A Positive Integer Is 40 More Than 29 Times Another. Their Product Is 10116 . Find The Two Integers.

Understanding the Problem: A Positive Integer and Its Relationship to Another

Restating the Problem

The problem presents a scenario involving two positive integers with a specific relationship. It states:

- One positive integer is 40 more than 29 times another integer.
- The product of these two integers is 10116.

Our goal is to determine the values of these two integers.

Breaking Down the Given Information

Let's define the integers to clearly understand and set up the problem:

- Let the second integer be x.
- The first integer then is described as 40 more than 29 times this second integer, so it can be expressed as $29x + 40$.

The information about their product gives us an equation:

$$
(29x + 40) \times x = 10116
$$

which simplifies to:

$$
x(29x + 40) = 10116
$$

Formulating the Mathematical Equation

Deriving the Quadratic Equation

Expanding the product:

$$
29x^2 + 40x = 10116
$$

Bring all terms to one side to set the equation to zero:

$$
29x^2 + 40x - 10116 = 0
$$

This is a quadratic equation in standard form:

$$
ax^2 + bx + c = 0
$$

where:

- $a = 29$
- $b = 40$
- $c = -10116$

Understanding the Quadratic Equation

To find the values of x, we will use the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

Before applying the formula, it's essential to compute the discriminant:

$$
D = b^2 - 4ac
$$

which will determine whether real solutions exist and help identify the potential integer solutions.

Calculating the Discriminant and Roots

Calculating the Discriminant

Plugging in the known values:

$$
D = 40^2 - 4 \times 29 \times (-10116)
$$

Calculate step-by-step:

- $40^2 = 1600$
- $4 \times 29 = 116$
- $116 \times (-10116) = -116 \times 10116$

Since the coefficient is negative, and the discriminant involves subtraction, note that:

$$
D = 1600 - 4 \times 29 \times (-10116) = 1600 + 116 \times 10116
$$

Now, compute 116×10116:

$$
116 \times 10116 = (100 + 16) \times 10116 = 100 \times 10116 + 16 \times 10116
$$

Calculate each:

- $100 \times 10116 = 1,011,600$
- $16 \times 10116 = 161,856$

Sum:

$$
1,011,600 + 161,856 = 1,173,456
$$

Therefore,

$$
D = 1600 + 1,173,456 = 1,175,056
$$

Evaluating the Square Root of the Discriminant

Next, find $\sqrt{D}$:

\[

\sqrt{1,175,056}
\]

Estimate:

- \(1,084^2 = 1,175,456 \)
- \(1,083^2 = 1,171,489 \)

Since \(1,175,056 \) is closer to \(1,175,456 \), the square root is approximately:

\[
\sqrt{1,175,056} \approx 1,084
\]

Check precisely:

\[
1,084^2 = (1,080 + 4)^2 = 1,080^2 + 2 \times 1,080 \times 4 + 4^2 = 1,166,400 + 8,640 + 16 = 1,175,056
\]

This confirms:

\[
\sqrt{D} = 1,084
\]

Calculating the Roots

Apply the quadratic formula:

\[
x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-40 \pm 1,084}{2 \times 29} = \frac{-40 \pm 1,084}{58}
\]

Calculate both solutions:

1. \(x = \frac{-40 + 1,084}{58} = \frac{1,044}{58} \)

2. \(x = \frac{-40 - 1,084}{58} = \frac{-1,124}{58} \)

Now, simplify each:

- \(\frac{1,044}{58} \):

Divide numerator and denominator by 2:

\[

\frac{522}{29}
\]

Since 29 is a prime number, check if 522 is divisible by 29:

- \(29 \times 18 = 522 \)

Thus:

\[
x = \frac{522}{29} = 18
\]

- \(\frac{-1,124}{58} \):

Divide numerator and denominator by 2:

\[
\frac{-562}{29}
\]

Check if 562 is divisible by 29:

- \(29 \times 19 = 551 \)

- \(29 \times 19 = 551 \), and \(562 - 551 = 11 \), so 562 is not divisible by 29, indicating \(x \) is not an integer here.

Since the problem states the integers are positive, and \(x \) must be positive, the valid solution is:

\[
x = 18
\]

Finding the Corresponding First Integer

Calculating the First Integer

Recall that:

\[
\text{First integer} = 29x + 40
\]

Substitute \(x = 18 \):

\[

29 \times 18 + 40
\]

Calculate:

- \(29 \times 18 = (30 - 1) \times 18 = 30 \times 18 - 1 \times 18 = 540 - 18 = 522 \)

Now add 40:

\[
522 + 40 = 562
\]

Verifying the Solution

Check the product:

\[
\text{First} \times \text{Second} = 562 \times 18
\]

Calculate:

- \(562 \times 18 = (560 + 2) \times 18 = 560 \times 18 + 2 \times 18 = (560 \times 18) + 36 \)

Calculate \(560 \times 18 \):

- \(560 \times 10 = 5600 \)
- \(560 \times 8 = 4480 \)

Sum:

\[
5600 + 4480 = 10,080
\]

Add 36:

\[
10,080 + 36 = 10,116
\]

which matches the given product, confirming our solution is correct.

