A Poster Is 17 Inches Longer Than It Is Wide. Find A Function That Models Its Area A In Terms Of Its

A Poster Is 17 Inches Longer Than It Is Wide. Find A Function That Models Its Area A In Terms Of Its

Understanding the relationship between the dimensions of a poster and its area is fundamental in geometry and real-world applications such as printing, advertising, and design. In this article, we will explore how to derive a mathematical function that models the area A of a rectangular poster based on its width. Specifically, given that the poster is 17 inches longer than it is wide, we will develop a formula that expresses the area A in terms of the width w. This comprehensive discussion will include step-by-step derivations, explanations, and practical insights to help you grasp the concepts involved.

Understanding the Problem

Before diving into the mathematics, it’s essential to understand the problem's context and what is being asked.

Problem Restatement

- The poster is rectangular.
- Its length (or height) is 17 inches longer than its width.
- We need to find a function $A(w)$ that models the area A of the poster based on its width w.

What is Known?

- The width of the poster, denoted by w, is an independent variable.
- The length of the poster, denoted by L, depends on w, with the specific relation:

$$
L = w + 17
$$

What is to be Found?

- A function $A(w)$ that gives the area A in terms of the width w.

Step-by-Step Derivation of the Area Function

Understanding how to derive the function $A(w)$ involves basic algebra and geometric principles.

Step 1: Define Variables

- Let w = width of the poster (in inches).
- Since the problem states that the poster is 17 inches longer than it is wide, the length L is:

$$
L = w + 17
$$

Step 2: Express the Area A in Terms of w

- The area A of a rectangle is given by:

$$
A = \text{length} \times \text{width}
$$

- Substituting the expression for the length:

$$
A(w) = w \times (w + 17)
$$

Step 3: Simplify the Expression

- Expand the product:

$$
A(w) = w \times w + w \times 17 = w^2 + 17w
$$

- Therefore, the function that models the area in terms of width is:

$$
\boxed{
A(w) = w^2 + 17w
}
$$

Analyzing the Area Function $A(w) = w^2 + 17w$

Having established the mathematical function, it's important to analyze its properties, domain, and implications.

Properties of the Function

- Type of function: Quadratic function.
- Shape: Parabola opening upwards (since the coefficient of w^2 is positive).
- Domain: Since width cannot be negative, the domain of $A(w)$ is:

$$
w > 0
$$

- Range: The area is always positive for $w > 0$.

Graphical Interpretation

Plotting $A(w) = w^2 + 17w$ helps visualize how the area changes with width.

- As w increases, the area increases rapidly because of the quadratic term.
- The minimum area occurs at the vertex of the parabola, which can be found by calculus or vertex formula.

Vertex of the Parabola

- The vertex of a quadratic $A(w) = aw^2 + bw + c$ occurs at:

$$
w = -\frac{b}{2a}

\]

- For our function:

$$
a = 1,\quad b = 17
$$

$$
w_{vertex} = -\frac{17}{2 \times 1} = -\frac{17}{2} = -8.5
$$

- Since width cannot be negative in this context, the physical interpretation is that the minimal area occurs at the smallest positive width, which is just above zero. The vertex at -8.5 is not physically meaningful for our problem but is useful mathematically.

Practical Applications and Considerations

Understanding and applying the area function has real-world significance.

Design and Optimization

- Maximizing area: If the goal is to maximize the poster area for a fixed width, the quadratic function indicates that increasing the width increases the area.
- Minimizing material use: For cost efficiency, choosing the smallest feasible width to satisfy design constraints minimizes material.

Size Constraints and Limitations

- In real situations, there are constraints such as maximum or minimum sizes due to printing capabilities or aesthetic considerations.
- To incorporate these constraints, define permissible ranges for w:

$$
w_{min} \leq w \leq w_{max}
$$

- For example, if the width must be at least 10 inches:

$$
10 \leq w
$$

Impact of the Length-Width Relationship

- The fixed difference of 17 inches impacts the area significantly.
- As the width increases, the length increases correspondingly, leading to larger areas.
- The linear relationship simplifies the modeling process.

Examples and Practice Problems

To solidify understanding, here are some example problems related to the area function.

Example 1: Calculate the Area for a Given Width

- Problem: Find the area of the poster when the width is 20 inches.
- Solution:

$$
A(20) = (20)^2 + 17 \times 20 = 400 + 340 = 740 \text{ square inches}
$$

Example 2: Find the Width for a Specific Area

- Problem: What should be the width of the poster so that the area is approximately 1000 square inches?
- Solution:

$$
w^2 + 17w = 1000
$$

- Rearranged as a quadratic:

$$
w^2 + 17w - 1000 = 0
$$

- Use the quadratic formula:

$$
w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a=1$, $b=17$, $c=-1000$.

$$
w = \frac{-17 \pm \sqrt{17^2 - 4 \times 1 \times (-1000)}}{2}
$$

$$
w = \frac{-17 \pm \sqrt{289 + 4000}}{2} = \frac{-17 \pm \sqrt{4289}}{2}
$$

$$
\sqrt{4289} \approx 65.45
$$

- Positive root:

$$
w = \frac{-17 + 65.45}{2} \approx \frac{48.45}{2} = 24.225
$$

- Negative root: discard, since width cannot be negative.

