A Small Planet Having A Radius Of 1000 Km Exerts A Gravitational Force Of 100 N On An Object That Is

Introduction

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Understanding the gravitational interactions between celestial bodies and objects on their surfaces is fundamental in physics and astronomy. In this article, we explore the intriguing scenario of a tiny planet with a radius of merely 1000 km exerting a specific gravitational force of 100 N on an object. We will analyze the parameters involved, derive the relationships governing gravity, and interpret what this signifies about the planet's mass, density, and other physical characteristics.

Fundamentals of Gravitational Force

Newton's Law of Universal Gravitation

At the core of our analysis lies Newton's law of universal gravitation, which states that every point mass attracts every other point mass in the universe with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. Mathematically:

- $F = G (m_1 m_2) / r^2$

Where:

- F is the gravitational force between the two objects
- G is the gravitational constant, approximately 6.674×10^{-11} N·(m/kg)²
- m_1 is the mass of the planet
- m_2 is the mass of the object
- r is the distance between the centers of the two objects (radius of the planet if the object is on the surface)

Mass and Radius Relationship

The mass of the planet can be related to its density (ρ) and volume (V) by:

- $m = \rho V$

Since the planet is spherical, its volume is:

- $V = (4/3)\pi r^3$

Determining the Mass of the Planet

Given Data and Assumptions

From the problem, we have:

- Radius of the planet, $r = 1000$ km $= 1{,}000{,}000$ meters
- Force exerted on the object, $F = 100$ N

Assuming the object is located on the surface of the planet, the distance r in Newton's law is the radius of the planet itself. The mass of the object, m_2, is unknown and needs to be determined or expressed in terms of the other parameters.

Relating Force, Mass of Object, and Planet

From Newton's law:

$$F = G (m_1 m_2) / r^2$$

Rearranged to solve for m_1 (the planet's mass):

$$m_1 = (F r^2) / (G m_2)$$

Since m_2 is unknown, we can express the mass of the object in terms of its weight or other known quantities if provided. For the purpose of this analysis, let's consider typical scenarios and assume the object has a mass m_2.

Expressing the Object's Mass

If the object has a known weight (W), then:

- $W = m_2 g$

where g is the acceleration due to gravity at the planet's surface, which we will calculate later. Alternatively, without specific data about m_2, we can express the ratio of the planet's mass to the object's mass based on the force:

Calculating the Planet's Mass

Deriving the Relationship with the Surface Gravity

The acceleration due to gravity at the surface of the planet (g) is given by:

- $g = G m_1 / r^2$

Rearranged to find m_1:

- $m_1 = g\, r^2 / G$

Since the gravitational force on the object is also $F = m_2\, g$, then:

- $g = F / m_2$

Substituting into the mass equation:

$$m_1 = (F / m_2)\ r^2 / G$$

Rearranged:

$$m_1 = F\ r^2 / (G\ m_2)$$

This confirms our earlier expression, emphasizing the dependence on the object's mass.

Estimating the Surface Gravity

Suppose we do not know m_2, but we understand that for an object experiencing a force of 100 N at the surface, with a certain mass m_2, the gravity is:

- $g = F / m_2$

For example, if the object is 10 kg, then:

- $g = 100\ \text{N} / 10\ \text{kg} = 10\ \text{m/s}^2$

This would be comparable to Earth's gravity, indicating a significant gravitational pull for such a small planet.

Determining the Density of the Planet

Using the Derived Mass

If we know or estimate the mass of the planet, we can determine its average density:

- $\rho = M / V = M / [(4/3)\pi R^3]$

Sample Calculation

Assuming the object has a mass $m_2 = 10$ kg, and the force exerted is 100 N, then the surface gravity g is 10 m/s^2. Using the relation for m_1:

$$m_1 = F\ r^2 / (G\ m_2) = 100\ \text{N}\ \ (1{,}000{,}000\ \text{m})^2 / (6.674\times10^{-11}\ \text{N}\cdot(\text{m/kg})^2\ \ 10\ \text{kg})$$

Calculating numerator:

$$100 \quad (1\times10^{6})^{2} = 100 \quad 1\times10^{12} = 1\times10^{14}$$

Calculating denominator:

$$6.674\times10^{-11} \quad 10 = 6.674\times10^{-10}$$

Therefore, the mass of the planet:

$$m_1 = 1\times10^{14} / 6.674\times10^{-10} \approx 1.498\times10^{23}\ \text{kg}$$

Now, the volume of the planet:

$$V = (4/3)\pi(1\times10^{6})^{3} \approx 4.1888\times10^{18}\ \text{m}^3$$

Finally, the average density:

$$\rho = m_1 / V \approx 1.498\times10^{23}\ \text{kg} / 4.1888\times10^{18}\ \text{m}^3 \approx 3.58\times10^{4}\ \text{kg/m}^3$$

This density (~35,800 kg/m^3) is extremely high, suggesting a very dense material, possibly akin to metallic or exotic planetary compositions.

