What Is The Form Of The Particular Solution For The Given Differential Equation? Y''-2y Y=x^3(1-5x)

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When tackling differential equations, especially nonhomogeneous linear differential equations, one of the critical steps is determining the form of the particular solution. In this article, we will explore the specific process of finding the particular solution for the differential equation:

$$ Y'' - 2Y = x^3(1 - 5x) $$

This equation features a second derivative, a constant coefficient, and a polynomial right-hand side. Understanding the appropriate form of the particular solution is essential for solving the equation efficiently and accurately. We will discuss the theoretical background, step-by-step methods, and practical strategies to identify the correct form of the particular solution for this specific differential equation.

Understanding the Differential Equation: Structure and Components

Before delving into the form of the particular solution, it’s important to understand the structure of the given differential equation:

$$ Y'' - 2Y = x^3(1 - 5x) $$

Components of the Differential Equation:

- Homogeneous part: $Y'' - 2Y = 0$
- Nonhomogeneous part: The right-hand side $x^3(1 - 5x)$, which is a polynomial expression.

Key considerations:

- The differential operator is linear and constant coefficient.
- The nonhomogeneous term is a polynomial of degree 4 (since $x^3 \times 1 = x^3$ and $x^3 \times 5x = 5x^4$).

Step-by-Step Approach to Find the Particular Solution

To find the particular solution, we typically use the Method of Undetermined Coefficients or the Variation of Parameters. For polynomial right-hand sides, the method of undetermined coefficients is usually more straightforward.

Step 1: Find the Complementary (Homogeneous) Solution

Solve the homogeneous differential equation:

$$ Y'' - 2Y = 0 $$

The characteristic equation:

$$ r^2 - 2 = 0 $$

$$ r^2 = 2 $$

$$ r = \pm \sqrt{2} $$

Thus, the homogeneous solution:

$$ Y_h = C_1 e^{\sqrt{2}x} + C_2 e^{-\sqrt{2}x} $$

Step 2: Identify the Form of the Particular Solution

Since the right-hand side is a polynomial $x^3(1 - 5x)$, which simplifies to:

$$ x^3 - 5x^4 $$

the particular solution should be a polynomial of degree 4 (the highest degree of the RHS), multiplied by undetermined coefficients:

$$ Y_p = A x^4 + B x^3 + C x^2 + D x + E $$

However, it’s important to check whether any of these terms are solutions to the homogeneous equation. Since the homogeneous solution involves exponential functions, and the RHS is polynomial, no overlap occurs, and we can directly assume a polynomial of degree 4 for the particular solution.

Step 3: Compute Derivatives of the Trial Particular Solution

Calculate Y_p' and Y_p'':

$$
Y_p = A x^4 + B x^3 + C x^2 + D x + E
$$

$$
Y_p' = 4A x^3 + 3B x^2 + 2C x + D
$$

$$
Y_p'' = 12A x^2 + 6B x + 2C
$$

Step 4: Substitute into the Differential Equation

Substitute Y_p, Y_p', and Y_p'' into the original equation:

$$ Y'' - 2Y = x^3 - 5x^4 $$

which becomes:

$$ (12A x^2 + 6B x + 2C) - 2(A x^4 + B x^3 + C x^2 + D x + E) = x^3 - 5x^4 $$

Simplify the left side:

$$
12A x^2 + 6B x + 2C - 2A x^4 - 2B x^3 - 2C x^2 - 2D x - 2E
$$

Group like terms:

$$
(-2A x^4) + (-2B x^3) + (12A x^2 - 2C x^2) + (6B x - 2D x) + (2C - 2E)
$$

which simplifies to:

$$
-2A x^4 - 2B x^3 + (12A - 2C) x^2 + (6B - 2D) x + (2C - 2E)
$$

Set this equal to the RHS:

$$ x^3 - 5x^4 $$

Matching Coefficients to Determine Unknowns

Now, equate coefficients of corresponding powers of x:

