A Student Writes The Equation For A Line That Has A Slope Of -6 And Passes Through The Point (2, 8).

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Understanding how to find the equation of a line given specific information is a fundamental concept in algebra and coordinate geometry. When a student is tasked with writing the equation of a line that has a particular slope and passes through a given point, they are essentially applying the point-slope form of a linear equation. This process involves understanding key concepts such as the slope of a line, the coordinates of a point, and how to manipulate these elements into an algebraic expression that accurately describes the line.

In this article, we will explore in detail how a student can write the equation of a line with a slope of -6 passing through the point (2, 8). We will break down the steps involved, examine the relevant formulas, and provide practical examples to clarify the process. Whether you're a student learning this for the first time or someone seeking to reinforce your understanding, this guide aims to make the task clear and accessible.

Understanding the Basics of Line Equations

Before delving into the specific problem, it's essential to review the foundational concepts related to the equations of lines.

The Slope of a Line

- The slope, often denoted as m, measures the steepness of a line.
- It is calculated as the change in y over the change in x between two points on the line:

$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

- The slope can be positive (line rises from left to right), negative (line falls from left to right), zero (horizontal line), or undefined (vertical line).

The Coordinates of a Point

- A point on the coordinate plane is expressed as (x, y).
- In our problem, the point is given as (2, 8).

The Equation of a Line

- Several forms exist, but the most relevant for this scenario are:
- Point-Slope Form:

$$
y - y_1 = m(x - x_1)
$$

- Slope-Intercept Form:

$$
y = mx + b
$$

where b is the y-intercept.

Applying the Point-Slope Form to the Problem

Given:
- Slope, $m = -6$
- Point, $(x_1, y_1) = (2, 8)$

The point-slope form allows us to directly substitute these values to write the equation.

Step-by-Step Derivation

1. Write the point-slope form:

$$
y - y_1 = m(x - x_1)
$$

2. Substitute $y_1 = 8$, $x_1 = 2$, and $m = -6$:

$$
y - 8 = -6(x - 2)
$$

3. Simplify the equation:

$$
y - 8 = -6x + 12
$$

4. Solve for y to get the slope-intercept form:

$$
y = -6x + 12 + 8
$$

$$
y = -6x + 20
$$

This is the equation of the line passing through (2, 8) with a slope of -6.

Final Equation of the Line

$$
\boxed{
y = -6x + 20
}
$$

This equation succinctly captures the line's slope and position in the coordinate plane.

Understanding the Significance of the Equation

Now that the line's equation has been derived, it’s important to understand what it represents and how it can be used.

Graphing the Line

- The slope of -6 indicates that for every 1 unit increase in x, y decreases by 6 units.
- The y-intercept, 20, means the line crosses the y-axis at the point (0, 20).

Using the Equation

- To find points on the line, substitute different x-values into the equation and compute y.
- To graph the line, plot the y-intercept and use the slope to find additional points.

Applications

- This line can represent real-world scenarios such as rate of change, cost vs. units, or other relationships where the rate (slope) is negative.

Additional Methods for Writing Line Equations

While the point-slope form is straightforward, there are alternative methods to derive the line's equation, which are useful in different contexts.

Using the Slope-Intercept Form

- Once you know the slope and a point, you can directly write the equation in slope-intercept form without intermediate steps.
- For example, with $m = -6$ and the point (2, 8):

$$
y = -6x + b
$$

Plugging in the point:

$$
8 = -6(2) + b
$$

$$
8 = -12 + b
$$

$$
b = 8 + 12 = 20
$$

Hence, the equation is $y = -6x + 20$.

Using Two Points

- If two points on the line are known, the slope can be calculated, and then the line's equation can be written.
- For example, if the line passes through (2, 8) and (4, -4):
- Calculate the slope:

$$
m = \frac{-4 - 8}{4 - 2} = \frac{-12}{2} = -6
$$

- Use either point with the point-slope form to derive the equation.

Common Mistakes and Tips for Students

Understanding the process thoroughly helps avoid common errors. Here are some tips:

1. **Always verify the slope and point:** Make sure the values given are correct before proceeding.

