

A Sample Of 25 Provided A Sample Mean $\bar{X} = 14$ And A Sample Standard Deviation $S = 4.32$.a. Compute The

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Understanding how to analyze sample data is an essential skill in statistics. When you are given a sample size, the sample mean, and the sample standard deviation, you can perform various calculations to infer details about the population from which the sample was drawn. In this article, we will explore the process of computing key statistical measures based on a sample size of 25, with a sample mean ($\bar{X}$) of 14 and a sample standard deviation (S) of 4.32. We will delve into how to compute the standard error, confidence intervals, and other relevant statistics, providing a comprehensive guide for students and professionals alike.

Understanding the Basic Terms

Before jumping into calculations, it's important to understand the fundamental statistical concepts involved.

Sample Mean ($\bar{X}$)

The sample mean, denoted as $\bar{X}$, is the average value of the sample data. It provides a central value that represents the data set.

Sample Standard Deviation (S)

The sample standard deviation measures the dispersion or spread of data points around the mean. It indicates how much individual data points differ from the mean.

Sample Size (n)

The number of observations in the sample. In this case, $n = 25$.

Calculating the Standard Error of the Mean

One of the first steps in many statistical analyses is to estimate the standard error of the mean (SEM). The SEM indicates how much the sample mean ($\bar{X}$) is expected to fluctuate from the true population mean (μ).

Formula for Standard Error (SE)

The standard error of the mean is calculated as:

$$SE = \frac{S}{\sqrt{n}}$$

Where:

- S = Sample standard deviation
- n = Sample size

Applying the Formula

Given:

- S = 4.32
- n = 25

Calculating:

$$SE = \frac{4.32}{\sqrt{25}} = \frac{4.32}{5} = 0.864$$

This means the sample mean is expected to vary by approximately 0.864 units due to sampling variability.

Constructing a Confidence Interval for the Population Mean

Confidence intervals provide a range within which we expect the true population mean (μ) to lie with a specified level of confidence. Typically, 95% confidence intervals are used.

Understanding the Confidence Interval

A 95% confidence interval is calculated as:

$$\text{CI} = \bar{X} \pm t^* \times SE$$

Where:

- (t^*) is the critical value from the t-distribution based on the degrees of freedom ($df = n - 1$)
- SE is the standard error

Finding the Critical t-Value

Degrees of freedom:

$$df = n - 1 = 25 - 1 = 24$$

Using a t-distribution table or calculator, the critical t-value for 24 degrees of freedom at a 95% confidence level is approximately:

$$t^* \approx 2.064$$

Calculating the Confidence Interval

Applying the formula:

$$\text{Lower bound} = 14 - 2.064 \times 0.864 \approx 14 - 1.785 \approx 12.215$$

$$\text{Upper bound} = 14 + 2.064 \times 0.864 \approx 14 + 1.785 \approx 15.785$$

Result:

The 95% confidence interval for the population mean is approximately (12.22, 15.79).

Performing Hypothesis Testing for the Population Mean

Suppose you want to test whether the population mean (μ) is equal to a specific value, say 13, based on this sample. This involves setting up null and alternative hypotheses.

Null and Alternative Hypotheses

- Null hypothesis (H_0): $\mu = 13$
- Alternative hypothesis (H_1): $\mu \neq 13$

Test Statistic Calculation

The t-test statistic is calculated as:

$$t = \frac{\bar{X} - \mu_0}{SE}$$

Where:

- (μ_0) is the hypothesized population mean (13)

Calculating:

$$t = \frac{14 - 13}{0.864} \approx 1.157$$

Determining the p-Value

Using the t-distribution table or calculator for $df = 24$:

- The two-tailed p-value corresponding to $t \approx 1.157$ is approximately 0.255.

Interpreting the Results

Since the p-value (≈ 0.255) is greater than common significance levels (0.05 or 0.01), we fail to reject the null hypothesis. There is not enough evidence to conclude that the population mean differs from 13.

Additional Considerations in Statistical Analysis

When working with sample data, other factors and calculations may be relevant depending on the context.

Effect Size

Calculating effect size, such as Cohen's d, helps understand the practical significance of the findings.

$$d = \frac{\bar{X} - \mu_0}{S}$$

For the hypothesis test:

$$d = \frac{14 - 13}{4.32} \approx 0.231$$

This indicates a small effect size.

