. For The Function Y= Sin 2 X1.55sinx+cos 2 X , Calculate The Value Of Y For The Following Values Of

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Understanding the behavior and calculation of trigonometric functions is essential in various fields such as mathematics, physics, engineering, and computer science. In this comprehensive guide, we will explore the function:

$$ Y = \sin 2X \times 1.55 \sin x + \cos 2X $$

and learn how to evaluate its value for different given values of X. We will break down the function, analyze its components, and provide step-by-step methods to accurately compute Y. Whether you're a student preparing for exams or a professional applying this function in real-world scenarios, this article offers detailed insights to enhance your understanding.

Understanding the Function Components

Before diving into calculations, it's crucial to understand the individual components of the function:

$$ Y = \sin 2X \times 1.55 \sin x + \cos 2X $$

This function involves:

- Double angle sine and cosine functions: $\sin 2X$ and $\cos 2X$
- The sine function of x: $\sin x$
- A multiplicative constant: 1.55

Let's analyze each part:

1. The Double Angle Formulas

The double angle formulas for sine and cosine are fundamental in simplifying and understanding trigonometric functions:

- $\sin 2X = 2 \sin X \cos X$
- $\cos 2X = \cos^2 X - \sin^2 X$

Knowing these formulas helps in transforming the function for easier computation or for deriving alternative expressions.

2. The Role of the Constant 1.55

The constant multiplier 1.55 scales the value of $\sin 2X \times \sin x$. Recognizing constants in functions helps in understanding the amplitude and overall behavior of the function.

3. The Overall Structure

The function combines two terms:

- The product $\sin 2X \times 1.55 \sin x$
- The cosine term $\cos 2X$

The sum of these two terms provides the value of Y for a given X.

Step-by-Step Calculation Approach

To evaluate Y accurately, follow these steps:

Step 1: Identify the value of X

You will be given specific values of X. Ensure these are in degrees or radians as per the problem statement. Convert degrees to radians if necessary, since most mathematical functions in programming and calculators operate in radians:

$$ \text{Radians} = \text{Degrees} \times \frac{\pi}{180} $$

Step 2: Calculate $\sin 2X$ and $\cos 2X$

Using the double angle formulas or directly plugging into a calculator:

- $\sin 2X$
- $\cos 2X$

Step 3: Calculate $\sin x$

Compute the sine of x.

Step 4: Compute the product $\sin 2X \times 1.55 \sin x$

Multiply $\sin 2X$ by 1.55 and then by $\sin x$.

Step 5: Add $\cos 2X$ to the product

Sum the result from step 4 with $\cos 2X$ to get the value of Y:

$$ Y = (\sin 2X \times 1.55 \sin x) + \cos 2X $$

Calculating Y for Specific Values of X

Let's now apply the above method to evaluate Y for given X values. Assume the values are provided in degrees; convert to radians before calculation.

Example 1: $X = 30^\circ$

Step 1: Convert to radians:

$$ X = 30^\circ = 30 \times \frac{\pi}{180} = \frac{\pi}{6} \approx 0.5236 \text{ radians} $$

Step 2: Calculate $\sin 2X$ and $\cos 2X$:

- $2X = 2 \times 0.5236 = 1.0472 \text{ radians}$

- $\sin 2X = \sin 1.0472 \approx 0.8660$

- $\cos 2X = \cos 1.0472 \approx 0.5$

Step 3: Compute $\sin x$:

- $\sin 30^\circ = \sin 0.5236 \approx 0.5$

Step 4: Calculate the product:

- $1.55 \times \sin 2X \times \sin x = 1.55 \times 0.8660 \times 0.5 \approx 1.55 \times 0.4330 \approx 0.6721$

Step 5: Sum with $\cos 2X$:

- $Y = 0.6721 + 0.5 = 1.1721$

Result:

$$ Y \approx 1.1721 $$

Example 2: $X = 45^\circ$

Repeat the process:

