A) Let R(t) = (sin(t), Cos(t), 4 Sin(t) + 6 Cos(2t)) . Find The Projection Of R(t) Onto The Xz-plane

A) Let R(t) = (sin(t), Cos(t), 4 Sin(t) + 6 Cos(2t)) . Find The Projection Of R(t) Onto The Xz-plane

Understanding the projection of a vector function onto a plane is a fundamental concept in vector calculus and analytical geometry. In this case, we're given a vector-valued function R(t), and the goal is to find its projection onto the xz-plane. This process involves analyzing the components of R(t), understanding the geometric interpretation of projections, and performing the necessary calculations to obtain the projected curve.

This comprehensive guide will break down the problem into manageable steps, clarify the key concepts involved, and provide detailed explanations to ensure a thorough understanding. Whether you're a student revising for exams or a professional brushing up on vector projections, this content aims to serve as an authoritative resource.

Understanding the Vector Function R(t)

Definition of R(t)

The vector function R(t) is defined as:

$$R(t) = (\sin t, \cos t, 4 \sin t + 6 \cos 2t)$$

where t is a real parameter, typically representing time or an angle parameter.

Breaking down its components:
- x-component: $\sin t$
- y-component: $\cos t$
- z-component: $4 \sin t + 6 \cos 2t$

This function describes a curve in three-dimensional space, with each value of t corresponding to a point (x, y, z).

Understanding Planes and Projections

What is the XZ-plane?

The xz-plane is a two-dimensional plane in three-dimensional space, defined by the set of points where the y-coordinate is zero:

$$ y = 0 $$

It includes all points $(x, 0, z)$, where x and z can vary freely.

Projection of a Point onto the XZ-plane

Projecting a point (x, y, z) onto the xz-plane involves "dropping" the point perpendicularly onto this plane along the y-axis. The projected point (x', y', z') has:
- The same x-coordinate as the original point.
- The y-coordinate set to zero (since the plane's y-value is zero).
- The same z-coordinate as the original point.

Mathematically:
$$ \text{Projection of } (x, y, z) \text{ onto } y=0 \text{ is } (x, 0, z) $$

Calculating the Projection of R(t) onto the XZ-plane

Step 1: Identify the components of R(t)

From the given function:
- $x(t) = \sin t$
- $y(t) = \cos t$
- $z(t) = 4 \sin t + 6 \cos 2t$

Step 2: Understand the projection process

Since the projection onto the xz-plane involves setting the y-component to zero, the projected point $R_{proj}(t)$ will be:
$$ R_{proj}(t) = (x(t), 0, z(t)) $$

This means:
- The x-component remains $\sin t$.
- The y-component is eliminated (set to zero).
- The z-component remains $4 \sin t + 6 \cos 2t$.

Step 3: Write the projected vector function

The projected curve can be described as:
$$ R_{proj}(t) = (\sin t, 0, 4 \sin t + 6 \cos 2t) $$

This function traces the path of the original curve's shadow on the xz-plane as t varies.

Analyzing the Projected Curve

Understanding the parametric form

The projected curve is parametric, with:
- $x(t) = \sin t$
- $z(t) = 4 \sin t + 6 \cos 2t$

Note that $y(t)$ is no longer part of the projection.

Expressing z(t) in terms of x

To analyze the shape of the projection, it is helpful to express z as a function of x.

Since:
$$x = \sin t$$
then:
$$t = \arcsin x$$
(considering t within the principal range for simplicity).

Now, express $z(t)$ using x:
$$z(t) = 4 \sin t + 6 \cos 2t$$

Recall the double-angle identity:
$$\cos 2t = 1 - 2 \sin^2 t$$

Substituting:
$$z(t) = 4 \sin t + 6 (1 - 2 \sin^2 t)$$
$$z(t) = 4 \sin t + 6 - 12 \sin^2 t$$

Replacing $\sin t$ with x:
$$z = 4x + 6 - 12 x^2$$

Thus, the projected curve in the xz-plane is described by:
$$z = 4x + 6 - 12 x^2$$

Graphical and Geometric Interpretation

Shape of the projected curve

The equation:
$$z = -12 x^2 + 4 x + 6$$
is a quadratic in x, opening downward (since coefficient of x^2 is negative).

