A Uniformly Charged Disk Of Radius 35.0cm Carries Charge With A Density Of 7.90 10 C / M . Calculate

A Uniformly Charged Disk Of Radius 35.0cm Carries Charge With A Density Of 7.90 × 10-6 C/m. Calculate

Understanding the electrostatics of charged objects is fundamental in physics, especially when analyzing how charge distribution affects electric fields and potentials. In this article, we will explore how to compute the total charge on a uniformly charged disk, given its radius and surface charge density, and then extend the discussion to calculating the electric field at specific points. This comprehensive guide aims to clarify the concepts involved, provide step-by-step calculations, and highlight practical applications.

Introduction to Charge Distribution on a Disk

Before diving into calculations, it is important to grasp the basic concepts:

Surface Charge Density (□)

Surface charge density, denoted by □ (sigma), specifies how much charge is distributed over a surface area, measured in Coulombs per square meter (C/m2). For a uniformly charged disk, the charge density remains constant across the surface.

Total Charge (Q)

The total charge stored on the disk can be obtained by integrating the surface charge density over the entire surface area of the disk.

Given Data and Objective

Let's first clarify the given data:

- Radius of the disk, **R = 35.0 cm = 0.35 meters**
- Surface charge density, $\sigma = 7.90 \times 10^{-6}\ \mathrm{C/m^2}$

Objective:

- Calculate the total charge, Q, on the disk.
- (Optional extension) Determine the electric field at a point along the axis of the disk.

Calculating the Total Charge on the Disk

The fundamental step is to find the total charge stored on the disk, which involves integrating the surface charge density over the entire surface.

Step 1: Understanding the Surface Area of the Disk

The area of a disk is given by:

$$ A = \pi R^2 $$

Substituting the given radius:

$$ A = \pi \times (0.35\, \text{m})^2 $$

$$ A = \pi \times 0.1225\, \text{m}^2 $$

$$ A \approx 3.1416 \times 0.1225\, \text{m}^2 $$

$$ A \approx 0.38485\, \text{m}^2 $$

Step 2: Calculating the Total Charge (Q)

Since the charge distribution is uniform:

$$ Q = \sigma \times A $$

$$ Q = 7.90 \times 10^{-6}\, \text{C/m}^2 \times 0.38485\, \text{m}^2 $$

$$ Q \approx 3.043 \times 10^{-6}\, \text{C} $$

Result: The total charge on the disk is approximately 3.04 μC.

Understanding Electric Fields Due to a Charged Disk

Once the total charge is known, a natural extension is to evaluate the electric field produced by the disk at various points in space. This involves more advanced concepts such as electrostatics integrals and potential theory.

Electric Field Along the Axis of a Uniformly Charged Disk

The electric field at a point along the axis passing through the center of the disk, at a distance z from the center, is given by:

$$E_z = \frac{1}{2 \varepsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right) \times \frac{Q}{\pi R^2}$$

Alternatively, more precisely:

$$E_z = \frac{\sigma}{2 \varepsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)$$

Where:
- $\varepsilon_0 \approx 8.854 \times 10^{-12} \, \text{F/m}$ (permittivity of free space)
- σ is the surface charge density
- z is the distance from the disk along its axis

Note: This formula assumes the disk is infinitely thin and uniformly charged.

Calculating the Electric Field at a Specific Point

Suppose we want to find the electric field at a point located 10 cm (0.10 m) along the axis from the center of the disk:

1. Convert all quantities to SI units:

$$ R = 0.35\, \text{m} $$

$$ z = 0.10\, \text{m} $$

2. Use the formula:

$$ E_z = \frac{\sigma}{2 \varepsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right) $$

3. Substitute the values:

$$ E_z = \frac{7.90 \times 10^{-6}}{2 \times 8.854 \times 10^{-12}} \times \left(1 - \frac{0.10}{\sqrt{0.10^2 + 0.35^2}} \right) $$

Calculate denominator:

$$ 2 \times 8.854 \times 10^{-12} = 1.7708 \times 10^{-11} $$

Calculate numerator:

$$ 7.90 \times 10^{-6} \approx 7.90 \times 10^{-6} $$

Compute:

$$ \frac{7.90 \times 10^{-6}}{1.7708 \times 10^{-11}} \approx 446,000 \, \text{V/m} $$

Calculate the geometric factor:

$$ \sqrt{0.10^2 + 0.35^2} = \sqrt{0.01 + 0.1225} = \sqrt{0.1325} \approx 0.364 $$

Then,

$$ 1 - \frac{0.10}{0.364} \approx 1 - 0.2747 \approx 0.7253 $$

Finally,

$$ E_z \approx 446,000 \times 0.7253 \approx 323,668\, \text{V/m} $$

Result: The electric field at 10 cm along the axis is approximately 324 kV/m directed away from the disk.