- Convert to radians:

$$ 45^\circ = \frac{\pi}{4} \approx 0.7854 \text{ radians} $$

- $2X = 2 \times 0.7854 = 1.5708$

- $\sin 2X = \sin 1.5708 \approx 1$

- $\cos 2X = \cos 1.5708 \approx 0$

- $\sin 45^\circ = \sin 0.7854 \approx 0.7071$

- Product:

$$ 1.55 \times 1 \times 0.7071 \approx 1.55 \times 0.7071 \approx 1.0967 $$

- Final Y:

$$ Y = 1.0967 + 0 \approx 1.0967 $$

Example 3: $X = 60^\circ$

- Convert to radians:

$$ 60^\circ = \frac{\pi}{3} \approx 1.0472 \text{ radians} $$

- $2X = 2 \times 1.0472 = 2.0944$

- $\sin 2X = \sin 2.0944 \approx 0.8660$

- $\cos 2X = \cos 2.0944 \approx -0.5$

- $\sin 60^\circ = \sin 1.0472 \approx 0.8660$

- Product:

$$ 1.55 \times 0.8660 \times 0.8660 \approx 1.55 \times 0.75 \approx 1.1625 $$

- Final Y:

$$ Y = 1.1625 - 0.5 = 0.6625 $$

Additional Tips for Accurate Calculations

- Always verify whether your calculator or computational tool is set to degrees or radians.
- Use high-precision calculations for more accurate results, especially for small differences.
- Remember the periodic nature of sine and cosine functions; values repeat every 2π radians.
- Be cautious with sign conventions, particularly when dealing with negative angles or quadrants.

Applications of the Function $Y= \sin 2X \times 1.55 \sin x + \cos 2X$

This function, combining multiple trigonometric elements, finds applications in various fields:

- Signal Processing: Modeling wave interactions where phase shifts and amplitudes are involved.
- Physics: Analyzing oscillations and wave interference patterns.
- Engineering: Designing systems with periodic behaviors, such as filters or control systems.
- Mathematics Education: Teaching concepts of double angles, product-to-sum formulas, and function evaluation.

Conclusion

Calculating the value of \(Y\) for the given function involves understanding the properties of sine and cosine functions, converting angles to the correct units, and applying the appropriate formulas. By following a systematic approach—identifying angle measures, calculating individual trigonometric components, and combining results—you can accurately evaluate \(Y\) for any specified \(X\).

Mastery of these techniques enhances problem-solving skills in trigonometry and prepares you for tackling more complex mathematical challenges. Remember to practice with various values and explore the behavior of the function across different intervals to gain deeper insights into its characteristics.

Keywords: trigonometric functions, sine, cosine, double angle formulas, function evaluation, mathematical calculations, radians, degrees, signal processing, physics, engineering

Frequently Asked Questions

What is the general form of the function $Y = \sin 2x + 1.55 \sin x + \cos 2x$?

The function combines multiple trigonometric terms: double angle sine and cosine, as well as a scaled sine of x, which can be simplified or evaluated for specific x-values.

How do I evaluate Y when $x = 0$ in the function $Y = \sin 2x + 1.55 \sin x + \cos 2x$?

Substitute $x = 0$: $\sin 0 = 0$, $\cos 0 = 1$, so $Y = \sin 0 + 1.55 \sin 0 + \cos 0 = 0 + 0 + 1 = 1$.

What is the value of Y when $x = \pi/4$ in the function $Y = \sin 2x + 1.55 \sin x + \cos 2x$?

At $x = \pi/4$: $\sin 2(\pi/4) = \sin(\pi/2) = 1$, $\sin(\pi/4) \approx 0.7071$, $\cos 2(\pi/4) = \cos(\pi/2) = 0$, so $Y \approx 1 + 1.55\ 0.7071 + 0 \approx 1 + 1.096 + 0 \approx 2.096$.

How can I simplify the function $Y = \sin 2x + 1.55 \sin x + \cos 2x$ for easier calculation?

Use double angle identities: $\sin 2x = 2 \sin x \cos x$ and $\cos 2x = \cos^2 x - \sin^2 x$, which can help rewrite Y in terms of sin x and cos x for easier evaluation.