Sample Variability and Reliability

A smaller standard deviation and larger sample size generally lead to more reliable estimates and narrower confidence intervals.

Summary of Key Calculations

Calculation	Result	Description
Standard Error (SE)	0.864	Variability of the sample mean
95% Confidence Interval	(12.22, 15.79)	Range for the true population mean
t-Statistic for $\mu=13$	1.157	Test statistic for hypothesis testing
p-Value	0.255	Probability of observing the data under H_0
Effect Size (Cohen's d)	0.231	Magnitude of the difference

Conclusion: Practical Applications of These Statistical

Measures

Understanding how to compute these key statistical measures allows researchers and data analysts to make informed decisions about their data. Whether constructing confidence intervals to estimate population parameters or conducting hypothesis tests to evaluate claims, these calculations form the backbone of statistical inference. By applying the formulas and methods outlined in this article, you can confidently analyze similar datasets and interpret their implications.

Final Thoughts

In summary, given a sample size of 25 with a mean of 14 and a standard deviation of 4.32, you can perform a variety of important statistical analyses. Calculating the standard error provides insight into the precision of the sample mean. Constructing confidence intervals offers a range for the true population mean, while hypothesis testing helps assess specific claims about the population parameter. Mastering these calculations enhances your ability to analyze data critically and make evidence-based decisions across numerous fields, including research, business, and social sciences.

By understanding and applying these fundamental concepts, you are well-equipped to handle similar statistical problems and interpret data with confidence.

Frequently Asked Questions

What is the sample mean provided in the problem?

The sample mean provided is 14.

What is the sample standard deviation given in the problem?

The sample standard deviation given is 4.32.

How do you compute the standard error of the mean for this sample?

The standard error (SE) is calculated as the sample standard deviation divided by the square root of the sample size: $SE = S / \sqrt{n}$. With $S = 4.32$ and $n = 25$, $SE = 4.32 / \sqrt{25} = 4.32 / 5 = 0.864$.

What is the purpose of calculating the standard error in this context?

The standard error measures the variability of the sample mean estimate and helps in constructing confidence intervals or conducting hypothesis tests about the population mean.

How would you construct a 95% confidence interval for the population mean based on this data?

Using the sample mean (14), standard error (0.864), and a t-value for 24 degrees of freedom at 95% confidence (approximately 2.064), the confidence interval is: $14 \pm (2.064 \cdot 0.864) \approx 14 \pm 1.786$, or (12.214, 15.786).

What additional information would you need to perform a hypothesis test on the population mean?

You would need to specify the null hypothesis value (e.g., μ_0), the significance level (e.g., $\alpha = 0.05$), and determine whether to use a t-test or z-test based on whether the population standard deviation is known. Since only the sample standard deviation is provided, a t-test is appropriate.

Additional Resources

A Sample Of 25 Provided A Sample Mean $\bar{X} = 14$ And A Sample Standard Deviation $S = 4.32$: A Comprehensive Breakdown and Guide

Understanding how to analyze a sample data set is fundamental in statistics, especially when drawing inferences about a larger population. In this article, we will explore the process of interpreting a sample that has a size of 25, with a sample mean of 14 and a sample standard deviation of 4.32. This example offers an excellent opportunity to walk through key statistical concepts such as point estimates, confidence intervals, and hypothesis testing, providing a detailed, step-by-step guide suitable for students, educators, and professionals alike.

Setting the Stage: The Significance of the Sample Data

When we have a sample of data, such as a sample of 25 observations with a mean ($\bar{X}$) of 14 and a standard deviation (S) of 4.32, these figures serve as point estimates of the corresponding population parameters. The sample mean provides an estimate of the population mean (μ), and the sample standard deviation estimates the population standard deviation (σ). However, because these are based on a sample, they inherently contain some degree of uncertainty, which statistical tools help us quantify.

Core Concepts and Terminology

Before diving into calculations, it's essential to familiarize yourself with key terms:

- Sample Size (n): Number of observations in the sample (here, $n=25$).
- Sample Mean ($\bar{X}$): Average of the sample data (here, 14).
- Sample Standard Deviation (S): Measure of variability within the sample (here, 4.32).
- Population Mean (μ): The true average of the entire population (unknown, often estimated).
- Standard Error (SE): An estimate of the standard deviation of the sampling distribution of the

sample mean.

