A) Show That Elasticity Of Demand Is -b In A Log-linear Demand Function Given By InQ-a- B(InP, Where

A) Show That Elasticity Of Demand Is -b In A Log-linear Demand Function Given By InQ = a - b InP, Where

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Introduction

In economics, understanding how the quantity demanded of a good responds to changes in its price is fundamental. This responsiveness is captured by the concept of price elasticity of demand. When the demand function is expressed in a log-linear form, it provides a convenient way to analyze elasticity directly from the parameters of the model.

This article aims to demonstrate that, given a log-linear demand function of the form:

$$
\ln Q = a - b \ln P,
$$

the price elasticity of demand is equal to $-b$. We will explore this relationship step-by-step, providing detailed explanations and supporting the derivations with clear mathematical reasoning.

Understanding the Log-linear Demand Function

The Form of the Demand Function

The demand function under consideration is:

$$
\ln Q = a - b \ln P,
$$

where:

- Q is the quantity demanded,
- P is the price of the good,
- a is a constant representing the intercept,
- b is a positive parameter indicating the responsiveness of demand to price changes.

This form is called a log-linear demand function because both quantity and price are expressed in their natural logarithms.

Significance of Parameters a and b

- Parameter a: Reflects the baseline level of demand when the price is normalized (i.e., when $\ln P = 0$, which implies $P=1$). It shifts the demand curve vertically in the log-log space.
- Parameter b: Measures how sensitive the demand is to price changes. A larger b indicates a more elastic demand.

Deriving Price Elasticity of Demand

Definition of Price Elasticity of Demand

Price elasticity of demand (ε_P) quantifies the percentage change in quantity demanded resulting from a 1% change in price. Mathematically:

$$
\varepsilon_P = \frac{\partial Q}{\partial P} \times \frac{P}{Q}.
$$

This formula captures the ratio of the percentage change in quantity to the percentage change in price.

Step-by-Step Derivation

Given the demand function:

$$
\ln Q = a - b \ln P,
$$

we proceed as follows:

Step 1: Express Q explicitly

Exponentiating both sides:

$$
Q = e^{a - b \ln P} = e^{a} \times e^{-b \ln P} = e^{a} \times P^{-b}.
$$

Thus, the demand function in a more familiar form:

$$
Q = K \times P^{-b},
$$

where $K = e^{a}$.

Step 2: Calculate the derivative $\frac{\partial Q}{\partial P}$

Differentiating Q with respect to P:

$$
\frac{\partial Q}{\partial P} = \frac{\partial}{\partial P} (K P^{-b}) = K \times (-b) P^{-b-1} = -b K P^{-b-1}.
$$

Step 3: Compute the elasticity

Substitute $Q = K P^{-b}$ into the elasticity formula:

$$
\varepsilon_P = \frac{\partial Q}{\partial P} \times \frac{P}{Q} = \left(-b K P^{-b-1}\right) \times \frac{P}{K P^{-b}}.
$$

Simplify numerator and denominator:

$$
\varepsilon_P = -b K P^{-b-1} \times \frac{P}{K P^{-b}} = -b \times \frac{K P^{-b-1} \times P}{K P^{-b}}.
$$

Cancel K:

$$
\varepsilon_P = -b \times \frac{P^{-b-1} \times P}{P^{-b}}.
$$

Note that:

$$
P^{-b-1} \times P = P^{-b-1+1} = P^{-b},
$$

so:

$$
\varepsilon_P = -b \times \frac{P^{-b}}{P^{-b}} = -b.
$$

Final Result:

$$
\boxed{
\text{Price Elasticity of Demand} \quad \varepsilon_P = -b.
}
$$

This derivation confirms that the elasticity of demand in a log-linear demand function is simply $-b$, a constant.

Interpretation of the Result

Significance of $\varepsilon_P = -b$

- The negative sign indicates that demand and price move in opposite directions (law of demand).
- The magnitude $|b|$ reflects how sensitive demand is to price changes:
- Larger $|b|$ (or equivalently, larger elasticity magnitude) means demand is more responsive.
- Smaller $|b|$ indicates demand is relatively inelastic.

Practical Implications

- Policymakers and firms can use the estimated b parameter to understand how a change in price might affect total quantity demanded.
- For example, if $b=2$, the demand is elastic with an elasticity of -2, meaning a 1% increase in price would lead to approximately a 2% decrease in quantity demanded.