This indicates:
- The projection is a parabola in the xz-plane.
- The vertex of this parabola can be found to understand the maximum point.

Finding the vertex of the parabola

The standard form:

$$ z = ax^2 + bx + c $$

where:

- $a = -12$
- $b = 4$
- $c = 6$

The x-coordinate of the vertex:

$$ x_{vertex} = -\frac{b}{2a} = -\frac{4}{2 \times (-12)} = -\frac{4}{-24} = \frac{1}{6} $$

Corresponding z-value:

$$ z_{vertex} = -12 \left(\frac{1}{6}\right)^2 + 4 \times \frac{1}{6} + 6 $$

$$ z_{vertex} = -12 \times \frac{1}{36} + \frac{4}{6} + 6 $$

$$ z_{vertex} = -\frac{12}{36} + \frac{2}{3} + 6 $$

$$ z_{vertex} = -\frac{1}{3} + \frac{2}{3} + 6 $$

$$ z_{vertex} = \frac{1}{3} + 6 = \frac{1 + 18}{3} = \frac{19}{3} $$

Therefore, the vertex is at:

$$ \left(\frac{1}{6}, \frac{19}{3} \right) $$

which corresponds to the point $(x, z) = (1/6, 19/3)$.

Summary and Key Takeaways

1. The projection of the vector function $R(t) = (\sin t, \cos t, 4 \sin t + 6 \cos 2t)$ onto the xz-plane is obtained by setting the y-component to zero, resulting in $R_{proj}(t) = (\sin t, 0, 4 \sin t + 6 \cos 2t)$.

2. Expressing $z(t)$ in terms of $x = \sin t$ simplifies the analysis, leading to the explicit parabola $z = -12 x^2 + 4 x + 6$.

3. The projected curve is a downward-opening parabola with a vertex at $(x, z) = (1/6, 19/3)$, indicating the maximum point of the projection.

4. This analysis helps visualize how the original spatial curve appears when "flattened" onto the xz-plane, which is useful in graphical representations, physics simulations, and understanding motion trajectories.

Additional Considerations and Applications

Applications in Physics and Engineering

Understanding projections like this is crucial in fields such as:
- Kinematics: Analyzing motion trajectories in different planes.
- Robotics: Planning paths in three-dimensional space and their shadows or projections.
- Computer Graphics: Rendering 3D objects onto 2D screens.

Extensions and Further Study

Students and professionals interested in extending this work can explore:
- Projections onto other planes, such as the yz-plane or xy-plane.
- The effect of different parametrizations or functions on the shape of the projected curve.
- Using calculus to analyze the curvature or other properties of

Frequently Asked Questions

What is the given vector function R(t)?

R(t) = (sin(t), cos(t), 4 sin(t) + 6 cos(2t))

What is the goal when finding the projection of R(t) onto the xz-plane?

The goal is to find the component of R(t) that lies in the xz-plane, effectively removing the y-component.

How do you find the projection of a vector onto the xz-plane?

To project onto the xz-plane, set the y-component of R(t) to zero, resulting in (x(t), 0, z(t)).

What is the projected vector of R(t) onto the xz-plane?

The projection is (sin(t), 0, 4 sin(t) + 6 cos(2t)).

Why is it sufficient to set the y-component to zero for the projection onto the xz-plane?

Because the xz-plane is characterized by y = 0, so the projection retains only the x and z components, nullifying y.

Can you express the projection explicitly in terms of t?

Yes, the projection is (sin(t), 0, 4 sin(t) + 6 cos(2t)).

How does the projection help in understanding the behavior of R(t)?