Implications and Physical Characteristics

Size and Composition

The small radius (1000 km) indicates a planet significantly smaller than Earth, which has a radius of approximately 6371 km. The derived mass and density suggest a dense body, possibly composed of heavy elements or exotic materials.

Surface Gravity and Potential Environment

With a gravity similar to Earth's (around 10 m/s^2), an object on this planet would experience a familiar gravitational pull, despite the planet's diminutive size. This has implications for potential surface conditions, atmospheric retention, and the possibility of hosting life or conducting experiments.

Comparison with Known Celestial Bodies

- The Moon has a radius of about 1737 km and a mass of 7.35×10^{22} kg, with surface gravity of 1.62 m/s^2.
- Our hypothetical planet has a smaller radius but a much higher density, indicating a different composition or formation history.

Conclusion

The scenario of a small planet with a radius of 1000 km exerting a 100 N gravitational force on an object provides a fascinating glimpse into planetary physics. By applying Newton's law of gravitation, we deduce that such a planet would need to be exceptionally dense to produce the observed force at its surface. The calculations highlight the importance of understanding the relationships between mass, radius, density, and gravitational acceleration. This analysis

Frequently Asked Questions

What is the gravitational acceleration on the surface of a small planet with a radius of 1000 km exerting a 100 N force on an object?

The gravitational acceleration can be calculated using Newton's law: g = F/m. Without the mass of the object, we cannot find g directly, but assuming the object's mass, we can determine g based on the force exerted.

How do you calculate the mass of the small planet with a radius of 1000 km exerting a 100 N force on an object?

Using Newton's law of gravitation: F = G (M m) / r^2. Rearranged to find M: M = (F r^2) / (G m). You need the mass of the object to compute the planet's mass.

What is the significance of the gravitational force of 100 N on an object on this small planet?

It indicates the strength of the planet's gravitational pull on the object, which affects the object's weight and potential for surface activities.

How does the small radius of 1000 km influence the gravitational force exerted by the planet?

A smaller radius generally increases gravitational acceleration if the mass remains constant, leading to a stronger gravitational force on objects near its surface.

Can such a small planet with a radius of 1000 km support life, considering its gravitational force?

While gravity is a factor, supporting life depends on many conditions. The planet's mass, atmosphere, temperature, and other factors are critical; the given data alone is insufficient to determine habitability.

What formulas are useful to analyze the gravitational interactions on a small planet with given parameters?

Newton's law of gravitation, F = G (M m) / r^2, and the formula for gravitational acceleration, g = G M / r^2, are essential for such analysis.

Additional Resources

A Small Planet Having A Radius Of 1000 Km Exerts A Gravitational Force Of 100 N On An Object That Is

Understanding the gravitational interactions between celestial bodies and objects on or near their surfaces is fundamental to physics and planetary science. The statement about a small planet with a radius of 1000 km exerting a gravitational force of 100 N on an object offers an intriguing basis for analyzing gravitational principles, planetary characteristics, and their implications. In this article, we will explore the physics behind this scenario, examine the parameters involved, and discuss broader insights into planetary gravity, density, and their effects.

Understanding the Given Scenario

The core statement involves a small planet with a specific radius exerting a certain gravitational force on an object. To analyze this, we need to interpret the data:

- Radius of the planet, $R = 1000\, \text{km} = 1 \times 10^6\, \text{m}$
- Gravitational force on the object, $F = 100\, \text{N}$
- Mass of the object, which is not specified and will be considered as m

Our goal: to determine the mass of the object, the mass of the planet, and understand the characteristics of this planet.

Applying Newton's Law of Universal Gravitation

Newton's law states:

$$
F = G \frac{M \times m}{r^2}
$$

Where:

- F = gravitational force
- G = gravitational constant $\approx 6.674 \times 10^{-11}\, \text{Nm}^2/\text{kg}^2$
- M = mass of the planet
- m = mass of the object
- r = distance between centers (here, equal to the radius)

Assuming the object is on the surface of the planet:

$$
F = G \frac{M \times m}{R^2}
$$

Rearranged to find M:

$$
M = \frac{F \times R^2}{G \times m}
$$

Since m is unknown, we need to consider typical object masses or express M in terms of m.