What is the significance of the coefficients in the function Y = sin 2x + 1.55 sin x + cos 2x?

The coefficient 1.55 scales the contribution of sin x, affecting the amplitude of the function's variation, while the sine and cosine terms determine the periodic behavior.

Can the function Y = sin 2x + 1.55 sin x + cos 2x be optimized to find maximum or minimum values?

Yes, by expressing Y in terms of a single trigonometric function or using calculus techniques, you can find critical points where Y attains maximum or minimum values.

How do I calculate Y for x = π/2 in the function Y = sin 2x + 1.55 sin x + cos 2x?

At x = π/2: sin 2(π/2) = sin π = 0, sin(π/2) = 1, cos 2(π/2) = cos π = -1. So, Y = 0 + 1.55 1 + (-1) = 0 + 1.55 - 1 = 0.55.

Are there specific x-values where Y reaches its maximum in the given function?

Maximum points can be found by analyzing the derivative of Y or by rewriting the function in a simplified form to identify peaks, typically where the combined sine and cosine terms constructively interfere.

Additional Resources

Mathematical Function Analysis and Calculation: Y = sin(2x) - 1.55 sin x + cos(2x)

When it comes to understanding complex mathematical functions, especially those involving multiple trigonometric components, it's essential to break down each element to comprehend their behavior and how they interact. The function in question—Y = sin(2x) - 1.55 sin x + cos(2x)—serves as an excellent example of a composite trigonometric function that combines different angles and functions, offering rich insights into its behavior across various inputs.

In this article, we will conduct a comprehensive analysis of this function, explaining each part in detail, calculating its value for specified inputs, and providing practical interpretations. Whether you're a student, educator, or enthusiast, this deep dive aims to clarify the intricacies of this function and equip you with the tools to analyze similar mathematical expressions effectively.

Understanding the Components of the Function

Before jumping into calculations, it's vital to understand the building blocks of the function:

$Y = \sin(2x) - 1.55 \sin x + \cos(2x)$

Key components:

- $\sin(2x)$: The sine of double the angle, which exhibits periodicity with a period of π (pi).
- $\sin x$: The sine of the angle x, with a period of 2π.
- $\cos(2x)$: The cosine of double the angle, sharing the same period as $\sin(2x)$.

These components are interconnected through their arguments and coefficients, producing a complex waveform that varies with x.

Mathematical Identities and Simplifications

To analyze the function more deeply, we can leverage fundamental trigonometric identities:

Double-Angle Formulas:
- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x$

Using these, we could rewrite the function as:

$Y = 2 \sin x \cos x - 1.55 \sin x + (\cos^2 x - \sin^2 x)$

This form helps us see the relationships between the components and simplifies calculations for specific x-values.

Alternative Expression:
$Y = 2 \sin x \cos x + \cos^2 x - \sin^2 x - 1.55 \sin x$

This presentation highlights the mixture of linear and quadratic terms of sine and cosine functions, offering insights into the function's oscillation and amplitude.

Calculating Y for Specific Values of x

To make this analysis practical, let's compute Y for selected x-values. For clarity, we will consider x in radians, as is standard in mathematical functions.

Given values:

- $x = 0$
- $x = \pi/4$ (~0.7854 radians)
- $x = \pi/2$ (~1.5708 radians)
- $x = \pi$ (~3.1416 radians)
- $x = 3\pi/2$ (~4.7124 radians)
- $x = 2\pi$ (~6.2832 radians)

1. When $x = 0$

- $\sin(0) = 0$
- $\cos(0) = 1$
- $\sin(2\times0) = 0$
- $\cos(2\times0) = 1$

Calculations:

$Y = \sin(0) - 1.55\sin(0) + \cos(0)$

$Y = 0 - 0 + 1 = 1$

2. When $x = \pi/4$ (~0.7854)

- $\sin(\pi/4) = \sqrt{2}/2 \approx 0.7071$
- $\cos(\pi/4) = \sqrt{2}/2 \approx 0.7071$
- $\sin(2\times\pi/4) = \sin(\pi/2) = 1$
- $\cos(2\times\pi/4) = \cos(\pi/2) = 0$

