A Triangle Has Sides Equal To 5 Cm, 10 Cm And 7 Cm. Find Its Angles. Options : A 111.8, 40.5, 27.7 B

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Understanding the Problem: Finding the Angles of a Triangle

When given the lengths of the sides of a triangle, calculating its interior angles is a fundamental problem in geometry. In this specific case, the triangle has sides measuring 5 cm, 10 cm, and 7 cm. Our goal is to determine the measures of its three angles.

This problem is an excellent example of applying the Law of Cosines, which relates the sides of a triangle to its angles. The Law of Cosines is especially useful when dealing with non-right triangles or when you know all three sides but not the angles.

Overview of Triangle Properties and Relevant Theorems

Triangle Inequality Theorem

- This theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the remaining side.
- For our triangle:
- 5 + 7 > 10 → 12 > 10 ✓
- 5 + 10 > 7 → 15 > 7 ✓
- 7 + 10 > 5 → 17 > 5 ✓
- All conditions are satisfied, so the given sides can form a triangle.

Law of Cosines

- It relates the lengths of sides to the cosine of the included angle:

$$
c^2 = a^2 + b^2 - 2ab \cos C
$$

where:
- a, b, and c are sides
- C is the angle opposite side c

- To find an angle, rearranged as:

$$
\cos C = \frac{a^2 + b^2 - c^2}{2ab}
$$

- We will apply this formula to find each angle.

Step-by-Step Calculation of Triangle Angles

The sides are:
- $a = 5\, \text{cm}$
- $b = 7\, \text{cm}$
- $c = 10\, \text{cm}$

We will find the angles:
- A, opposite side $a = 5\, \text{cm}$
- B, opposite side $b = 7\, \text{cm}$
- C, opposite side $c = 10\, \text{cm}$

Calculating Angle C (Opposite the 10 cm side)

Using the Law of Cosines:
$$
\cos C = \frac{a^2 + b^2 - c^2}{2ab}
$$

Plugging in values:
$$
\cos C = \frac{5^2 + 7^2 - 10^2}{2 \times 5 \times 7} = \frac{25 + 49 - 100}{70} = \frac{-26}{70} \approx -0.3714
$$

Calculating C:
$$
C = \arccos(-0.3714) \approx 111.8^\circ
$$

Calculating Angle A (Opposite the 5 cm side)

$$
\cos A = \frac{b^2 + c^2 - a^2}{2bc}
$$

$$
\cos A = \frac{7^2 + 10^2 - 5^2}{2 \times 7 \times 10} = \frac{49 + 100 - 25}{140} = \frac{124}{140} \approx 0.8857
$$

$$
A = \arccos(0.8857) \approx 27.7^\circ
$$

Calculating Angle B (Opposite the 7 cm side)

$$
\cos B = \frac{a^2 + c^2 - b^2}{2ac}
$$

$$
\cos B = \frac{5^2 + 10^2 - 7^2}{2 \times 5 \times 10} = \frac{25 + 100 - 49}{100} = \frac{76}{100} = 0.76
$$

$$
B = \arccos(0.76) \approx 40.5^\circ
$$

Summary of Calculated Angles

Angle	Measure (degrees)	Approximate Value
A	Opposite 5 cm side	27.7°
B	Opposite 7 cm side	40.5°
C	Opposite 10 cm side	111.8°

The sum of these angles:

$$
27.7^\circ + 40.5^\circ + 111.8^\circ \approx 180^\circ
$$

which confirms the validity of our calculations.

Interpreting the Results and Choosing the Correct Option

Given the options:
- A) 111.8, 40.5, 27.7
- B) (not specified here)

Our calculated angles match option A precisely:
- $C \approx 111.8^\circ$
- $B \approx 40.5^\circ$
- $A \approx 27.7^\circ$

Therefore, the correct answer is Option A.

Practical Applications of Triangle Angle Calculations

Understanding how to find angles from side lengths is essential in various fields:
- Architecture and construction: Ensuring structures meet design specifications.
- Navigation: Calculating courses and bearings.
- Engineering: Designing mechanical parts with precise angles.
- Mathematics Education: Building foundational knowledge in geometry.

Additional Tips for Solving Similar Problems

- Always verify the triangle inequality before proceeding.
- Use the Law of Cosines when all three sides are known.
- For right triangles, Pythagoras’ theorem simplifies calculations.
- Convert angles to degrees if using a calculator in degree mode.
- Double-check calculations to avoid common errors.

Conclusion

Calculating the angles of a triangle based on given side lengths involves a clear understanding of the Law of Cosines. In our specific problem with sides measuring 5 cm, 10 cm, and 7 cm, the angles are approximately 27.7°, 40.5°, and 111.8°, respectively. Recognizing these values allows us to confidently select the correct option, which in this case is Option A.

Mastering these techniques enhances your problem-solving skills in geometry and prepares you for more complex mathematical challenges. Whether for academic purposes, engineering, or everyday applications, understanding how to derive angles from side lengths is a vital skill in the mathematical toolkit.

Frequently Asked Questions

How can the triangle's angles help in solving real-world problems involving this triangle?

Knowing the angles allows for precise calculations of distances, areas, and other geometric properties, which can be useful in construction, navigation, and design applications where such triangles are involved.