It allows us to analyze the motion or shape of R(t) in the xz-plane, ignoring the y-component's influence.

Is the projection of R(t) onto the xz-plane a curve? If so, what is its nature?

Yes, it is a parametric curve describing the path of the projection in the xz-plane, influenced by the trigonometric functions of t.

What are some potential applications of projecting a vector function onto a coordinate plane?

Applications include simplifying analysis of motion in physics, visualizing trajectories, and studying components of vector fields in specific planes.

Additional Resources

Projection of R(t) = (sin(t), cos(t), 4 sin(t) + 6 cos(2t)) onto the XZ-plane: An Analytical Exploration

The study of parametric curves and their projections is a fundamental aspect of multivariable calculus, offering profound insights into the geometric behavior of curves in three-dimensional space. In this detailed examination, we focus on the vector function R(t) = (sin(t), cos(t), 4 sin(t) + 6 cos(2t)) and analyze its projection onto the XZ-plane. This process involves understanding the nature of the vector function, the concept of projection in a geometric context, and the specific steps to determine the projected curve. Through a structured approach, we will uncover the underlying relationships, interpret the geometric implications, and contextualize the significance of the projection within broader mathematical frameworks.

Understanding the Vector Function R(t)

Parametric Representation of R(t)

The vector function R(t) describes a three-dimensional curve parametrized by the variable t. Its components are:

- x(t) = sin(t)
- y(t) = cos(t)
- z(t) = 4 sin(t) + 6 cos(2t)

This parametric form captures the position of a point moving through space as t varies over real numbers. Each component offers insights into the behavior of the curve:

- The x and y components resemble the familiar parametric equations of a circle, since $\sin(t)$ and $\cos(t)$ satisfy the Pythagorean identity and trace a unit circle in the XY-plane.
- The z component combines sinusoidal functions, indicating oscillations along the Z-axis with frequencies tied to t.

Periodic and Geometric Properties

Given the sinusoidal nature of the components:

- $x(t) = \sin(t)$ and $y(t) = \cos(t)$ are periodic with period 2π, describing a circle of radius 1 in the XY-plane.
- $z(t) = 4 \sin(t) + 6 \cos(2t)$ combines two sinusoidal functions with different frequencies: $\sin(t)$ (frequency 1) and $\cos(2t)$ (frequency 2). This indicates that $z(t)$ oscillates with a fundamental period 2π, but with a more complex waveform.

The interaction of these components results in a space curve that exhibits both rotational symmetry and oscillatory behavior along the Z-axis, which is critical in understanding the projection onto the XZ-plane.

Concept of Projection onto the XZ-plane

What Is a Projection in Geometric Terms?

Projection, in geometric and vector calculus contexts, refers to the process of "flattening" a three-dimensional object onto a plane, capturing the shadow or outline of the original object from a particular perspective. Specifically, the projection of a point (x, y, z) onto the XZ-plane involves:

- Keeping the x and z components unchanged.
- Eliminating the y component (since the YZ component is orthogonal to the XZ-plane).

Mathematically, the projection $P(t)$ of $R(t) = (x(t), y(t), z(t))$ onto the XZ-plane is:

$$P(t) = (x(t), 0, z(t))$$

This effectively "drops" the point vertically onto the XZ-plane, ignoring its y-coordinate.

Implications of the Projection

- The projected curve reveals the relationship between the x and z components as t varies.
- It provides a two-dimensional representation of the original three-dimensional trajectory, helping visualize the pattern and oscillations without the complexity of the y-dimension.

- Such projections are essential in fields like computer graphics, physics, and engineering, where understanding the shadow or outline of a shape simplifies analysis.