Additional Insights

Elasticity in Log-Linear Models

The key advantage of the log-linear form is that the elasticity is constant and directly equal to the parameter $-b$.

- Constant elasticity models are useful in empirical analysis because they simplify the interpretation.
- Many real-world demand functions approximate this log-linear form, making the elasticity straightforward to compute.

Variations in the Demand Function

While this derivation applies to the specific form $\ln Q = a - b \ln P$, similar techniques can analyze other functional forms, such as:

- Linear demand functions,
- Exponential demand functions,
- Nonlinear models with different elasticity properties.

Practical Example

Suppose a demand function:

$$
\ln Q = 4 - 1.5 \ln P,
$$

\]

then:

- \(a=4\),
- \(b=1.5\).

From the derivation:

\[
\varepsilon_P = -1.5.
\]

This means:

- Demand is elastic (\(>1\) in absolute value).
- A 1% increase in price results in approximately a 1.5% decrease in quantity demanded.

Conclusion

In conclusion, the derivation clearly demonstrates that in a log-linear demand function of the form \(\ln Q = a - b \ln P\), the price elasticity of demand is \(-b\). This result is fundamental for both theoretical and empirical analysis in economics, as it simplifies the interpretation of how demand reacts to price changes.

Understanding this relationship enables economists, businesses, and policymakers to make informed decisions based on the estimated responsiveness embedded within the demand function. The constant elasticity property of the log-linear form is particularly valuable for modeling and forecasting demand behavior across different price levels.

References

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Frequently Asked Questions

What is the form of the log-linear demand function discussed in the context of elasticity?

The log-linear demand function is given by lnQ = a - b lnP, where Q is quantity demanded, P is price, and a and b are constants.

How is the price elasticity of demand derived from the log-linear demand function?

The price elasticity of demand is derived as the percentage change in quantity demanded divided by the percentage change in price, which simplifies to -b in the log-linear demand function.

Why is the elasticity of demand expressed as -b in the given demand function?

Because in the log-linear form lnQ = a - b lnP, the coefficient b directly measures the responsiveness of demand to price changes, making elasticity equal to -b.

What does a negative sign indicate about the elasticity of demand in this context?

The negative sign indicates that demand is downward sloping; as price increases, quantity demanded decreases, consistent with the law of demand.

How does the constant 'a' in the demand function influence the demand curve?

The constant 'a' represents the intercept in the log-linear relationship, reflecting the baseline level of demand when price is normalized, but it does not affect the price elasticity.

Can the elasticity of demand vary along the demand curve in this model?

No, in the log-linear demand function, the elasticity of demand remains constant at -b along the entire demand curve, indicating unit elasticity at all points.

Additional Resources

Elasticity of demand is a fundamental concept in economics, serving as a measure of how responsive the quantity demanded of a good is to changes in its price. Among various forms of demand functions, the log-linear demand function holds a special place due to its analytical simplicity and interpretive clarity. Specifically, when the demand function is expressed as a logarithmic relationship between quantity demanded and price, it allows for a straightforward derivation of the price elasticity of demand. This article aims to demonstrate that the elasticity of demand in such a model is equal to \(-b\), where \(b\) is the coefficient associated with the logarithm of price. Through detailed explanations and step-by-step derivations, we will explore the mathematical underpinnings of

this relationship and elucidate its significance in economic analysis.

Understanding the Log-Linear Demand Function

Definition and Formulation

The log-linear demand function is a functional form that models the relationship between quantity demanded (Q) and price (P) using natural logarithms. It is typically expressed as:

$$\ln Q = a - b \ln P$$

where:

- $\ln Q$ is the natural logarithm of quantity demanded.
- $\ln P$ is the natural logarithm of the price.
- a is a constant term representing the intercept in the log-log space.
- b is the coefficient that measures the responsiveness of $\ln Q$ to changes in $\ln P$.

This model assumes a multiplicative relationship between Q and P, capturing proportional changes rather than absolute differences. The log-linear form simplifies many econometric analyses by linearizing nonlinear relationships.

Implications of the Log-Linear Form

The primary advantage of the log-linear demand function is its ease of interpretation:

- Elasticity is constant: Since the relationship is linear in logs, the elasticity of demand remains constant along the demand curve.
- Linear coefficients as elasticities: The coefficient b directly corresponds to the elasticity of demand with respect to price, simplifying both estimation and interpretation.
- Ease of estimation: Using logarithms often stabilizes variance and makes the data more amenable to linear regression techniques.