Power of x	Left Side Coefficient	Right Side Coefficient	Equation

x^4	$-2A$	-5	$-2A = -5$
x^3	$-2B$	1	$-2B = 1$
x^2	$12A - 2C$	0	$12A - 2C = 0$
x^1	$6B - 2D$	0	$6B - 2D = 0$
Constant term	$2C - 2E$	0	$2C - 2E = 0$

Step 5: Solve for the Coefficients

From the equations:

1. $-2A = -5 \Rightarrow A = \frac{5}{2}$

2. $-2B = 1 \Rightarrow B = -\frac{1}{2}$

3. $12A - 2C = 0 \Rightarrow 12 \times \frac{5}{2} - 2C = 0$

$$
30 - 2C = 0 \Rightarrow 2C = 30 \Rightarrow C = 15
$$

4. $6B - 2D = 0 \Rightarrow 6 \times (-\frac{1}{2}) - 2D = 0$

$$
-3 - 2D = 0 \Rightarrow 2D = -3 \Rightarrow D = -\frac{3}{2}
$$

5. $2C - 2E = 0 \Rightarrow 2 \times 15 - 2E = 0$

$$
30 - 2E = 0 \Rightarrow 2E = 30 \Rightarrow E = 15
$$

Final Particular Solution

Substituting the calculated coefficients into the polynomial:

$$
Y_p = A x^4 + B x^3 + C x^2 + D x + E
$$

$$
Y_p = \frac{5}{2} x^4 - \frac{1}{2} x^3 + 15 x^2 - \frac{3}{2} x + 15
$$

Complete General Solution:

\[

$$
Y(x) = Y_h + Y_p = C_1 e^{\sqrt{2}x} + C_2 e^{-\sqrt{2}x} + \frac{5}{2} x^4 - \frac{1}{2} x^3 + 15 x^2 - \frac{3}{2} x + 15
$$

Key Takeaways for the Form of Particular Solutions in Differential Equations

Understanding the form of the particular solution involves several important considerations:

1. Nature of the Nonhomogeneous Term

- If RHS is a polynomial, the particular solution is typically a polynomial of the same degree.
- If RHS involves exponential functions, sines, or cosines, the particular solution often includes similar functions multiplied by polynomials, exponentials, or sine/cosine terms.

2. Degree of the Polynomial

- For RHS polynomial of degree n, assume a polynomial of degree n (or n plus possibly higher if overlaps occur).

3. Avoiding Overlap with Homogeneous Solutions

- If the RHS resembles solutions of the homogeneous equation, the form of the particular solution must be multiplied by x or higher powers to account for duplication.

4. Use of Undetermined Coefficients

- Assign undetermined coefficients to the terms in the assumed particular solution.
- Substitute into the differential equation to solve for these coefficients.

Why Is Knowing

Frequently Asked Questions

What is the general approach to find the particular

solution of the differential equation $y'' - 2y = x^3(1 - 5x)$?

The method involves first solving the homogeneous equation $y'' - 2y = 0$ to find the complementary solution, then guessing a particular solution based on the form of the right-hand side, typically using the method of undetermined coefficients or variation of parameters.

What is the form of the homogeneous solution for $y'' - 2y = 0$?

The characteristic equation is $r^2 - 2 = 0$, giving $r = \pm\sqrt{2}$. Therefore, the homogeneous solution is $y_h = C_1 e^{\sqrt{2} x} + C_2 e^{-\sqrt{2} x}$.

How do we determine the form of the particular solution for the right-hand side $x^3(1 - 5x)$?

Since the right-hand side is a polynomial of degree 4 (after expansion), the particular solution should be a polynomial of degree 4. We assume a polynomial of the form $y_p = A x^4 + B x^3 + C x^2 + D x + E$.

Why do we choose a polynomial form for the particular solution in this differential equation?

Because the nonhomogeneous term x^3(1 - 5x) simplifies to a polynomial in x, and polynomials are typical particular solutions when the RHS is polynomial, unless they overlap with the homogeneous solution's form.

Do we need to modify the polynomial form if any of its terms overlap with the homogeneous solution?