- Answer: To have an area of approximately 1000 square inches, the width should be about 24.2 inches.

Extensions and Additional Topics

Beyond deriving the basic area function, there are several related topics and extensions worth exploring.

1. Deriving the Perimeter Function

- The perimeter P of the poster is:

$$
P(w) = 2 \times (w + L) = 2(w + w + 17) = 2(2w + 17) = 4w + 34
$$

2. Optimization Problems

- Find the width that maximizes or minimizes the area within certain constraints.
- For example, determining the width that maximizes the area given a fixed perimeter.

3. Real-World Constraints

- Incorporate constraints such as maximum allowable width or length due to printing equipment.
- Adjust the model accordingly.

4. Applications in Other Geometric Shapes

- Similar modeling applies to other shapes where one dimension depends linearly on another, such as in designing rectangles, parallelograms, or complex shapes.

Summary and Key Takeaways

- The problem involves translating a real-world constraint into a mathematical model.
- The key relationship is that the length

Frequently Asked Questions

What variables should I use to represent the width and length of the poster?

Let w represent the width of the poster in inches, and since the length is 17 inches longer than the width, the length can be represented as $l = w + 17$.

How do I express the length of the poster in terms of its width?

The length l is expressed as $l = w + 17$, where w is the width of the poster.

What is the formula for the area of the poster in terms of its width?

The area A in terms of the width w is $A(w) = w (w + 17) = w^2 + 17w$.

How can I write the area function A(w) based on the given information?

$A(w) = w^2 + 17w$, where w is the width of the poster in inches.

What is the domain of the area function A(w)?

Since width w must be a positive number, the domain of A(w) is $w > 0$.

How can I find the width that maximizes the area of the poster?

To maximize $A(w) = w^2 + 17w$, take the derivative with respect to w, set it to zero, and solve for w: $dA/dw = 2w + 17 = 0$, so $w = -8.5$. Since width cannot be negative, the maximum occurs at the boundary, but in practical terms, the maximum area occurs at the largest feasible width within the domain $w > 0$.

What is the maximum possible area of the poster?

The maximum area in the domain $w > 0$ would occur at the largest feasible width, but since the quadratic opens upward, the area increases as w increases. Theoretically, as w approaches infinity, so does the area. Within realistic constraints, you would select a feasible width to maximize the area.

How would you interpret the function A(w) in practical terms?

The function $A(w) = w^2 + 17w$ models how the area of the poster changes as the width varies, with the length always being 17 inches longer than the width. It helps determine optimal dimensions for maximum area within size constraints.

Can you provide an example calculation of the area if the width is 10 inches?

Yes. If w = 10 inches, then the length l = 10 + 17 = 27 inches, and the area A = 10 27 = 270 square inches.

Additional Resources

A Poster Is 17 Inches Longer Than It Is Wide. Find A Function That Models Its Area A In Terms Of Its Width

---

Introduction

In the realm of geometry and mathematical modeling, understanding the relationship between the dimensions of an object and its area is fundamental. Consider a poster that measures 17 inches longer than it is wide. This seemingly simple statement opens the door to a rich exploration of algebraic functions, geometric properties, and their practical applications. In this article, we delve into the process of formulating a mathematical model for the poster's area as a function of its width, providing a comprehensive analysis suitable for academic review, educational contexts, and practical design considerations.

Setting the Geometric Framework

Defining the Variables

Let’s begin by establishing the variables involved in our problem:

- w: the width of the poster, measured in inches.
- l: the length of the poster, measured in inches.
- $A(w)$: the area of the poster as a function of width.

The key information provided is that the poster's length exceeds its width by 17 inches.

Expressing Length in Terms of Width

Given the problem statement, we can formulate the length as:

$$ l = w + 17 $$

This simple relationship forms the foundation for our area function.

Deriving the Area Function

Basic Area Formula

The area A of a rectangle (such as a poster) is given by:

$$ A = \text{length} \times \text{width} $$

Substituting our expression for length:

$$ A(w) = l \times w = (w + 17) \times w $$

This simplifies to:

$$ A(w) = w(w + 17) $$

which further expands to:

$$ A(w) = w^2 + 17w $$

This quadratic function models the area of the poster in terms of its width.