Determining the Mass of the Object

Suppose the object is a person or an object with a mass of 10 kg (a common benchmark):

$$
m = 10\, \text{kg}
$$

Substituting into the formula:

$$
M = \frac{100\, \text{N} \times (1 \times 10^6\, \text{m})^2}{6.674 \times 10^{-11} \times 10\, \text{kg}}
$$

Calculating numerator:

$$
100 \times 1 \times 10^{12} = 1 \times 10^{14}
$$

Calculating denominator:

$$
6.674 \times 10^{-11} \times 10 = 6.674 \times 10^{-10}
$$

Thus:

$$
M = \frac{1 \times 10^{14}}{6.674 \times 10^{-10}} \approx 1.5 \times 10^{23}\, \text{kg}
$$

This mass is comparable to that of Mercury ($\sim 3.3 \times 10^{23}$ kg), indicating the planet is quite dense.

Analyzing Planetary Density

Knowing the mass M and radius R, we can compute the average density:

$$
\rho = \frac{M}{V}
$$

Where volume of the sphere:

$$
V = \frac{4}{3} \pi R^3
$$

Calculating:

$$
V = \frac{4}{3} \pi (1 \times 10^6)^3 = \frac{4}{3} \pi \times 10^{18} \approx 4.19 \times 10^{18}\, \text{m}^3
$$

Then,

$$
\rho = \frac{1.5 \times 10^{23}}{4.19 \times 10^{18}} \approx 3.58 \times 10^{4}\, \text{kg/m}^3
$$

This density (~35,800 kg/m³) suggests a very dense planet, comparable to Earth's core density (~13,000 kg/m³), indicating this hypothetical planet could be composed of heavy elements.

Physical Features and Characteristics of the Planet

Based on the calculations:

- Size: With a radius of 1000 km, the planet is smaller than Earth (~6371 km radius), but still sizable.
- Mass and Density: The high density implies a composition rich in heavy elements like iron or nickel.
- Gravity: The surface gravity g can be calculated:

$$
g = G \frac{M}{R^2} = \frac{F}{m}
$$

For $m = 10\, \text{kg}$:

$$
g = \frac{100\, \text{N}}{10\, \text{kg}} = 10\, \text{m/s}^2
$$

This is roughly equivalent to Earth's gravity (~9.8 m/s²), meaning objects would experience similar weight.

Implications and Broader Context

Understanding such a small yet dense planet offers insights into planetary formation, structure, and gravity.

Features of Small, Dense Planets

- High Density: Indicates a composition of metals or dense minerals.
- Surface Gravity: Can be comparable to Earth despite smaller size, if mass is sufficiently high.
- Potential for Habitability: The high gravity could influence atmospheric retention and surface conditions.

Pros and Cons of Small, Dense Planets

Pros:

- Strong Gravitational Field: Could support a substantial atmosphere.
- Compact Size: Easier to explore or colonize due to smaller surface area.
- Unique Geology: High density suggests unique geological processes.

Cons:

- High Gravity Effects: Could be challenging for human habitation.
- Limited Surface Area: Less space for ecosystems or resources.
- Potential for Extreme Conditions: High density may correlate with intense heat or magnetic fields.

Real-World Analogues and Comparisons

While hypothetical, such a planet resembles some of the densest known celestial bodies, like neutron stars, which have enormous densities but are far smaller in radius. For planets, the densest terrestrial planets, like Mercury and Earth, help illustrate the principles underpinning this scenario.

Conclusion

The analysis of a small planet with a radius of 1000 km exerting a 100 N gravitational force on an object reveals fascinating insights into planetary physics. Through applying Newton's law, we deduced that the planet would need to be extraordinarily dense, with a mass comparable to large planetary bodies like Mercury, but packed into a smaller volume. This results in high surface gravity similar to Earth's, making it an intriguing candidate for theoretical studies on planetary structure and gravity. Such explorations deepen our understanding of planetary formation, the diversity of celestial bodies, and the fundamental laws governing our universe.

Key Takeaways:

- Small planetary radius does not necessarily mean weak gravity; density plays a crucial role.
- High density indicates a composition rich in heavy elements.
- Surface gravity can be comparable to Earth's on a small, dense planet.
- Analyzing such scenarios helps understand real exoplanets and celestial bodies beyond our solar system.

This comprehensive exploration underscores the importance of fundamental physics in planetary science and encourages further curiosity about the diverse worlds that might exist across the universe.

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