Yes, if any terms of the polynomial particular solution are solutions to the homogeneous equation, we multiply those terms by x to increase their degree, ensuring linear independence. In this case, since the homogeneous solutions are exponential functions, this isn't necessary.

What is the next step after assuming the polynomial form for the particular solution?

Substitute the assumed particular solution into the differential equation, compute its derivatives, and equate coefficients of like powers of x to solve for the unknowns A, B, C, D, and E.

Can the method of undetermined coefficients be used directly here?

Yes, because the RHS is a polynomial, the method of undetermined coefficients is suitable. Alternatively, variation of parameters can also be used, but it is more complex.

What is the importance of checking for overlaps between the particular solution form and the homogeneous solution?

It prevents duplication that would lead to terms that do not satisfy the particular solution criteria. Since the homogeneous solutions are exponential functions, and the particular solution is polynomial, no overlap occurs, so no modification is necessary.

What is the final form of the particular solution for the given differential equation?

The particular solution is a polynomial y_p = A x^4 + B x^3 + C x^2 + D x + E, where the coefficients are determined by substituting into the original equation and solving the resulting algebraic system.

How does understanding the form of the particular solution aid in solving differential equations?

Knowing the form guides the choice of an appropriate trial function, simplifies calculations, and ensures an efficient path to the complete solution of the differential equation.

Additional Resources

Understanding the Form of the Particular Solution for the Differential Equation $y'' - 2 y' = x^3 (1 - 5x)$

In the realm of differential equations, the process of finding solutions often involves a strategic decomposition into homogeneous and particular parts. The focus of this discussion centers on determining the appropriate form of the particular solution for the differential equation:

$$
y'' - 2 y' = x^3 (1 - 5x)
$$

This task is fundamental in differential equations, especially for nonhomogeneous equations where

the right-hand side (forcing function) guides the choice of the particular solution's structure. Understanding the method to identify this form is essential for both students and practitioners aiming to solve such equations accurately and efficiently.

Fundamentals of Differential Equations and Particular Solutions

Homogeneous vs. Nonhomogeneous Equations

Differential equations are classified into two main categories:

- Homogeneous Differential Equations: Those where the right-hand side (forcing term) is zero, i.e., $y'' + p(x) y' + q(x) y = 0$. Solutions involve characteristic equations and their roots.

- Nonhomogeneous Differential Equations: Those with a non-zero forcing term, such as $y'' + p(x) y' + q(x) y = f(x)$. The general solution comprises the homogeneous solution plus a particular solution that accounts for $f(x)$.

The primary goal in solving nonhomogeneous equations is to find a particular solution y_p that satisfies the entire equation, which, combined with the homogeneous solution y_h, yields the general solution:

$$
y = y_h + y_p
$$

Methodology for Finding the Particular Solution

The method of undetermined coefficients often guides the selection of y_p. This method involves making an educated guess about the form of y_p based on the nature of $f(x)$, then determining the unknown coefficients by substitution.

Alternatively, the variation of parameters method offers a more general approach, especially when $f(x)$ does not fit the typical forms suitable for undetermined coefficients. However, in cases where $f(x)$ is a polynomial or a combination of polynomials, exponentials, sines, or cosines, undetermined coefficients are usually preferred.

Analysis of the Given Differential Equation

Identifying the Type and Structure of the Equation

The differential equation under investigation is:

$$
y'' - 2 y' = x^3 (1 - 5x)
$$

This is a second-order linear nonhomogeneous differential equation with variable coefficients. The left side resembles a constant-coefficient differential operator, but note that the coefficients are constants, making it straightforward to analyze.

The homogeneous part:

$$
y'' - 2 y' = 0
$$

has characteristic equation:

$$
r^2 - 2 r = 0
$$

which factors into:

$$
r (r - 2) = 0
$$

leading to roots:

$$
r = 0 \quad \text{and} \quad r = 2
$$

Thus, the homogeneous solution:

$$
y_h = C_1 + C_2 e^{2x}
$$

The right side, $x^3 (1 - 5x)$, is a polynomial expression, which suggests that the particular solution may be a polynomial or a combination of polynomial terms.

