The Height Of An Equilateral Triangle Is The Square Root Of 12. Find The Perimeter Of The Triangle.

The Height Of An Equilateral Triangle Is The Square Root Of 12. Find The Perimeter Of The Triangle.

Understanding the properties of equilateral triangles is fundamental in geometry. When given specific measures such as the height, it becomes possible to determine other critical dimensions, including the side length and the perimeter. In this comprehensive guide, we will explore how to find the perimeter of an equilateral triangle when its height is known to be the square root of 12. We will break down the problem step-by-step, delve into the mathematical principles involved, and provide detailed explanations to enhance your understanding.

Understanding Equilateral Triangles

What Is an Equilateral Triangle?

An equilateral triangle is a triangle where all three sides are of equal length. Consequently, all three internal angles are also equal, each measuring 60 degrees. Because of its symmetry and equal sides, the equilateral triangle exhibits unique properties that make calculations straightforward once one dimension is known.

Key Properties of Equilateral Triangles

- All sides are equal in length.
- All interior angles measure 60 degrees.
- The height, median, angle bisector, and perpendicular bisector from a vertex coincide, meaning they are the same line.
- The height divides the triangle into two 30-60-90 right triangles.

Understanding these properties is essential because they form the basis for calculating other dimensions when a specific measurement, like height, is given.

Given Data and Problem Restatement

The problem states:

- Height (h) of the equilateral triangle = $\sqrt{12}$
- Find the perimeter (P) of the triangle

Our goal is to determine the perimeter based on the given height.

Before proceeding, it's helpful to simplify the given height:

Simplifying the Height

$$
\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2 \sqrt{3}
$$

Thus,

$$
h = 2 \sqrt{3}
$$

Mathematical Foundations for Calculating Side Length and Perimeter

To find the perimeter, we need to determine the length of one side, denoted as s.

Since the height divides the equilateral triangle into two 30-60-90 right triangles, we can use properties of these special triangles.

Properties of 30-60-90 Right Triangles

In a 30-60-90 triangle:

- The side opposite the 30° angle is half the hypotenuse.
- The side opposite the 60° angle is $\sqrt{3}$ times the side opposite the 30° angle.
- The hypotenuse is the length of the side of the original equilateral triangle.

Applying these to the divided triangle:

- The hypotenuse corresponds to the side of the equilateral triangle (s)
- The longer leg (adjacent to 60°) corresponds to the height (h)
- The shorter leg (adjacent to 30°) corresponds to half the side length ($s/2$)

Calculating the Side Length of the Equilateral Triangle

Step 1: Express Height in Terms of Side Length

In the 30-60-90 triangle formed:

$$
h = \text{longer leg} = \sqrt{3} \times \text{shorter leg} = \sqrt{3} \times \frac{s}{2}
$$

Rearranged:

$$
h = \frac{\sqrt{3}}{2} s
$$

Step 2: Solve for Side Length s

Given $h = 2 \sqrt{3}$:

$$
2 \sqrt{3} = \frac{\sqrt{3}}{2} s
$$

Multiply both sides by 2 to clear denominator:

$$
2 \times 2 \sqrt{3} = \sqrt{3} \times s
$$

$$
4 \sqrt{3} = \sqrt{3} \times s
$$

Divide both sides by $\sqrt{3}$:

$$
\frac{4 \sqrt{3}}{\sqrt{3}} = s
$$

$$
s = 4
$$

Therefore, the side length of the equilateral triangle is 4 units.

Calculating the Perimeter of the Triangle

Since all sides are equal:

$$
P = 3 \times s = 3 \times 4 = 12
$$

The perimeter of the triangle is 12 units.

Summary of the Calculation Process

Here's a quick overview of the steps involved:

1. Simplify the given height: $\sqrt{12} = 2 \sqrt{3}$.
2. Identify the relationship between height and side length in an equilateral triangle: $h = \frac{\sqrt{3}}{2} s$.
3. Solve for side length: $s = \frac{2h}{\sqrt{3}}$.
4. Substitute the height value and compute: $s = \frac{2 \times 2 \sqrt{3}}{\sqrt{3}} = 4$.
5. Calculate the perimeter: $P = 3s = 12$.

Additional Insights and Applications

Understanding how to derive the side length from the height of an equilateral triangle has practical applications in various fields such as engineering, architecture, and design. It allows for precise measurements and calculations when designing structures that incorporate equilateral triangular components.