Calculations:

$Y = 1 - 1.55\times0.7071 + 0$

$Y \approx 1 - 1.55\times0.7071 + 0$

$Y \approx 1 - 1.095 + 0 = -0.095$

3. When $x = \pi/2$ (~1.5708)

- $\sin(\pi/2) = 1$
- $\cos(\pi/2) = 0$
- $\sin(2\times\pi/2) = \sin(\pi) = 0$
- $\cos(2\times\pi/2) = \cos(\pi) = -1$

Calculations:

$Y = 0 - 1.55\times1 + (-1)$

$Y = 0 - 1.55 - 1 = -2.55$

4. When $x = \pi$ (~3.1416)

- $\sin(\pi) = 0$
- $\cos(\pi) = -1$
- $\sin(2\pi) = 0$
- $\cos(2\pi) = 1$

Calculations:

$Y = 0 - 1.55\times0 + 1 = 1$

5. When $x = 3\pi/2$ (~4.7124)

- $\sin(3\pi/2) = -1$
- $\cos(3\pi/2) = 0$
- $\sin(2\times3\pi/2) = \sin(3\pi) = 0$
- $\cos(2\times3\pi/2) = \cos(3\pi) = -1$

Calculations:

$Y = 0 - 1.55\times(-1) + (-1)$

$Y = 0 + 1.55 - 1 = 0.55$

6. When $x = 2\pi$ (~6.2832)

- $\sin(2\pi) = 0$
- $\cos(2\pi) = 1$
- $\sin(4\pi) = 0$
- $\cos(4\pi) = 1$

Calculations:

$Y = 0 - 1.55\times0 + 1 = 1$

Analysis of Calculated Values and Function Behavior

Key observations from the calculations:

- The value of Y oscillates between positive and negative, reflecting the periodic nature of the sine and cosine functions.
- Maximum value observed at $x = 0$ and $x = 2\pi$, where $Y = 1$.
- Minimum value occurs at $x = \pi/2$, with $Y \approx -2.55$.
- The function exhibits symmetry around specific points, consistent with the properties of sine and cosine.

Behavioral insights:

- The dominant negative dip at $x = \pi/2$ suggests that the $-1.55 \sin x$ term significantly influences the function's minima.
- The peaks at $x = 0$ and 2π demonstrate the influence of the cosine terms, which are positive at these points.
- The oscillation amplitude appears to be around 3 units, considering the negative minimum and positive maximum.

Practical Implications and Applications

Understanding such a function is not merely an academic exercise. Functions combining multiple trigonometric components are prevalent in various fields:

Physics
- Modeling wave interference patterns
- Analyzing oscillations in mechanical systems

Engineering
- Signal processing, where combining sine and cosine functions models complex waveforms
- Control systems involving sinusoidal inputs

Computer Graphics
- Generating periodic textures or animations

Signal Analysis
- Fourier analysis often involves decomposing signals into sine and cosine components similar to this function

Educational Context
- Serves as an excellent example for students learning about double-angle identities and the behavior of combined trigonometric functions.

Conclusion and Final Remarks

The function $Y = \sin(2x) - 1.55 \sin x + \cos(2x)$ embodies the intricate interplay of fundamental trigonometric functions. Through detailed analysis, application of identities, and explicit calculations at key points, we've unraveled its behavior across different inputs.

Its oscillatory nature, characterized by peaks and troughs, mirrors the fundamental properties of sine and cosine waves, yet the coefficients and combined angles produce a unique waveform. Recognizing these patterns empowers practitioners and students to interpret similar complex functions effectively.

In practical applications, mastering the analysis of such functions enhances one's ability to model real-world phenomena, optimize systems, and deepen understanding of periodic behaviors inherent in nature and technology.

Whether you're solving specific problems or exploring the theoretical underpinnings of waveforms, this comprehensive approach provides a solid foundation for further exploration in trigonometry and its myriad applications.

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