Determining the Form of the Particular Solution

Step 1: Recognizing the Forcing Function $f(x)$

The nonhomogeneous term:

$$
f(x) = x^3 (1 - 5x) = x^3 - 5x^4
$$

is a polynomial of degree 4. When dealing with polynomial forcing functions, the undetermined coefficients method entails guessing a polynomial particular solution with degree equal to the degree of $f(x)$.

Since $f(x)$ is of degree 4, the initial guess for the particular solution should be:

$$
y_p = A x^4 + B x^3 + C x^2 + D x + E
$$

where A, B, C, D, E are constants to be determined.

Step 2: Checking for Resonance or Overlap with Homogeneous Solution

A crucial consideration is whether the form of the particular solution overlaps with the homogeneous solution. The homogeneous solution:

**\[
y_h = C_1 + C_2 e^{2x}
\]**

contains only constants and exponentials. Since the forcing function is polynomial, there's no resonance or overlap with the homogeneous solutions, so no adjustments are needed.

Step 3: Formulating the Particular Solution

Given the above, the form of the particular solution is:

**\[
\boxed{
y_p = A x^4 + B x^3 + C x^2 + D x + E**

}
\]

This polynomial form is standard for polynomial nonhomogeneous terms in linear differential equations with constant coefficients, provided there's no overlap with the homogeneous solution.

Additional Considerations and Variations

When the Forcing Function is a Polynomial

In most cases, when the forcing function $f(x)$ is a polynomial of degree n, the particular solution is taken as a polynomial of degree n. This is because derivatives of polynomial functions are also polynomials, ensuring that substituting y_p into the differential equation yields polynomial expressions, making coefficient comparison straightforward.

Implications of the Homogeneous Solution

Since the homogeneous solution contains only constant and exponential terms, the polynomial particular solution remains unaffected by resonance considerations, simplifying the process.

Alternative Forms for Non-Polynomial Forcing Terms

If the nonhomogeneous term were, for example, exponential, sine, or cosine functions, the particular solution's form would vary accordingly. For instance:

- For $f(x) = e^{ax}$, the particular solution would include terms like $P(x) e^{ax}$.

- For $f(x) = \sin bx$ or $\cos bx$, it would involve sinusoidal functions multiplied by polynomials.

Summary of the Form of the Particular Solution

Based on the polynomial nature of the forcing function $x^3(1 - 5x)$, the form of the particular solution for the differential equation:

$$
y'' - 2 y' = x^3 (1 - 5x)
$$

is:

$$
\boxed{
y_p = A x^4 + B x^3 + C x^2 + D x + E
}
$$

where A, B, C, D, E are constants to be determined through substitution into the original differential equation. This form is both comprehensive and standard, aligning with the classical method of undetermined coefficients, and ensures a systematic approach to solving the nonhomogeneous differential equation.

Final Remarks and Practical Approach

The process of identifying the particular solution's form is a vital step in solving differential equations. It combines an understanding of the nature of the nonhomogeneous term with the characteristics of the homogeneous solutions. For the given differential equation, the polynomial forcing function directly indicates a polynomial particular solution.

In practice, once the form is established, the next steps involve:

1. Calculating derivatives of the proposed particular solution y_p.

2. Substituting y_p, y_p', and y_p'' into the original differential equation.

3. Equating coefficients of like powers of x on both sides to find the unknown constants.

This systematic approach ensures a complete solution that encompasses both the homogeneous and particular solutions, ultimately providing a comprehensive understanding of the differential equation's behavior.

In conclusion, the form of the particular solution for the differential equation \(y'' - 2 y' = x^3 (1 -

5x) \) is a polynomial of degree 4, explicitly expressed as:

\[
\boxed{
y_p = A x^4 + B x^3 + C x^2 + D x + E
}
\]

This choice is rooted in the polynomial nature of the forcing function and is consistent with the principles of the method of undetermined coefficients, ensuring a manageable and effective pathway toward a complete solution.

[What Is The Form Of The Particular Solution For The Given Differential Equation Y 2y Y X 3 1 5x]()

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