# A Risk Averse Decision Maker With U(x) = (x Faces A Lottery L That Delivers $2500 Or $100 With Equal

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## Understanding the Concept of Risk Aversion and Utility Functions

To comprehend how a risk-averse decision maker approaches a lottery offering a 50/50 chance at either $2,500 or $100, it is essential to understand the foundational principles of risk aversion and utility theory.

### What Is Risk Aversion?

Risk aversion describes a preference for certainty over uncertainty with the same expected value. A risk-averse individual prefers a guaranteed amount over a gamble with a higher or equal expected payoff but with variability.

Key characteristics of risk-averse individuals:
- They dislike uncertainty.
- They prefer a certain outcome even if the expected value of a risky prospect is higher.
- Their utility functions are concave, reflecting diminishing marginal utility of wealth.

### The Utility Function U(x) = √x

In the context of this discussion, the utility function is specified as U(x) = √x. This mathematical form indicates how the decision maker values wealth:
- It is concave, illustrating risk aversion.
- As wealth increases, the additional utility gained decreases.

Implications of this utility function:
- The decision maker exhibits diminishing marginal utility.
- They are willing to accept certain lower payoffs rather than risky higher payoffs when the risk involves significant variability.

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# Analyzing the Lottery L

The lottery L provides two possible outcomes:
- A 50% chance to receive $2,500.
- A 50% chance to receive $100.

Expected monetary value (EMV) of lottery L:
- EMV = 0.5 $2,500 + 0.5 $100 = $1,250 + $50 = $1,300

Despite the high expected monetary value, risk-averse individuals evaluate lotteries through their utility rather than monetary expectation.

## Calculating the Expected Utility of Lottery L

Expected utility (EU) is calculated as:
- EU = 0.5 U($2,500) + 0.5 U($100)

Using $U(x) = \sqrt{x}$:
- U($2,500) = √2,500 ≈ 50
- U($100) = √100 = 10

Thus:
- EU = 0.5 50 + 0.5 10 = 25 + 5 = 30

## Determining the Utility of a Certain Amount

To decide whether the individual prefers the lottery or a certain amount, compare the utility of a certain amount x to the expected utility of the lottery.

If the individual considers accepting a certain amount x:
- Utility of $x = \sqrt{x}$

They will accept the lottery if:
- $\sqrt{x} \leq 30$

Solving for x:
- $\sqrt{x} = 30$
- $x = 30^2 = 900$

Interpretation:
- The individual is indifferent between participating in the lottery and receiving a certain amount of $900.
- Because their utility from $900 matches the expected utility of the gamble, they would accept any certain amount less than or equal to $900.
- They would reject any certain amount exceeding $900.

This demonstrates the risk-averse nature: even though the lottery has an expected monetary value of $1,300, the utility analysis shows they value it equivalent to $900 in certain terms.

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# Implications of Risk Aversion in Decision-Making

Understanding how risk-averse individuals evaluate lotteries has several practical implications.

## 1. Preference for Certainty

- Risk-averse individuals prefer guaranteed outcomes over uncertain ones with higher expected values.
- For example, they might prefer a certain $900 over a 50/50 gamble between $2,500 and $100, despite the gamble's higher expected monetary value.

## 2. Utility-Based Decision Making

- Decisions are driven by utility maximization rather than monetary gains.
- The concavity of the utility function causes the individual to weigh the utility of outcomes more heavily than their monetary value.

## 3. Risk Premium Concept

- The difference between the expected monetary value and the certain amount the individual is willing to accept is called the risk premium.
- In this case:
- Risk premium = EMV of lottery - certain amount willing to accept
- = $1,300 - $900 = $400

This means the individual would require at least $900 in certain wealth to be indifferent to the lottery that offers an expected $1,300.

## 4. Practical Applications

- Insurance: Risk-averse individuals prefer paying a premium to avoid potential large losses.
- Investment choices: They prefer less risky assets, even if the expected return is lower.
- Business decisions: Risk-averse managers may opt for safer projects with lower but guaranteed returns.

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# Graphical Representation of Utility and Risk Preferences

Visual tools help in understanding the risk preferences of the individual.

## Utility Curve for $U(x) = \sqrt{x}$

- The utility curve is concave, reflecting diminishing marginal utility.
- It shows that each additional dollar adds less utility than the previous one.

## Indifference Curves and Budget Constraints

- An indifference curve connects all combinations of wealth that yield the same utility.
- The individual’s risk preference is represented by the point where the expected utility of the lottery intersects the utility of a certain wealth level.

## Risk Premium Illustration

- The gap between the expected monetary value line and the certainty equivalent point illustrates the risk premium, highlighting the individual's risk aversion.

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# Conclusion: Decision-Making Strategy for a Risk-Averse Individual

In summary, a risk-averse decision maker with a utility function $U(x) = \sqrt{x}$ evaluates risky prospects by comparing their expected utility to the utility of certain outcomes. Despite the lottery L offering an expected monetary value of $1,300, the individual’s utility-based analysis indicates they are indifferent to a certain amount of approximately $900. This demonstrates their risk-averse nature: they prefer certainty and accept lower expected returns to avoid variability.

