

A Sample Of Size 50 Is To Be Taken From An Infinite Population Whose Mean And Standard Deviation Are

A Sample Of Size 50 Is To Be Taken From An Infinite Population Whose Mean And Standard Deviation Are

Understanding how to properly select and analyze a sample from a population is fundamental in statistics. When dealing with an infinite population—one that is so large that it can be considered unbounded—certain assumptions and methods come into play that influence how we interpret sample data. In this article, we will explore the process of sampling, the statistical implications of sample size, and how to analyze the resulting data effectively, especially when the population mean and standard deviation are known or unknown.

Introduction to Sampling from an Infinite Population

Sampling forms the backbone of statistical inference, allowing us to estimate population parameters based on a subset of data. When the population is infinite, the sampling process simplifies some aspects but introduces others that require careful attention.

What Is an Infinite Population?

- An infinite population is one with an unbounded number of elements.
- Common examples include measurements like height, weight, or time, which, in theory, can take infinitely many values.
- In practice, the population can be considered infinite when the population size is extremely large relative to the sample size.

Why Is Sampling Important?

- Sampling reduces the cost and time involved in data collection.
- It allows for statistical inference about the entire population based on a manageable subset.
- Proper sampling methods ensure the representativeness and accuracy of estimates.

Understanding Sample Size and Its Significance

The sample size plays a critical role in determining the reliability and accuracy of statistical estimates.

The Significance of a Sample Size of 50

- A sample size of 50 is generally considered sufficiently large for many inferential statistical techniques, especially when the population variance is known or can be estimated accurately.
- It strikes a balance between being manageable and providing adequate statistical power.
- In the context of the Central Limit Theorem (CLT), a sample of size 50 typically ensures the sampling distribution of the mean is approximately normal, even if the population distribution is not normal.

Implications of Sample Size

1. **Precision:** Larger samples tend to produce more precise estimates of the population parameter.
2. **Variance:** The standard error decreases as the sample size increases, improving estimate reliability.
3. **Cost and Practicality:** Larger samples may be more costly and time-consuming, so a balance must be struck.

Population Parameters: Mean and Standard Deviation

The analysis of a sample largely depends on whether the population parameters—mean (μ) and standard deviation (σ)—are known or unknown.

When Population Mean and Standard Deviation Are Known

- Statistical inference can be performed using the Z-distribution (standard normal distribution).
- Confidence intervals for the population mean are calculated directly, leveraging known σ .

When Population Parameters Are Unknown

- Sample statistics (sample mean $\bar{x}$) and sample standard deviation s) are used as estimates.
- The Student's t-distribution is used for inference, especially for smaller sample sizes.

In the context of the problem—"A Sample Of Size 50 Is To Be Taken From An Infinite Population Whose Mean And Standard Deviation Are"—it is essential to clarify whether these parameters are known or unknown for proper analysis.

Sampling Distribution of the Sample Mean

Understanding the distribution of the sample mean is crucial for making inferences.

Central Limit Theorem (CLT)

- States that, for sufficiently large sample sizes, the sampling distribution of the sample mean will be approximately normal, regardless of the population's distribution.
- Typically, a sample size of 30 or more is considered adequate for the CLT to hold.
- Since our sample size is 50, the CLT assures the normality of the sampling distribution of $\bar{x}$.

Standard Error of the Mean

- The standard deviation of the sampling distribution, called the standard error (SE), is given by:

$$\text{SE} = \frac{\sigma}{\sqrt{n}}$$

- Where:
 - σ is the population standard deviation.
 - $n = 50$ is the sample size.

Implication of Known σ

- If σ is known, the sampling distribution of $\bar{x}$ follows a normal distribution with mean μ and standard deviation $\sigma / \sqrt{n}$.
- This allows for straightforward confidence interval calculations and hypothesis testing using the Z-distribution.

Calculating Confidence Intervals for the Population Mean

Confidence intervals provide a range within which the true population mean is likely to fall, with a specified level of confidence.

When σ Is Known

- The confidence interval for μ is:

$$\bar{x} \pm Z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$

- Where:
 - $\bar{x}$ = sample mean

- $(Z_{\alpha/2})$ = critical value from the standard normal distribution for the desired confidence level

Example Calculation

- Suppose the sample mean is 100, the population standard deviation (σ) is 15, and we want a 95% confidence interval.
- Find $(Z_{0.025})$ (for 95%, $(\alpha=0.05)$), which is approximately 1.96.
- The margin of error (ME):

$$(\text{ME} = 1.96 \times \frac{15}{\sqrt{50}} \approx 1.96 \times 2.12 \approx 4.16)$$

- The confidence interval:

$$(100 \pm 4.16 \rightarrow (95.84, 104.16))$$

When (σ) Is Unknown

- Use the sample standard deviation (s) and the t-distribution:

$$(\bar{x} \pm t_{\alpha/2, df} \times \frac{s}{\sqrt{n}})$$

- Where:

- $(t_{\alpha/2, df})$ = critical value based on degrees of freedom $(df = n - 1)$

Hypothesis Testing Using the Sample

Hypothesis testing allows us to make decisions about the population based on sample data.