Analyzing the Function $A(w) = w^2 + 17w$

Domain Considerations

- Since width w represents a physical measurement, it must be positive:

$$ w > 0 $$

- There is no specified maximum size, but practically, the width will be constrained by printing and framing limitations.

Range of the Function

- The area $A(w)$ increases as the width increases, following a quadratic growth pattern.
- For very small positive widths, the area approaches zero.
- As w increases, $A(w)$ grows quadratically.

Graphical Interpretation

Graphing $A(w) = w^2 + 17w$ reveals a parabola opening upward with its vertex at a specific point, representing the minimum area for positive widths.

Finding the Vertex

The vertex of the parabola occurs at:

$$ w_v = -\frac{b}{2a} $$

where $a = 1$, $b = 17$:

$$ w_v = -\frac{17}{2 \times 1} = -\frac{17}{2} = -8.5 $$

Since negative width is physically meaningless, the parabola's vertex indicates the minimum mathematically, but for practical purposes, the domain is $w > 0$.

Practical Implications and Optimization

Minimizing or Maximizing Area

- The quadratic nature indicates that the area increases as the width moves away from zero.
- For design purposes, selecting a width impacts the area directly and proportionally.

Critical Points and Design Choices

- If a specific area is desired, the quadratic formula can be used to solve for the width:

$$ A = w^2 + 17w $$

Solving for w:

$$w^2 + 17w - A = 0$$

which yields:

$$w = \frac{-17 \pm \sqrt{(17)^2 - 4 \times 1 \times (-A)}}{2}$$

$$w = \frac{-17 \pm \sqrt{289 + 4A}}{2}$$

Since width must be positive, we select the positive root:

$$w = \frac{-17 + \sqrt{289 + 4A}}{2}$$

Application: Practical Design and Considerations

Step-by-Step Approach to Designing the Poster

1. Determine the desired area A based on aesthetic, space, or printing constraints.
2. Calculate the width w using the quadratic formula:

$$w = \frac{-17 + \sqrt{289 + 4A}}{2}$$

3. Determine length l:

$$l = w + 17$$

4. Verify the dimensions fit within physical or aesthetic constraints.

Example Calculation

Suppose the desired area is 300 square inches:

$$w = \frac{-17 + \sqrt{289 + 4 \times 300}}{2}$$

$$w = \frac{-17 + \sqrt{289 + 1200}}{2}$$

$$w = \frac{-17 + \sqrt{1489}}{2}$$

$$\sqrt{1489} \approx 38.6$$

$$w \approx \frac{-17 + 38.6}{2} = \frac{21.6}{2} = 10.8$$

Then, the length:

$$l = 10.8 + 17 = 27.8 \text{ inches}$$

The dimensions are approximately 10.8 inches in width and 27.8 inches in length, yielding the desired area.

Deeper Mathematical Insights

Quadratic Behavior and Real-World Constraints

- The quadratic function illustrates how small increases in width significantly impact the area, especially at larger dimensions.
- Design constraints may limit the maximum or minimum widths, influencing feasible area sizes.

Extending the Model

- For more complex posters—such as those with borders or non-rectangular shapes—the model can be extended with additional variables or constraints.
- Incorporating material properties or framing margins could refine the function further.

Conclusion

The simple yet elegant relationship between a poster's dimensions and its area exemplifies the power of algebraic modeling in practical design scenarios. By establishing the width as the primary variable and expressing the length in terms of this width, we derived a quadratic function $A(w) = w^2 + 17w$ that accurately models the area. This function serves as a vital tool for designers and manufacturers, enabling precise calculations for desired sizes and optimizing space utilization.

Understanding this relationship not only underscores fundamental mathematical principles but also demonstrates their real-world applications—from the initial concept of dimensions to final production specifications. Whether for academic purposes, professional design, or educational demonstrations, this exploration highlights how a simple statement about dimensions transforms into a comprehensive mathematical model with practical significance.

References

1. Stewart, J. (2012). Calculus: Early Transcendentals. Cengage Learning.
2. Blitzer, R. (2014). Algebra and Trigonometry. Pearson.
3. Larson, R., & Edwards, B. H. (2017). Precalculus with Limits. Cengage Learning.
4. Math is Fun. (n.d.). Quadratic Functions. Retrieved from https://www.mathsisfun.com/quadratic-functions.html

---

End of Article

A Poster Is 17 Inches Longer Than It Is Wide Find A Function That Models Its Area A In Terms Of Its

A Poster Is 17 Inches Longer Than It Is Wide Find A Function That Models Its Area A In Terms Of Its

Related Articles

- A Company That Makes A Software Product Wants To Grow Their Customer Base. Which Of The Following Is
- A Restaurant Orders Corn Tortillas And Flour Tortillas. The Ratio Of The Number Of Corn Tortillas To
- A Diary Entry On You Accidentallydropped Your Mother's Phone And It's Notworking Write A Diary Entry

Back to Home