Applications of Charged Disks in Physics and Engineering

Charged disks are more than theoretical constructs; they have practical applications in various fields:

- **Electrostatic Sensors:** Devices that utilize charged disks to measure electric fields or charges.
- **Capacitors:** Thin circular plates with charge distributions similar to disks are fundamental in capacitor design.
- **Particle Accelerators:** Charged disks and plates are used to generate electric fields for particle acceleration.
- **Electrostatic Precipitators:** Devices that use charged surfaces to remove particles from a gas stream.

Understanding the physics behind these objects allows engineers and scientists to optimize their design and functionality.

Summary and Key Takeaways

- The total charge on a uniformly charged disk can be calculated directly from its surface charge density and area.
- For a disk with radius 35.0 cm and surface charge density of 7.90 × 10-6 C/m2, the total charge is approximately 3.04 μC.
- The electric field produced by such a disk at a point along its axis depends on the distance from the disk and can be calculated using established formulas involving the surface charge density and geometry.
- Practical applications of charged disks span multiple fields, emphasizing the importance of understanding their electrostatic properties.

Conclusion

Analyzing a uniformly charged disk involves fundamental principles of electrostatics, including surface charge density, geometric calculations, and electric field evaluation. By mastering these concepts, students and professionals can better understand how charge distributions influence electric fields and potential, paving the way for innovations in technology and scientific research.

For further exploration, one can extend the calculations to determine potentials at various points, analyze the effects of non-uniform charge distributions, or simulate real-world scenarios using computational tools.

References:

- Griffiths, D. J. Introduction to Electrodynamics, 4th Edition. Pearson, 2013.

- Serway, R. A., & Jewett, J. W. Physics for Scientists and Engineers. Cengage Learning, 2014.

- HyperPhysics, Electric Fields from Distributions. Georgia State University.

Frequently Asked Questions

What is the total charge on a uniformly charged disk with a radius of 35.0 cm and a surface charge density of 7.90 × 10 C/m²?

The total charge Q is given by Q = □ × A, where A = □r^2. Converting radius to meters: 0.35 m. So, A = □ × $(0.35)^2$ □ 0.3848 m^2. Therefore, Q = 7.90 × 10 C/m^2 × 0.3848 m^2 □ 3.045 × 10 C.

How does the surface charge density affect the electric field at the center of the disk?

The electric field at the center of a uniformly charged disk is directly proportional to the surface charge density □. A higher □ results in a stronger electric field, calculated using E = □ / (2□□) for points just above the disk's center.

What is the expression for the electric potential at the center of a uniformly charged disk?

The electric potential V at the center of a disk with total charge Q is V = (1 / (4□□□)) × (Q / R), where R is the radius of the disk. Using Q from the given data, V can be calculated accordingly.

How can the electric field at a point along the axis of the charged disk be determined?

The electric field at a distance x along the axis of a uniformly charged disk is given by E = (□ / 2□□) × (1 - x / □(x^2 + R^2)). This formula accounts for the contribution of all charge elements on the disk.

What is the significance of the surface charge density in calculating the electrostatic effects of the disk?

The surface charge density □ determines the magnitude of the electric field and potential generated by the disk. It quantifies how much charge is spread over a given area, directly influencing the electrostatic interactions.

If the charge density is doubled, how does that affect the total charge and electric field?

Doubling the charge density □ doubles the total charge Q, since Q = □ × area. Consequently, the electric field (which is proportional to □) also doubles, increasing the electrostatic influence of the disk.

Additional Resources

A Uniformly Charged Disk Of Radius 35.0cm Carries Charge With A Density Of 7.90 × 10 C / M . Calculate

Understanding the behavior of electric fields generated by charged objects is fundamental in physics and engineering. One classic problem involves determining the electric field produced by a uniformly charged disk—a scenario relevant to capacitors, sensors, and various electronic devices. In this article, we will analyze a disk with a radius of 35.0 centimeters, uniformly charged with a surface charge density of 7.90 × 10 C/m, and walk through the detailed process of calculating the electric field at various points, particularly along the axis of the disk.

Introduction to the Problem

Imagine a circular disk with a radius of 35.0 centimeters, which is approximately 0.35 meters, uniformly coated with electric charge. The surface charge density, a measure of how much charge resides per unit area, is given as 7.90 × 10 C/m. This setup is common in electrostatics and provides a tangible way to visualize the influence of charge distributions on the surrounding space.

The primary goal here is to determine the electric field at specific points—most notably along the central axis of the disk—that result from this uniform charge distribution. Such calculations are vital for designing devices where control over electric fields is essential, like in electrostatic precipitators or particle accelerators.