Formulating Hypotheses

- Null Hypothesis (H_0): The population mean is equal to a specified value (μ_0).
- Alternative Hypothesis (H_a): The population mean differs from μ_0 (two-tailed), or is greater/less than μ_0 (one-tailed).

Test Statistic

- When σ is known:

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

- When σ is unknown, replace with s and use t-statistic:

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

Decision Criteria

1. Calculate the test statistic based on sample data.
2. Compare with critical value(s) from the normal or t-distribution.
3. Reject H_0 if the test statistic exceeds the critical value(s), indicating statistical significance.

Practical Applications

Frequently Asked Questions

What is the significance of sample size 50 when sampling from

an infinite population?

A sample size of 50 allows for reasonably accurate estimates of the population parameters while maintaining manageable data collection efforts, especially when sampling from an infinite population where the sample size is small relative to the population.

How does the sample size of 50 affect the precision of the sample mean estimate?

A sample size of 50 reduces the standard error of the mean, increasing the precision of the estimate, especially if the population standard deviation is known or estimated accurately.

If the population mean and standard deviation are known, how can we construct a confidence interval for the population mean based on a sample of size 50?

Using the sample mean and the known population standard deviation, a confidence interval can be constructed as: sample mean $\pm$ Z(population standard deviation / $\sqrt{50}$), where Z is the Z-score corresponding to the desired confidence level.

Why is the sample size of 50 considered sufficient for applying the Central Limit Theorem in this context?

Because a sample size of 50 is generally large enough for the sampling distribution of the sample mean to be approximately normal, even if the population distribution is not normal, due to the Central Limit Theorem.

How does knowing the population standard deviation impact the choice of statistical tests for the sample?

Knowing the population standard deviation allows the use of Z-tests for hypothesis testing and confidence intervals, which are more straightforward and accurate than t-tests when the population standard deviation is known.

What are the implications of sampling from an infinite population regarding sample independence?

Sampling from an infinite population ensures that each sample is independent, as the probability of selecting any individual remains constant, and the population is effectively limitless.

How can we estimate the population mean if the population

standard deviation is unknown with a sample size of 50?

If the population standard deviation is unknown, we use the sample standard deviation and apply the t-distribution with 49 degrees of freedom to estimate the population mean and construct confidence intervals.

What is the impact of the population mean and standard deviation on the sampling process and analysis?

Knowing the population mean and standard deviation simplifies statistical analysis, allows for precise confidence intervals, and enables the use of Z-statistics, enhancing the accuracy and interpretability of the results.

In what scenarios would a sample size of 50 be considered inadequate when sampling from an infinite population?

A sample size of 50 may be inadequate if the population has high variability, requires very high precision, or if the data distribution is highly skewed or non-normal, necessitating larger samples for reliable inference.

Additional Resources

A Sample Of Size 50 Is To Be Taken From An Infinite Population Whose Mean And Standard Deviation Are

In the realm of statistical inference, understanding how to accurately analyze and interpret data from a sample is fundamental. Whether in scientific research, quality control, or market analysis, the process often begins with selecting a representative subset from an entire population. Today, we explore a common yet critical scenario: A sample of size 50 is to be taken from an infinite population whose mean and standard deviation are known. This situation forms the backbone of many statistical procedures, providing the foundation for estimating population parameters, testing hypotheses, and making informed decisions.

This article delves into the nuances of sampling from an infinite population, the implications of knowing the population mean and standard deviation, and the statistical tools that enable reliable inference. We aim to present a comprehensive, reader-friendly analysis that balances technical rigor with clarity.

Understanding the Context: Infinite Populations and Their Significance

What Is an Infinite Population?

An infinite population refers to a theoretical or practical scenario where the population size is so large that it can be considered limitless for all practical purposes. Examples include:

- The number of grains of sand on a beach.**
- The total number of molecules in a gas sample.**
- The vast number of manufactured products produced over time.**

In such cases, the population size (N) approaches infinity, simplifying many statistical calculations. When sampling from an infinite population, the probability of selecting the same individual twice is negligible, and the population parameters (mean, standard deviation) are considered fixed and known.

Why the Assumption of Known Population Parameters Matters

Knowing the population mean (μ) and standard deviation (σ) is a significant advantage. It allows statisticians to:

- Use specific techniques that assume known parameters.**
- Calculate exact probabilities and confidence intervals more straightforwardly.**
- Reduce estimation uncertainty, leading to more**

precise inferences.

In practice, however, population parameters are often unknown, and their estimation from samples becomes necessary. But in theoretical scenarios and controlled experiments, assuming known parameters helps understand the fundamental properties of sampling distributions.

Sampling Methodology: Drawing a Sample of Size 50

Random Sampling and Its Importance

The process of selecting a sample should be random to ensure that each individual in the population has an equal chance of being chosen. Random sampling helps:

- Minimize bias.**
- Ensure the sample accurately reflects the population.**
- Justify the use of probability-based inference methods.**

For an infinite population, simple random sampling implies that each of the $n=50$ units is selected independently and with equal probability, without replacement considerations becoming problematic due to the population's vast size.