Practical Examples:

- Designing Triangular Frameworks: When given the height of an equilateral triangular support, engineers can determine the length of each side to ensure stability.
- Art and Architecture: Architects may use these calculations to create aesthetically pleasing and structurally sound triangular elements.
- Educational Purposes: Teaching students about special triangles and their properties using real-world examples.

Conclusion: Key Takeaways

- The height of an equilateral triangle directly relates to its side length through the formula $h = \frac{\sqrt{3}}{2} s$.
- Given the height as $\sqrt{12}$, simplifying yields $2 \sqrt{3}$.
- Solving for the side length using this height results in $s = 4$.
- The perimeter, being thrice the side length, is therefore 12 units.

By understanding these relationships, you can solve various problems involving equilateral triangles efficiently. This knowledge is fundamental in geometry and enhances problem-solving skills related to figures with symmetrical properties.

FAQs About Equilateral Triangles and Their Dimensions

Q1: How is the height of an equilateral triangle related to its side length?
A1: The height h of an equilateral triangle with side length s is given by $h = \frac{\sqrt{3}}{2} s$.

Q2: What are the common properties of an equilateral triangle?
A2: All sides are equal, all angles are 60°, and the height, median, and angle bisector from any vertex coincide.

Q3: Can the methods used here be applied to other types of triangles?
A3: The specific relationships depend on the triangle type. For right triangles, Pythagoras' theorem applies; for scalene triangles, different formulas are used.

Q4: Why is understanding special triangles like 30-60-90 important?
A4: They provide simplified relationships that facilitate quick calculations in various geometric problems.

In summary, knowing the height of an equilateral triangle allows you to accurately determine its side length and perimeter. This exercise demonstrates how understanding fundamental geometric properties enables solving practical and theoretical problems efficiently.

Frequently Asked Questions

What is the given height of the equilateral triangle in the problem?

The height of the equilateral triangle is given as ⍰ 12.

How can the height of an equilateral triangle be expressed in terms of its side length?

The height (h) of an equilateral triangle with side length a is given by h = (⍰ 3/2) a.

What is the value of the height given in the problem?

The height is ⍰ 12, which simplifies to 2⍰ 3.

How do we find the side length of the equilateral triangle using its height?

Set (⍰ 3/2) a = 2⍰ 3 and solve for a.

What is the step to find the side length of the triangle?

Multiply both sides by 2 to get ⍰ 3 a = 4⍰ 3, then divide both sides by ⍰ 3 to find a = 4.

What is the side length of the triangle based on the calculation?

The side length of the triangle is 4 units.

How do we find the perimeter of the equilateral triangle?

Multiply the side length by 3: Perimeter = 3 a.

What is the perimeter of the triangle in this problem?

The perimeter is 3 4 = 12 units.

Why is the side length of the triangle 4 units?

Because solving the height formula for a gives a = 4, based on the given height of 2⍰ 3.

What is the final answer to the problem?

The perimeter of the triangle is 12 units.

Additional Resources

Perimeter Calculation of an Equilateral Triangle with a Given Height

When delving into the fascinating world of geometry, few shapes are as symmetrical and elegant as the equilateral triangle. Known for its uniformity—where all sides and angles are equal—it serves as an essential building block in various fields, from architecture to engineering, and even in art and design. In this comprehensive exploration, we will analyze a specific problem: "The height of an equilateral triangle is the square root of 12. Find the perimeter of the triangle." This journey will not only provide the mathematical solution but also illuminate the underlying principles and reasoning steps involved.

Understanding the Basics of an Equilateral Triangle

Before diving into calculations, it's crucial to revisit the fundamental properties of an equilateral triangle:

- Equal Sides: All three sides are of equal length, typically denoted as s.
- Equal Angles: Each interior angle measures exactly 60°.
- Symmetry: The triangle exhibits perfect symmetry about any of its medians, altitudes, or angle bisectors.
- Altitude, or Height: The perpendicular segment from a vertex to the opposite side (or its extension) is called the altitude or height, denoted as h.

These properties simplify calculations because knowing one property often allows us to find others systematically.

Analyzing the Given Data: Height of the Triangle

The problem states: "The height of the equilateral triangle is the square root of 12." Symbolically,

$$ h = \sqrt{12} $$

Let's first simplify this radical for a clearer understanding.