2. **Be attentive to signs:** Remember that the slope is negative, which affects the graph and the equation's structure.

3. **Substitute correctly:** When plugging in the point into formulas, double-check the coordinates to avoid mistakes.

4. **Simplify carefully:** Ensure algebraic steps are accurate when solving for y or b.

5. **Practice with different points and slopes:** This enhances understanding and confidence.

Practical Examples and Practice Problems

To reinforce learning, here are some practice problems similar to our initial scenario:

Example 1: Write the equation of a line with a slope of -6 passing through (2, 8).

- Solution: As shown above, the equation is $y = -6x + 20$.

Example 2: Find the equation of a line with a slope of 3 passing through (1, -2).

- Solution:
- Use point-slope form:

$$
y - (-2) = 3(x - 1)
$$

$$
y + 2 = 3x - 3
$$

$$
y = 3x - 5
$$

Practice Problem:

- Write the equation of a line with a slope of -4 passing through the point (3, 7).

Answer:
- Using point-slope form:

$$
y - 7 = -4(x - 3)
$$

$$
y - 7 = -4x + 12
$$

$$
y = -4x + 19
$$

Conclusion: Mastering Line Equations

Writing the equation of a line given its slope and a point is a fundamental skill in algebra that lays the foundation for understanding more complex topics like linear functions, systems of equations, and graphing. By mastering the point-slope and slope-intercept forms, students can efficiently derive and analyze lines in various contexts.

In our specific example, a student successfully derived the line's equation as $y = -6x + 20$ by applying the point-slope formula with the given point and slope. This process highlights the importance of understanding the relationships between slope, points, and line equations, which are essential tools in mathematics and real-world problem-solving.

Continued practice with different scenarios will deepen comprehension and build confidence in handling linear equations, an indispensable part of the mathematical toolkit.

Frequently Asked Questions

How do you write the equation of a line with a slope of -6 passing through the point (2, 8)?

Use the point-slope form: $y - y_1 = m(x - x_1)$. Plug in $m = -6$ and $(x_1, y_1) = (2, 8)$: $y - 8 = -6(x - 2)$. Simplify to slope-intercept form if needed.

What is the slope-intercept form of the line with slope -6 passing through (2, 8)?

Starting from y - 8 = -6(x - 2), expand to y - 8 = -6x + 12, then add 8 to both sides: y = -6x + 20.

Why do we use the point-slope form to find the line’s equation in this scenario?

Because we know the slope and a point on the line, the point-slope form directly incorporates both, making it straightforward to find the equation.

Can you verify that the point (2, 8) satisfies the equation y = -6x + 20?

Yes, substitute x = 2: y = -6(2) + 20 = -12 + 20 = 8. Since y = 8 matches the point’s y-coordinate, the point lies on the line.

What is the significance of the slope being -6 in the line’s equation?

A slope of -6 indicates the line is decreasing; for each unit increase in x, y decreases by 6 units, showing a steep downward trend.

How would the equation change if the line passed through a different point, say (3, 10), but still had slope -6?

Using point-slope form: y - 10 = -6(x - 3). Simplify to get y = -6x + 28.

Additional Resources

Understanding the Equation of a Line: A Deep Dive into Slope-Point Form

When exploring the fascinating realm of coordinate geometry, one of the fundamental skills students learn is how to formulate the equation of a line given specific information. In this detailed analysis, we'll examine a practical example: a student writing the equation for a line that has a slope of -6 and passes through the point (2, 8). This process not only reinforces core mathematical concepts but also highlights the importance of precise formulation and understanding the underlying principles.

Introduction to the Equation of a Line

Before delving into the specifics of our example, it’s crucial to understand the basic forms and concepts involved in describing lines algebraically.

What is a line?
In the Cartesian plane, a line is a collection of points extending infinitely in both directions, characterized by a specific relationship between the x and y coordinates of these points.

Why is the equation important?
The equation of a line provides a compact, algebraic way to describe all points lying on that line. It enables us to predict, analyze, and graph the line efficiently.

Main forms of line equations:
- Slope-intercept form: $y = mx + b$
- Point-slope form: $y - y_1 = m(x - x_1)$
- Standard form: $Ax + By = C$

In our case, the point-slope form is particularly useful because it directly incorporates the slope and a specific point the line passes through.