- Confidence Interval: A range of values within which the true population parameter is expected to fall with a certain probability.
- Hypothesis Testing: A method to assess claims or assumptions about the population.

Step 1: Calculating the Standard Error of the Mean

The standard error (SE) quantifies how much the sample mean (14) is expected to fluctuate from the true population mean (μ). It is calculated as:

$$\text{SE} = \frac{S}{\sqrt{n}} = \frac{4.32}{\sqrt{25}} = \frac{4.32}{5} = 0.864$$

This value indicates that if we repeatedly took samples of size 25 from the population, the sample means would typically vary by approximately 0.864 around the true population mean.

Step 2: Constructing a Confidence Interval for the Population Mean

Confidence intervals (CIs) provide a range of plausible values for the population parameter, considering sampling variability. For a sample size of 25, and assuming the data is approximately normally distributed, we typically use the t-distribution to account for additional uncertainty introduced by estimating σ with S .

Key points:

- Degrees of freedom (df) = $n - 1 = 24$.
- The critical t-value depends on the confidence level (commonly 95%).

Finding the t-Critical Value

For a 95% confidence level and $df=24$:

- From t-distribution tables or statistical software, $t_{\{0.025, 24\}} \approx 2.064$.

Calculating the Confidence Interval

The formula:

$$\text{CI} = \bar{X} \pm t_{\{\alpha/2, df\}} \times \text{SE}$$

Plugging in the numbers:

$$\text{CI} = 14 \pm 2.064 \times 0.864$$

\]

\[

\text{CI} = 14 \pm 1.785

\]

Thus, the 95% confidence interval for the population mean μ is approximately:

\[

(12.215, 15.785)

\]

Interpretation: We are 95% confident that the true population mean lies between approximately 12.22 and 15.78.

Step 3: Conducting a Hypothesis Test

Suppose we want to test whether the population mean μ is equal to a specific value, say 13. How do we proceed?

Null Hypothesis (H_0): $\mu = 13$

Alternative Hypothesis (H_1): $\mu \neq 13$

This is a two-tailed test.

Calculating the Test Statistic

The t-statistic for testing the mean is:

\[

$t = \frac{\bar{X} - \mu_0}{SE} = \frac{14 - 13}{0.864} \approx 1.157$

\]

Comparing Against Critical Values

- For $\alpha=0.05$ and $df=24$, the critical t-value (two-tailed) is approximately ± 2.064 .
- Since $|1.157| < 2.064$, we fail to reject the null hypothesis at the 5% significance level.

Conclusion: There is not enough evidence to conclude that the population mean differs from 13.

Additional Analyses and Considerations

Effect of Sample Size

- Larger sample sizes reduce the standard error, leading to narrower confidence intervals and more precise estimates.

- For example, increasing the sample size to 100 would reduce SE to:

$$SE = \frac{4.32}{\sqrt{100}} = \frac{4.32}{10} = 0.432$$

which would tighten the confidence interval significantly.

Assumptions and Validity

- The calculations above assume that the data are approximately normally distributed, which is reasonable for $n=25$ due to the Central Limit Theorem.
- If the data are skewed or contain outliers, alternative methods or transformations might be necessary.

Practical Applications

- Estimating the average performance in an educational setting.
- Assessing average customer satisfaction scores.
- Comparing sample data to hypothesized population parameters.

Summary: Key Takeaways

- The sample mean provides an estimate of the population mean, but uncertainty is quantified through the standard error.
- Confidence intervals offer a range of plausible values for the population mean, with common levels being 90%, 95%, or 99%.
- Hypothesis testing helps determine whether observed sample data provide sufficient evidence to support or refute claims about the population.

Final Thoughts

Analyzing a sample with a mean of 14 and a standard deviation of 4.32 from 25 observations involves understanding the variability inherent in sampling, utilizing the t-distribution for inference, and carefully interpreting the results. Whether estimating the population parameter or testing specific hypotheses, these tools enable statisticians and analysts to make informed decisions rooted in data. By mastering these fundamental concepts and calculations, you enhance your ability to interpret and apply statistical data effectively in a wide array of fields.

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