Additional Resources

Triangle Side Lengths and Angle Calculation: An In-Depth Analysis

Understanding the properties of triangles is fundamental in geometry, with applications spanning architecture, engineering, and various scientific fields. In this article, we will meticulously analyze a triangle with side lengths of 5 cm, 10 cm, and 7 cm, aiming to determine its internal angles. This exploration combines theoretical principles with practical calculations, providing comprehensive insights into the triangle's structure.

Understanding the Triangle's Sides and the Context of the Problem

The given triangle has three sides:

- Side a = 5 cm
- Side b = 10 cm
- Side c = 7 cm

The problem asks us to find the angles corresponding to these sides, with options provided as approximate values:

- Option A: 111.8°, 40.5°, 27.7°

The core challenge lies in determining which angles correspond to which sides and verifying the options against calculated results.

Fundamental Concepts Used in Triangle Angle Calculations

Before diving into calculations, let’s review the key principles relevant to our problem:

2.1 Triangle Inequality Theorem

This theorem states that for any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side. Validating this ensures our side lengths can indeed form a triangle:

- 5 + 7 = 12 > 10 ✔
- 5 + 10 = 15 > 7 ✔
- 7 + 10 = 17 > 5 ✔

Since all these inequalities hold, the triangle exists.

2.2 Law of Cosines

The Law of Cosines is central to calculating angles when side lengths are known:

$$
\cos C = \frac{a^2 + b^2 - c^2}{2ab}
$$

Similarly, for angles A and B:

$$
\cos A = \frac{b^2 + c^2 - a^2}{2bc}
$$
$$
\cos B = \frac{a^2 + c^2 - b^2}{2ac}
$$

This law is particularly useful when the triangle isn't right-angled and side lengths are known.

2.3 Sum of Angles in a Triangle

The sum of internal angles in any triangle is 180°. Once two angles are known, the third can be found by subtraction:

$$
A + B + C = 180^\circ
$$

Step-by-Step Calculation of the Triangle’s Angles

To proceed systematically, we will:

1. Assign sides to variables:
- a = 5 cm
- b = 10 cm
- c = 7 cm

2. Use the Law of Cosines to find each angle.

Calculating Angle C (opposite side c = 7 cm)

Applying the Law of Cosines:

$$
\cos C = \frac{a^2 + b^2 - c^2}{2ab}
$$

Plugging in the values:

$$
\cos C = \frac{5^2 + 10^2 - 7^2}{2 \times 5 \times 10} = \frac{25 + 100 - 49}{100} = \frac{76}{100} = 0.76
$$

Calculating the angle:

$$
C = \cos^{-1}(0.76) \approx 40.54^\circ
$$

Thus, the angle opposite side 7 cm is approximately 40.5°.

Calculating Angle B (opposite side b = 10 cm)

Using the Law of Cosines:

$$
\cos B = \frac{a^2 + c^2 - b^2}{2ac}
$$

Plugging in the numbers:

$$
\cos B = \frac{5^2 + 7^2 - 10^2}{2 \times 5 \times 7} = \frac{25 + 49 - 100}{70} = \frac{-26}{70} \approx -0.3714
$$

Calculating the angle:

$$
B = \cos^{-1}(-0.3714) \approx 111.86^\circ
$$

The angle opposite side 10 cm is approximately 111.8°.

Calculating Angle A (opposite side a = 5 cm)

Alternatively, since the sum of angles is 180°, and we've computed angles B and C, we can find A:

$$
A = 180^\circ - B - C \approx 180^\circ - 111.8^\circ - 40.5^\circ = 27.7^\circ
$$

The angle opposite side 5 cm is approximately 27.7°.

Summary of Findings and Correspondence with Options

Side Length	Opposite Angle	Approximate Degree
5 cm	A	27.7°

10 cm	B	111.8°
7 cm	C	40.5°

These results align with Option A:

- 111.8°, 40.5°, 27.7°

Matching our calculated angles to the options confirms the correctness.

Implications and Practical Applications

Understanding the angles corresponding to given sides is not merely an academic exercise; it has real-world relevance:

- Engineering & Construction: Accurate angle measurements ensure structural integrity.
- Navigation & Surveying: Determining angles from side measurements aids in mapping and positioning.
- Robotics & Automation: Precise calculations inform movement and joint angles.

In this context, the ability to compute angles from side lengths enhances problem-solving efficiency across multiple disciplines.

Additional Insights and Tips for Triangle Calculations

To master similar problems, consider these tips:

- Always verify the side lengths satisfy the triangle inequality.
- Use the Law of Cosines for non-right triangles with known sides.
- Convert cosine values to degrees carefully, using a calculator with the correct mode.
- Cross-check calculations by summing angles to ensure they total 180°.
- Remember, the Law of Sines can be used when angles are known or when specific side-angle pairs are given.

Conclusion

In analyzing the triangle with sides 5 cm, 10 cm, and 7 cm, we have meticulously applied the Law of Cosines to determine internal angles. The results—approximately 27.7°, 40.5°, and 111.8°—correspond to Option A, confirming its accuracy.

This exercise underscores the importance of methodical calculation and understanding fundamental geometric principles. Mastery of such techniques enables professionals and students alike to approach complex problems confidently, whether in academics, engineering, or applied sciences.

Final note: Precise calculations and careful interpretation are essential in geometry, ensuring practical applications are based on accurate measurements and understanding.

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