Understanding the Given Data

Our scenario involves two pieces of information:

- Slope (m): -6
- Point (x_1, y_1): (2, 8)

Interpreting the data:
- The slope of -6 indicates the line descends steeply; for every increase of 1 unit in x, y decreases by 6 units.
- The point (2, 8) is a known point on the line, providing a reference for the line's position.

Formulating the Equation: Step-by-Step Process

Creating the line’s equation involves applying the point-slope formula, which is straightforward and intuitive:

$$
y - y_1 = m(x - x_1)
$$

Step 1: Substitute known values

$$
y - 8 = -6(x - 2)
$$

Step 2: Simplify the equation

Distribute the slope:

$$
y - 8 = -6x + 12
$$

Add 8 to both sides to solve for y:

$$
y = -6x + 12 + 8
$$

$$
y = -6x + 20
$$

Result: The slope-intercept form of the line is:

$$
\boxed{y = -6x + 20}
$$

This formula succinctly captures the entire line, indicating it has a slope of -6 and a y-intercept at 20.

Key Features of the Resultant Line Equation

Understanding the features of this line enhances comprehension:

1. Slope (m = -6)

- The slope indicates the line’s steepness and direction.
- A negative slope signifies a line descending from left to right.
- The magnitude (6) shows a steep decline.

2. Y-intercept (b = 20)

- The point where the line crosses the y-axis.
- Represents the y-value when x = 0.

- Graphically, this is the point (0, 20).

3. Graphical Representation

Plotting the line involves:

- Marking the y-intercept at (0, 20).
- Using the slope to find another point:
Starting from (0, 20), move 1 unit right (x increases by 1), then move down 6 units (since slope is -6), placing the second point at (1, 14).
- Drawing a straight line through these points.

Verifying the Line Passes Through the Given Point

Validation is an essential step in confirming the correctness of the derived equation.

Check: Does the point (2, 8) satisfy $y = -6x + 20$?

Substitute x = 2:

$$
y = -6(2) + 20 = -12 + 20 = 8
$$

Yes, the point (2, 8) satisfies the equation, confirming its validity.

Alternative Forms and Their Use Cases

While the slope-intercept form is elegant and easy to interpret, other forms offer advantages in different contexts.

1. Standard Form: $Ax + By = C$

Convert $y = -6x + 20$:

$$
6x + y = 20
$$

This form is useful in systems of equations and when analyzing relationships between x and y.

2. Point-Slope Form: $y - y_1 = m(x - x_1)$

Already used, it’s particularly handy when a specific point and slope are known, especially for quick derivations.

Common Mistakes and How to Avoid Them

When deriving the equation, students often encounter pitfalls:

- Incorrect substitution: Ensure the point coordinates and slope are correctly inserted.
- Sign errors: Pay attention to the signs when distributing and simplifying.
- Misidentifying the slope: Confirm that the slope matches the problem statement.
- Forgetting to validate: Always substitute the known point into the derived equation.

Practical Applications and Significance

Understanding how to write the equation of a line from given data extends beyond academic exercises. It has real-world significance:

- Engineering: Designing structures with specific slopes.
- Economics: Modeling linear relationships between variables.
- Physics: Describing motion along a straight path.
- Data Science: Fitting linear models to data points.

In educational contexts, mastering this skill promotes analytical thinking and problem-solving prowess.

Conclusion: The Power of Precise Formulation

The process of writing the equation of a line given a slope and a point exemplifies the elegance of algebraic problem-solving. Starting from fundamental principles, applying the point-slope form ensures clarity and accuracy, culminating in the familiar slope-intercept form that vividly describes the line’s behavior.

By meticulously substituting known values, simplifying systematically, and verifying results, students develop not only mathematical proficiency but also confidence in tackling more complex geometric problems. This example underscores the importance of foundational knowledge, attention to detail, and the ability to translate real-world data into precise mathematical expressions—skills that are invaluable across various scientific and technical disciplines.

A Student Writes The Equation For A Line That Has A Slope Of 6 And Passes Through The Point 2 8

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