Key takeaways include:
- Risk aversion is characterized by concave utility functions.
- Utility maximization guides decision-making rather than monetary expectations alone.
- The concept of risk premium quantifies the individual's preference for certainty.
- Practical applications span insurance, investments, and business strategies.

Understanding these principles helps shape better financial decisions, policy-making, and risk management strategies tailored for risk-averse individuals. Recognizing the psychological and utility-based aspects of decision-making allows for more accurate predictions of behavior in uncertain environments.

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Meta Description:
Discover how a risk-averse decision maker evaluates a lottery with a 50/50 chance at $2,500 or $100 using a concave utility function. Learn about utility theory, risk premiums, and real-world

applications in investment and insurance.

# Frequently Asked Questions

## What is the utility function of a risk-averse decision maker in this scenario?

The decision maker's utility function is $U(x) = \sqrt{x}$, indicating risk aversion because the utility increases at a decreasing rate as wealth increases.

## What is the expected monetary value (EMV) of the lottery L?

The EMV of the lottery is (0.5 \$2500) + (0.5 \$100) = \$1250 + \$50 = \$1300.

## Would a risk-averse decision maker prefer the lottery L or a certain amount of $1300?

Typically, a risk-averse decision maker would prefer the certain amount of $1300 over the lottery because the expected utility of the lottery is less than the utility of a certain $1300.

## How do you determine the certainty equivalent for this lottery for a risk-averse individual?

The certainty equivalent is the guaranteed amount that provides the same utility as the expected utility of the lottery, calculated by solving U(CE) = expected utility of the lottery.

## What is the expected utility of the lottery L for the risk-averse decision maker?

Expected utility = $0.5 \sqrt{2500} + 0.5 \sqrt{100} = 0.5\ 50 + 0.5\ 10 = 25 + 5 = 30$.

## What is the certainty equivalent (CE) for the lottery L?

Solve $\sqrt{CE} = 30$, so $CE = 30^2$ = \$900. This is the amount the decision maker values equally to the lottery.

## Based on the utility and certainty equivalent, does the decision maker accept the lottery?

No, because the certain amount of $900 (the CE) is less than the expected monetary value of $1300, indicating they prefer the certain amount over the risky lottery.

### How does risk aversion affect the decision to accept or reject the lottery?

Risk aversion causes the decision maker to prefer certainty over the risky lottery if the certainty equivalent is less than the expected monetary value, leading them to reject the lottery in this case.

### What real-world scenarios illustrate a risk-averse decision maker facing similar choices?

Examples include choosing fixed investments over risky stocks, opting for insurance policies, or accepting guaranteed job salaries instead of commission-based earnings.

## Additional Resources

A Risk Averse Decision Maker With U(x) = √x Faces A Lottery L That Delivers \$2500 Or \$100 With Equal Probability

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Introduction

Decision-making under risk is a foundational topic in economics and behavioral sciences, shaping our understanding of how individuals evaluate uncertain prospects. Among the myriad of decision models, the concept of risk aversion — where individuals prefer certain outcomes over uncertain ones with equivalent or higher expected values — remains central. When a risk-averse decision maker (DM) encounters a simple binary lottery, their choice reveals deep insights into their utility preferences, attitudes toward risk, and the underlying calculus that guides their decisions.

This article delves into a scenario involving a risk-averse individual with a specific utility function, $U(x) = \sqrt{x}$, faced with a straightforward lottery $L$, which offers a 50/50 chance of receiving either \$2,500 or \$100. We explore the decision-making process, quantify the individual's subjective valuation of the lottery, and analyze the implications of their risk preferences. This examination provides a comprehensive understanding of how utility functions of the form $U(x) = \sqrt{x}$ influence choices in risky contexts, offering broader insights into risk attitudes and decision theories.

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Theoretical Foundations

Risk Aversion and Utility Functions

Risk aversion is characterized by a concave utility function, indicating diminishing marginal utility of wealth. In mathematical terms, a utility function $U(x)$ is concave if its second derivative $U''(x)$ is negative for all relevant $x$. This property ensures that individuals prefer certain outcomes over risky ones with the same expected monetary value, reflecting a preference to avoid uncertainty.

The utility function $U(x) = \sqrt{x}$ is a classic example of a concave utility, capturing moderate risk aversion. As wealth increases, the additional utility gained from incremental increases diminishes, aligning with observed behavioral patterns in risk-averse individuals.

### Expected Utility Theory

Expected Utility Theory (EUT) posits that rational decision-makers evaluate risky prospects by calculating the expected utility, $EU$, of each alternative:

$$
EU = \sum_{i} p_i U(x_i)
$$

where $p_i$ is the probability of outcome $x_i$.

In the context of our lottery $L$, which offers two outcomes with equal probability:

$$
EU(L) = 0.5 \times U(2500) + 0.5 \times U(100)
$$

The decision hinges on whether this expected utility exceeds the utility of a certain amount, often the sure payoff, or vice versa.