Sample Size and Its Implications

A sample size of 50 strikes a balance between feasibility and statistical reliability. While larger samples tend to produce more precise estimates, smaller samples may be more practical in real-world scenarios. The key points include:

- The Central Limit Theorem (CLT) applies as the sample size increases, ensuring the sampling distribution of the mean tends toward a normal distribution.**
- For $n=50$, the CLT generally provides a good approximation, especially when the population distribution is not heavily skewed.**

Sampling Distribution of the Sample Mean

Theoretical Foundations

When drawing a sample of size $n=50$ from a population with known mean μ and standard deviation σ , the sampling distribution of the sample mean ($\bar{x}$) has key characteristics:

- The mean of $\bar{x}$ is equal to the population mean, μ .**
- The standard deviation of $\bar{x}$, called the standard error**

(SE), is given by:

$$\text{SE} = \sigma / \sqrt{n}$$

This standard error reflects the variability of the sample mean across different samples.

Distribution Type: Normal or Approximate Normal

Since the population parameters are known and the sample size is reasonably large, the distribution of $\bar{x}$ is approximately normal (by CLT). Specifically:

- If the population distribution is normal, $\bar{x}$ follows a normal distribution exactly.**
- If the population distribution is unknown or non-normal, the CLT ensures the sampling distribution of $\bar{x}$ approaches normality as n increases, which is the case here.**

This normal approximation allows for straightforward computation of probabilities, confidence intervals, and hypothesis tests.

Estimation and Hypothesis Testing Using the Sample

Estimating the Population Mean

Given the known population standard deviation, the most precise estimate of the population mean from the sample is the sample mean itself. To construct a confidence interval (CI), we use:

$$\text{CI} = \bar{x} \pm Z \times (\sigma / \sqrt{n})$$

Where:

- $\bar{x}$ = the observed sample mean.**
- Z = the critical value from the standard normal distribution corresponding to the desired confidence level (e.g., 1.96 for 95% confidence).**
- $\sigma/\sqrt{n}$ = standard error.**

Example: For a 95% confidence level,

$$\text{CI} = \bar{x} \pm 1.96 \times (\sigma / \sqrt{50})$$

This interval estimates the range within which the true population mean μ lies with 95% certainty, assuming the sample is representative.

Conducting a Hypothesis Test for the Population Mean

Suppose we want to test whether the population mean μ equals a specific value μ_0 . The hypotheses are:

- Null hypothesis (H_0): $\mu = \mu_0$**

- **Alternative hypothesis (H_1): $\mu \neq \mu_0$ (two-tailed test)**

Using the known σ , the test statistic (Z) is:

$$Z = (\bar{x} - \mu_0) / (\sigma / \sqrt{n})$$

We compare the calculated Z to critical values ($\pm Z$) to decide whether to reject H_0 .

Practical Applications and Limitations

Real-World Scenarios

Understanding the statistical principles discussed is crucial in many fields:

- **Manufacturing: Ensuring product quality by sampling items from an infinite production stream.**
- **Environmental Science: Estimating mean pollutant levels from an ongoing, large-scale monitoring system.**
- **Market Research: Gauging customer satisfaction across a vast population of consumers.**

In all these cases, knowing the population parameters and applying the correct sampling techniques allows for precise estimates and reliable inferences.

Limitations and Considerations

While the theoretical framework is robust, practical limitations include:

- Assumption of Known σ :** In real-world scenarios, the population standard deviation is often unknown, requiring estimation from the sample.
- Sampling Bias:** Actual sampling methods may introduce bias, undermining representativeness.
- Population Variability:** If the population distribution is heavily skewed or non-normal, the normal approximation may be less accurate, especially with smaller samples.

Conclusion: The Power of Statistical Inference in Large-Scale Sampling

Sampling from an infinite population with known mean and standard deviation offers a powerful avenue for making precise statistical inferences. The key lies in leveraging the properties of the sampling distribution of the sample mean, applying the Central Limit Theorem, and understanding the importance of sample size and randomness.

By carefully selecting a sample of size 50 and applying

the appropriate statistical tools, researchers and analysts can confidently estimate population parameters, test hypotheses, and make informed decisions. While idealized in theory, these principles underpin much of the practical work in science, industry, and social sciences, illustrating the profound impact of statistical reasoning in our data-driven world.

In summary, taking a sample of size 50 from an infinite population with known parameters is not just a routine task; it encapsulates the core ideas of statistical inference, balancing theoretical elegance with practical utility. As data continues to grow in importance, mastering these concepts remains essential for anyone seeking to interpret and act upon information with confidence.

[A Sample Of Size 50 Is To Be Taken From An Infinite Population Whose Mean And Standard Deviation Are](#)

A Sample Of Size 50 Is To Be Taken From An Infinite Population Whose Mean And Standard Deviation Are

Related Articles

- **[Choose The Word That BEST Completes The Sentence."His Response Was So _____ That I](#)**

Thought He Must

- Aaron, Bob, And Carl, As Joint Tenants With Right Of Survivorship, Own 15 Acres Of Land. A Purchaser
- An Article Discusses The Market For Autographs By Mickey Mantle, The Superstar Centerfielder For The

[Back to Home](#)