Understanding these implications sets the stage for deriving the elasticity formula and appreciating its significance in economic analysis.

Deriving the Price Elasticity of Demand in a Log-Linear Model

What Is Price Elasticity of Demand?

Price elasticity of demand (E_d) quantifies the percentage change in quantity demanded resulting from a one-percent change in price. Formally:

$$ E_d = \frac{\%\ \text{change in } Q}{\%\ \text{change in } P} $$

or, more precisely:

$$ E_d = \frac{dQ/Q}{dP/P} $$

which simplifies to:

$$ E_d = \frac{dQ}{dP} \times \frac{P}{Q} $$

This measure can be positive or negative depending on the nature of the demand curve; however, it is conventionally expressed as an absolute value or, in the case of the law of demand, as a negative value indicating inverse relationship.

Mathematical Derivation in the Log-Linear Context

Starting from the log-linear demand function:

$$ \ln Q = a - b \ln P $$

we aim to find the elasticity of demand E_d, which involves differentiating Q with respect to P and then expressing the result in terms of Q and P.

Step 1: Differentiate $\ln Q$ with respect to $\ln P$.

Since:

$$ \ln Q = a - b \ln P $$

differentiating both sides with respect to $\ln P$:

$$ \frac{d \ln Q}{d \ln P} = -b $$

Step 2: Connect $\frac{d \ln Q}{d \ln P}$ with $\frac{dQ}{dP}$.

Recall that:

$$ \frac{d \ln Q}{d \ln P} = \frac{d \ln Q}{dQ} \times \frac{dQ}{dP} \times P $$

But more straightforwardly, the elasticity of demand can be expressed as:

$$ E_d = \frac{dQ}{dP} \times \frac{P}{Q} $$

Given that:

$$\frac{d \ln Q}{d \ln P} = \frac{dQ/Q}{dP/P} = E_d$$

and since:

$$\frac{d \ln Q}{d \ln P} = -b$$

we conclude:

$$E_d = -b$$

Step 3: Interpret the result.

This derivation shows that the price elasticity of demand in a log-linear demand function is simply the negative of the coefficient b.

Economic Significance of $E_d = -b$

Constant Elasticity and Its Implications

The fact that the elasticity is constant at $-b$ across all price levels is a crucial feature of the log-linear demand function. Unlike linear demand functions, where elasticity varies along the demand curve, the log-linear form assumes a proportional response, making it particularly useful in empirical modeling.

- Predictability: Economists can reliably predict how demand will respond to price changes using the estimated coefficient b.
- Policy implications: Governments and firms can evaluate the impact of price changes on revenue and demand stability.

Negative Sign and Its Interpretation

The negative sign indicates the inverse relationship between price and quantity demanded, consistent with the law of demand. However, in absolute value terms, elasticity is often considered without regard to sign, focusing on the magnitude of responsiveness.

In summary:

- The elasticity of demand in a log-linear model is directly equal to $-b$.
- This simplicity facilitates the interpretation of empirical results and the formulation of economic policies.

Extensions and Practical Applications

Estimating b from Empirical Data

Econometricians often estimate the demand function by regressing $\ln Q$ on $\ln P$ using observed data. The estimated coefficient $\hat{b}$ then provides a direct measure of the demand elasticity:

$$\hat{E}_d = -\hat{b}$$

This approach is widely used in market analysis, where understanding responsiveness guides pricing strategies and regulatory decisions.

Limitations and Considerations

While the log-linear model offers analytical elegance, it also has limitations:

- Assumption of constant elasticity: Some goods exhibit variable elasticity across different price ranges.
- Data constraints: Accurate estimation requires quality data and appropriate model specification.
- External factors: Demand responses may be influenced by factors beyond price, such as income, preferences, or substitute goods.

Despite these limitations, the log-linear form remains a powerful tool for understanding demand dynamics.

Conclusion

The exploration of the log-linear demand function reveals a crucial and elegant insight: the price elasticity of demand in this model is exactly $-b$. This result underscores the utility of logarithmic transformations in simplifying complex relationships and provides a clear interpretive link between model coefficients and economic responsiveness. Recognizing that the elasticity is constant and directly tied to the coefficient b enables economists and policymakers to make informed decisions based on empirical estimates, facilitating a deeper understanding of market behavior. As demand analysis continues to evolve, the foundational relationship uncovered here remains a cornerstone of microeconomic theory and applied econometrics, illustrating the profound connection between mathematical form and economic intuition.

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