplified to slope-intercept form.

- Offers insights into the line's behavior, graphical representation, and applications.

This comprehensive approach underscores the significance of mastering linear equations, fostering analytical thinking, and bridging abstract mathematics with practical realities.

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