Summary of the Solution

- The second integer is $\boxed{18}$.
- The first integer is $\boxed{562}$.

Conclusion and Final Remarks

Key Takeaways

- Setting up the problem with variables simplifies complex relationships.
- Deriving a quadratic equation from the problem statement is a common approach in algebraic problems involving relationships and products.
- Calculating the discriminant is vital to determine the nature of the roots.
- Checking divisibility ensures solutions are valid integers, especially when the problem specifies positive integers.

Additional Considerations

While the problem's conditions lead to a single valid positive integer solution, it's essential to verify solutions to ensure consistency, particularly when dealing with quadratic equations that may yield extraneous roots. In this case, the solution $(562, 18)$ satisfies all the conditions, including the product and the relationship between the integers.

This problem exemplifies the power of algebra in solving word problems involving relationships between numbers. By translating words into equations, solving quadratics, and verifying solutions, we demonstrate a systematic approach to mathematical problem-solving that can be applied to a wide range of similar problems.

Frequently Asked Questions

What is the main problem asking us to find in the given scenario?

It is asking us to find the two positive integers where one is 40 more than 29 times the other, and their product is 10116.

How can we represent the two integers algebraically?

Let the smaller integer be x; then, the larger integer can be expressed as 29x + 40.

What equation can be formed based on the given relationship and product?

The equation is $x (29x + 40) = 10116$.

How do we simplify the quadratic equation derived from the problem?

Expanding gives $29x^2 + 40x - 10116 = 0$, which can be solved using quadratic methods.

What is the discriminant of the quadratic equation, and how is it calculated?

Discriminant $D = (40)^2 - 4\ 29\ (-10116) = 1600 + 4\ 29\ 10116$.

Is the discriminant a perfect square, and why is that important?

Calculating the discriminant shows whether the quadratic has rational solutions; a perfect square discriminant indicates real solutions.

What are the steps to solve for x once the quadratic equation is established?

Use the quadratic formula $x = [-b \pm \sqrt{D}] / 2a$, where $a=29$, $b=40$, and D is the discriminant.

After finding the value of x, how do we verify the solution?

Substitute x back into the expressions to find the larger integer and check if the product equals 10116.

What are the two integers identified from the solution?

The smaller integer is x, and the larger is $29x + 40$; substituting the found value of x provides the specific integers.

Why is it important that both integers are positive in this problem?

The problem specifies positive integers, so solutions must be positive, which helps eliminate extraneous or invalid solutions during calculations.

Additional Resources

Mathematical Puzzle Analysis: Discovering the Two Integers

When exploring the fascinating world of algebraic problems, few challenges are as engaging as those involving relationships between two unknown integers. Today, we delve into a particularly intriguing problem: A positive integer is 40 more than 29 times another. Their product is 10,116. Find the two integers. This problem not only tests fundamental algebra skills but also offers insight into problem-solving strategies, variable formulation, and the application of quadratic equations.

In this comprehensive analysis, we'll walk through each step meticulously, breaking down the problem's components, translating words into algebraic expressions, and solving systematically. Our goal is to not only find the solution but also to understand the reasoning process, making this a valuable guide for students, educators, and math enthusiasts alike.

Understanding the Problem

Before diving into equations, it’s crucial to interpret the problem statement carefully. Let's dissect the core information:

- Positive integer: The problem references two integers, both greater than zero.
- Relationship between the integers: One integer is 40 more than 29 times the other.
- Product of the two integers: The multiplication of the two integers results in 10,116.

This problem involves two key relationships:

1. The algebraic relationship between the two integers.
2. The multiplication (product) of these integers.

Defining Variables

The first step in any algebraic word problem is to assign variables to the unknown quantities. Let's denote:

- Let x be the smaller positive integer.
- Since the problem states "A positive integer is 40 more than 29 times

another", and considering the typical flow of such statements, we interpret:

> "A positive integer" (say, A) is 40 more than 29 times another integer (say, B).

Therefore, we can set:

- B as the other integer.
- A as the positive integer described.

Accordingly, the relation is:

$$
A = 29B + 40
$$

Given that both are positive integers, B must be positive as well.

Formulating the Equation for the Product

The key piece of information: Their product is 10,116. In algebraic terms:

$$
A \times B = 10,116
$$

Substituting the expression for A:

$$
(29B + 40) \times B = 10,116
$$

This simplifies to:

$$
29B^2 + 40B = 10,116
$$

Rearranged into standard quadratic form:

$$
29B^2 + 40B - 10,116 = 0
$$

This quadratic equation is the focal point for finding B.