Deriving the Projection of R(t) onto the XZ-plane

Step-by-Step Calculation

Given $R(t) = (\sin(t), \cos(t), 4 \sin(t) + 6 \cos(2t))$, the projection $P(t)$ onto the XZ-plane is:

$P(t) = (x(t), 0, z(t))$

Substituting the components:

- $x(t) = \sin(t)$
- $z(t) = 4 \sin(t) + 6 \cos(2t)$

Therefore, the projected curve is:

$P(t) = (\sin(t), 0, 4 \sin(t) + 6 \cos(2t))$

This parametric form encapsulates the entire projected shape.

Expressing z(t) in a More Analyzable Form

To better understand the nature of the projection, it's helpful to analyze $z(t)$:

$z(t) = 4 \sin(t) + 6 \cos(2t)$

Recall the double-angle identity:

$\cos(2t) = 2 \cos^2(t) - 1$

Thus,

$z(t) = 4 \sin(t) + 6 (2 \cos^2(t) - 1) = 4 \sin(t) + 12 \cos^2(t) - 6$

Expressing $z(t)$ entirely in terms of $\sin(t)$ and $\cos(t)$:

$z(t) = 4 \sin(t) + 12 \cos^2(t) - 6$

This form suggests that $z(t)$ depends on both $\sin(t)$ and $\cos^2(t)$. Since $\cos^2(t) = 1 - \sin^2(t)$, we can rewrite $z(t)$ as:

$z(t) = 4 \sin(t) + 12 (1 - \sin^2(t)) - 6$

Simplify:

$z(t) = 4 \sin(t) + 12 - 12 \sin^2(t) - 6$

which simplifies further to:

$z(t) = (4 \sin(t)) - (12 \sin^2(t)) + 6$

Or,

$z(t) = -12 \sin^2(t) + 4 \sin(t) + 6$

This quadratic form in $\sin(t)$ provides a basis for analyzing the relationship between $x(t) = \sin(t)$ and $z(t)$.

Graphical and Analytical Insights into the Projection

Relationship Between x and z in the Projected Curve

Since $x(t) = \sin(t)$, and $z(t)$ can be expressed as:

$z(t) = -12 \sin^2(t) + 4 \sin(t) + 6$

we can interpret this as a function $z = f(x)$:

$z = -12 x^2 + 4 x + 6$

where $x = \sin(t)$, with x ranging between -1 and 1.

This quadratic relationship describes a parabola in the xz-plane, opening downward because the coefficient of x^2 is negative.

Key features:

- Vertex of the parabola can be found by completing the square or using calculus.
- The domain of x is $[-1, 1]$, corresponding to the sine function's range.

Finding the Vertex of the Parabola

The quadratic:

$z(x) = -12 x^2 + 4 x + 6$

has:

- $a = -12$

- $b = 4$
- $c = 6$

The x-coordinate of the vertex:

$x_v = -b / (2a) = -4 / (2 -12) = -4 / -24 = 1/6$

The corresponding z-coordinate:

$z_v = -12 (1/6)^2 + 4 (1/6) + 6$

Calculate:

$z_v = -12 (1/36) + 4/6 + 6 = -12 / 36 + 2/3 + 6$

Simplify:

$-12/36 = -1/3$

So,

$z_v = -1/3 + 2/3 + 6 = (1/3) + 6 = 6 + 1/3 \approx 6.333...$

This indicates that the parabola reaches its maximum at $x = 1/6$, $z \approx 6.333$.

Given the domain $x \in [-1, 1]$, the parabola's shape and maximum point reveal how the projection traces a curved path in the xz-plane.

Geometric and Physical Interpretations

Understanding the Projection's Shape

The projected curve in the xz-plane is essentially a segment of a parabola restricted to $x \in [-1, 1]$. Its maximum height occurs at $x = 1/6$, with $z \approx 6.333$, indicating a "peak" in the curve. On either side of this maximum, z decreases, reaching values at the endpoints $x = -1$ and $x = 1$:

- For $x = -1$:

z = -12 (1) + 4 (-

[A Let R T Sin T Cos T 4 Sin T 6 Cos 2t Find The Projection Of R T Onto The Xz Plane]()

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