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## Scenario Specification

### The Lottery $L$

- Outcomes: \$2,500 or \$100
- Probabilities: 50% each
- Expected monetary value (EMV):

$$
EMV_L = 0.5 \times 2500 + 0.5 \times 100 = 1250 + 50 = \$1,300
$$

Despite its high expected monetary value, risk-averse individuals may not find the lottery attractive due to the variability and potential for low outcomes.

### The Decision Maker

- Utility function: $U(x) = \sqrt{x}$
- Initial wealth: $w$ (assumed to be sufficiently large to avoid boundary issues)
- Decision: To accept or reject the lottery based on subjective valuation.

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## Quantitative Analysis

### Computing the Expected Utility of the Lottery

Using the utility function:

$$
U(2500) = \sqrt{2500} = 50
$$
$$
U(100) = \sqrt{100} = 10
$$

Expected utility:

$$
EU_{L} = 0.5 \times 50 + 0.5 \times 10 = 25 + 5 = 30
$$

Indifference Point and Certainty Equivalent

To determine whether the decision maker would accept the lottery, we compare $EU_L$ with the utility of a certain equivalent, $CE$:

$$
U(CE) = EU_{L} = 30
$$

Solving for $CE$:

$$
\sqrt{CE} = 30 \Rightarrow CE = 30^2 = 900
$$

This certainty equivalent (CE) indicates the amount of guaranteed wealth the decision maker considers equally desirable as the lottery.

Interpretation: The individual perceives the risky lottery as equally attractive as a certain payment of \$900, despite its expected monetary value of \$1,300. This discrepancy underscores risk aversion: the subjective valuation (via utility) is lower than the expected monetary return.

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Implications of Risk Aversion

Utility vs. Expected Monetary Value

The divergence between expected monetary value (\$1,300) and certainty equivalent (\$900) exemplifies how risk-averse individuals undervalue risky prospects relative to their EMV. This phenomenon aligns with observed behaviors, such as refusing gambles with high expected gains due to variance.

Risk Premium

The risk premium is the difference between the EMV and the certainty equivalent:

$$
\text{Risk Premium} = EMV - CE = 1300 - 900 = \$400
$$

This signifies how much the individual would require in certain wealth to be indifferent to taking the risky lottery.

## Broader Context and Decision-Making Strategies

### Variance of the Lottery

The variance of the lottery outcomes quantifies the level of risk:

$$
\sigma^2 = 0.5 \times (2500 - 1300)^2 + 0.5 \times (100 - 1300)^2
$$

$$
= 0.5 \times (1200)^2 + 0.5 \times (-1200)^2 = 0.5 \times 1,440,000 + 0.5 \times 1,440,000 = 720,000
$$

The standard deviation:

$$
\sigma = \sqrt{720,000} \approx 848.53
$$

This high standard deviation underscores the risk inherent in the lottery, which risk-averse individuals tend to dislike.

### Utility of Wealth and Risk Preferences

The chosen utility function $U(x) = \sqrt{x}$ models moderate risk aversion, as evidenced by the non-zero risk premium. More risk-averse individuals might have utility functions with greater curvature (e.g., $U(x) = x^{\alpha}$ with $0 < \alpha < 1$), leading to higher risk premiums.

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## Practical Considerations and Limitations

### Assumptions

- The individual’s initial wealth $w$ is sufficiently large; the utility calculations focus on incremental changes.
- The utility function accurately captures the decision maker's preferences.
- The lottery outcomes are perceived objectively, with no subjective biases.

### Limitations of the Model

- Real-world decision makers often deviate from expected utility maximization, exhibiting behaviors like loss aversion or probability weighting.

- The utility function $U(x) = \sqrt{x}$ is a simplification; actual preferences may be more complex.
- The analysis assumes rationality and consistency over repeated choices.

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## Broader Implications and Applications

### Risk Management and Insurance

Individuals with utility functions similar to $U(x) = \sqrt{x}$ tend to purchase insurance or avoid risky gambles that could threaten their wealth, influencing market behaviors and product designs.

### Investment Decisions

Understanding risk preferences helps in portfolio allocation, determining asset mixes aligned with individual utility curves, and assessing acceptable levels of risk.

### Policy and Economic Modeling

Models incorporating risk-averse utility functions inform policy decisions, such as social safety nets, taxation, and welfare programs, ensuring they align with individuals’ risk attitudes.

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## Conclusion

The scenario of a risk-averse decision maker with $U(x) = \sqrt{x}$ facing a simple lottery exemplifies core principles of expected utility theory and behavioral economics. The calculation of the certainty equivalent (\$900) relative to the expected monetary value (\$1,300) highlights how risk aversion influences subjective valuation, leading individuals to prefer certainty over risky prospects with higher expected returns.

This analysis underscores the importance of utility functions in modeling real-world decision-making under risk. It also illustrates that risk attitudes are not solely about maximizing expected monetary gains but involve complex preferences shaped by diminishing marginal utility. Recognizing these nuances enhances our understanding of economic behavior, from individual choices to broader market phenomena.

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This comprehensive review underscores the nuanced interplay between utility, risk, and decision-making, providing a detailed framework for understanding how risk-averse individuals evaluate lotteries with varying payoffs.

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