Simplification of $\sqrt{12}$

Recall that:

$$ \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2 \sqrt{3} $$

Thus,

$$ h = 2 \sqrt{3} $$

This expression will be instrumental in deriving the side length of the triangle.

Relating Height to the Side Length of an Equilateral Triangle

In an equilateral triangle, the altitude, median, and angle bisector all coincide. The following relationship links the side length s to the height h:

$$ h = \frac{\sqrt{3}}{2} \times s $$

This formula emerges from the geometric properties of the triangle, often derived via the Pythagorean theorem.

Deriving the Relationship

In an equilateral triangle:

1. Draw the altitude from vertex A to side BC, meeting at point D.
2. This divides the base BC into two equal segments:

$$ BD = DC = \frac{s}{2} $$

3. Triangle ABD is a right-angled triangle with:

- Hypotenuse AB = s
- One leg: BD = $\frac{s}{2}$
- The other leg: height h

Applying the Pythagorean theorem:

$$ h^2 + \left(\frac{s}{2}\right)^2 = s^2 $$

Rearranged as:

$$ h^2 = s^2 - \frac{s^2}{4} = \frac{3s^2}{4} $$

Taking square roots:

$$ h = \frac{\sqrt{3}}{2} s $$

Calculating the Side Length from the Height

Given:

$$ h = 2 \sqrt{3} $$

and

$$ h = \frac{\sqrt{3}}{2} s $$

Set these equal:

$$ 2 \sqrt{3} = \frac{\sqrt{3}}{2} s $$

To find s, solve for s:

1. Multiply both sides by 2 to clear the denominator:

$$ 2 \times 2 \sqrt{3} = \sqrt{3} \times s $$

$$ 4 \sqrt{3} = \sqrt{3} \times s $$

2. Divide both sides by $\sqrt{3}$:

$$ \frac{4 \sqrt{3}}{\sqrt{3}} = s $$

$$ 4 = s $$

Thus, the side length of the equilateral triangle is:

$$ s = 4 $$

Determining the Perimeter of the Triangle

The perimeter P of an equilateral triangle is simply three times the length of one side:

$$ P = 3 \times s $$

Substituting the calculated side length:

$$ P = 3 \times 4 = 12 $$

Therefore, the perimeter of the triangle is 12 units.

Key Takeaways and Summary

- The height of an equilateral triangle relates to its side length via the formula:

$$ h = \frac{\sqrt{3}}{2} s $$

- Given the height as $\sqrt{12} = 2 \sqrt{3}$, we used algebraic substitution to find the side length:

$$ s = 4 $$

- The perimeter then follows straightforwardly:

$$ P = 3s = 12 $$

Additional Insights and Practical Applications

Understanding the relationship between the height and side length of an equilateral triangle has numerous real-world applications:

- Architecture: Equilateral triangles are often used in trusses and frameworks for their strength and aesthetic appeal.
- Design and Art: Symmetry and proportionality principles derived from such calculations ensure balanced and visually pleasing designs.
- Engineering: Precise measurements of components often require similar geometric reasoning.
- Mathematics Education: This problem exemplifies how radical expressions and algebraic manipulation are crucial tools in problem-solving.

Furthermore, mastering these fundamental relationships bolsters one's capacity to tackle more complex geometric problems, including those involving non-equilateral triangles or three-dimensional figures.

Conclusion

In conclusion, given the height of an equilateral triangle as \(\sqrt{12}\), we have systematically deduced its side length to be 4 units and its perimeter to be 12 units. This problem underscores the importance of fundamental geometric formulas and algebraic techniques in solving practical and theoretical questions alike. Whether you're a student, educator, or professional, a solid grasp of these relationships forms the backbone of geometric reasoning, paving the way for more advanced mathematical explorations.

The Height Of An Equilateral Triangle Is The Square Root Of 12 Find The Perimeter Of The Triangle

The Height Of An Equilateral Triangle Is The Square Root Of 12 Find The Perimeter Of The Triangle

Related Articles

- 2. The Ability Of The Judicial Branch To Determine The Constitutionality Of A Law Passed By Congress
- A Client Has A Job As A Tenured Professor Of Finance At A Successful University. Tenure Implies That
- *GNS221 GENERAL CLASS ASSIGNMENT*The Rail Road Was Not The First Institution To Impose Regularity On

Back to Home