Solving the Quadratic Equation

Let's analyze the quadratic:

$$
29B^2 + 40B - 10,116 = 0
$$

The coefficients are:

- $a = 29$
- $b = 40$
- $c = -10,116$

Applying the quadratic formula:

$$
B = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

Calculating the discriminant:

$$
\Delta = b^2 - 4ac = 40^2 - 4 \times 29 \times (-10,116)
$$

$$
\Delta = 1600 + 4 \times 29 \times 10,116
$$

Since the negative sign in c becomes positive in the discriminant calculation, the discriminant simplifies to:

$$
\Delta = 1600 + 4 \times 29 \times 10,116
$$

Calculating step-by-step:

1. $4 \times 29 = 116$
2. $116 \times 10,116 = ?$

Let's compute $116 \times 10,116$:

- $116 \times 10,000 = 1,160,000$
- $116 \times 116 = 13,456$

Adding these:

\[

1,160,000 + 13,456 = 1,173,456
\]

Therefore:

\[
\Delta = 1600 + 1,173,456 = 1,175,056
\]

Next, evaluate \(\sqrt{\Delta}\):

\[
\sqrt{1,175,056}
\]

Estimating the square root:

- \(1,084^2 = 1,175,456 \)

Compare:

\[
1,175,456 \text{ (approximate)} \quad \text{vs} \quad 1,175,056
\]

Since \(1,084^2 \) overshoots slightly, try:

- \(1,083.5^2 \):

\[
(1,083.5)^2 = (1,084 - 0.5)^2 = 1,084^2 - 2 \times 1,084 \times 0.5 + 0.5^2
\]

Calculate:

- \(1,084^2 = 1,175,456 \)
- \(2 \times 1,084 \times 0.5 = 1,084 \)
- \(0.5^2 = 0.25 \)

Now:

\[
1,175,456 - 1,084 + 0.25 = 1,175,456 - 1,084 + 0.25
\]

\[
= 1,174,372 + 0.25 = 1,174,372.25
\]

This is less than 1,175,056; thus, the square root is approximately 1,084.2.

Using calculator approximation, the square root:

$$
\sqrt{1,175,056} \approx 1,084.6
$$

Now, apply the quadratic formula:

$$
B = \frac{-40 \pm 1,084.6}{2 \times 29} = \frac{-40 \pm 1,084.6}{58}
$$

Calculating the two solutions:

- Positive root:

$$
B = \frac{-40 + 1,084.6}{58} = \frac{1,044.6}{58} \approx 18.02
$$

- Negative root:

$$
B = \frac{-40 - 1,084.6}{58} = \frac{-1,124.6}{58} \approx -19.4
$$

Since B must be a positive integer, the feasible solution is approximately 18.

Note: The approximation suggests B = 18, but to confirm precisely, we need to verify whether B = 18 satisfies the original quadratic equation.

Verification of B = 18

Plug B=18 into the quadratic:

$$
29 \times 18^2 + 40 \times 18 - 10,116
$$

Calculate:

- $18^2 = 324$
- $29 \times 324 = ?$

Calculate:

$$
29 \times 324 = (30 -1) \times 324 = 30 \times 324 - 324 = 9,720 - 324 = 9,396
$$

Next:

$$
40 \times 18 = 720
$$

Sum:

$$
9,396 + 720 = 10,116
$$

Subtract:

$$
10,116 - 10,116 = 0
$$

Thus, B = 18 is an exact solution.

Finding the Corresponding A

Recall:

$$
A = 29B + 40
$$

Plug in B = 18:

$$
A = 29 \times 18 + 40 = 9,396 + 40 = 9,436
$$

Now, verify the product:

$$
A \times B = 9,436 \times 18
$$

Calculate:

- \(9,436 \times 10 = 94,360 \)
- \(9,436 \times 8 = 75,488 \)

Sum:

\[
94,360 + 75,488 = 169,848
\]

But this seems inconsistent with the earlier product of 10,116. Wait, this indicates a problem: our previous calculations suggest that A is 9,436, which yields a product much larger than 10,116.

Let's revisit the earlier step.

Reconciling the Discrepancy: Correcting the Approach

The key oversight lies in the assumption about which integer is larger and the initial interpretation.

Original wording:

> "A positive integer is 40 more than 29 times another."

It is possible that:

- The positive integer (say, A) equals 40 more than 29 times another integer (B).

- Or, alternatively, another integer (B) is 40 more than 29 times A.

Let's test the second interpretation:

\[
B = 29A + 40
\

A Positive Integer Is 40 More Than 29 Times Another Their Product Is 10116 Find The Two Integers

A Positive Integer Is 40 More Than 29 Times Another Their Product Is 10116 Find The Two Integers

Related Articles

- A Cylindrical Drill With Radius 3 Is Used To Bore A Hole Through The Center Of A Sphere Of Radius 5.
- 2 2.A Ii. Iii. Attempt ANY ONE Of Two Extracts Given. Iv. I. Complete The Sentence Appropriately. It
- (b). A Double-slit Diffraction Pattern Is Formed Using A 546.1 Nm Mercury Green Light. Each Slit Has

Back to Home