Fundamental Concepts

Before diving into the calculations, it is essential to understand some core principles of electrostatics:

- Surface Charge Density (□): The amount of charge per unit area on a surface, measured in C/m^2.
- Electric Field (E): A vector field representing the force per unit charge experienced by a small positive test charge placed in the field.
- Superposition Principle: The total electric field due to a distribution of charges is the vector sum of the fields due to each charge element.
- Symmetry: For uniformly charged disks, symmetry simplifies the problem, allowing us to focus on the electric field along the axis perpendicular to the disk's plane.

Step 1: Calculating the Total Charge on the Disk

The first step is to determine the total charge stored on the disk, which hinges on its surface charge density and area.

Area of the Disk

Given:
- Radius $R = 35.0\, \text{cm} = 0.35\, \text{m}$

The area A of a circle:

$$
A = \pi R^2
$$

Calculating:

$$
A = \pi \times (0.35)^2 = \pi \times 0.1225 \approx 0.385 \, \text{m}^2
$$

Total Charge Q

Given the surface charge density:

$$
\sigma = 7.90 \times 10\, \text{C/m}
$$

(Note: The notation suggests $7.90 \times 10\, \text{C/m}$, which would normally be written as $7.90 \times 10^{-x} \, \text{C/m}$. Since the exact exponent is missing, we will assume it is $7.90 \times 10^{-10} \, \text{C/m}^2$ based on typical charge densities, but please verify the exponent. For this explanation, we will proceed with $7.90 \times 10^{-10} \, \text{C/m}^2$.)

Total charge:

$$
Q = \sigma \times A = (7.90 \times 10^{-10}) \times 0.385 \approx 3.04 \times 10^{-10}\, \text{C}
$$

This is the total charge distributed uniformly over the disk.

Step 2: Electric Field Along the Axis

The primary focus is calculating the electric field at a point along the axis perpendicular to the disk's center. This is a classic problem where symmetry simplifies the integral, and the problem reduces to a well-known formula.

Derivation of the Electric Field Equation

Imagine a point P located a distance z from the center of the disk along its axis. To find the electric field at this point:

- Consider a small ring element of the disk with radius r and thickness dr.
- The amount of charge in this ring:

$$
dq = \sigma \times dA = \sigma \times 2\pi r dr
$$

- The electric field contribution from this ring at point P:

Since the charge distribution is symmetric, the horizontal components cancel, leaving only the component along the axis:

$$
dE_z = \frac{1}{4\pi \varepsilon_0} \frac{dq \times z}{(r^2 + z^2)^{3/2}}
$$

where ε_0 is the vacuum permittivity, approximately $8.854 \times 10^{-12} \, \text{C}^2/(\text{N} \cdot \text{m}^2)$.

Integrating Over the Disk

Integrate from $r=0$ to $r=R$:

$$
E_z(z) = \frac{1}{4\pi \varepsilon_0} 2\pi \sigma z \int_0^R \frac{r dr}{(r^2 + z^2)^{3/2}}
$$

Simplify:

$$
E_z(z) = \frac{\sigma}{2 \varepsilon_0} z \int_0^R \frac{r dr}{(r^2 + z^2)^{3/2}}
$$

Perform the integral:

$$
\int \frac{r dr}{(r^2 + z^2)^{3/2}} = -\frac{1}{\sqrt{r^2 + z^2}}
$$

Evaluated from 0 to R:

$$
\int_0^R \frac{r dr}{(r^2 + z^2)^{3/2}} = \left[-\frac{1}{\sqrt{r^2 + z^2}} \right]_0^{R} = -\frac{1}{\sqrt{R^2 + z^2}} + \frac{1}{z}
$$

Putting it all together:

$$
E_z(z) = \frac{\sigma}{2 \varepsilon_0} z \left(\frac{1}{z} - \frac{1}{\sqrt{R^2 + z^2}} \right)

$$

Simplify:

$$
E_z(z) = \frac{\sigma}{2 \varepsilon_0} \left(1 - \frac{z}{\sqrt{R^2 + z^2}} \right)
$$

This is the electric field along the axis at a distance z from the center of the disk.

Step 3: Numerical Calculation

Suppose we want to find the electric field at a point close to the disk's center, say at $z = 10\, \text{cm} = 0.10 \, \text{m}$.

Parameters:

- $R = 0.35\, \text{m}$
- $z = 0.10\, \text{m}$
- $\sigma = 7.90 \times 10^{-10}\, \text{C/m}^2$ (assumed)
- $\varepsilon_0 = 8.854 \times 10^{-12}\, \text{C}^2/(\text{N} \cdot \text{m}^2)$

Calculation:

$$
E_z = \frac{7.90 \times 10^{-10}}{2 \times 8.854 \times 10^{-12}} \left(1 - \frac{0.10}{\sqrt{(0.35)^2 + (0.10)^2}} \right)
$$

Calculate denominator:

$$
2 \times \varepsilon_0 = 2 \times 8.854 \times 10^{-12} = 1.7708 \times 10^{-11}
$$

Calculate the ratio:

$$
\frac{7.90 \times 10^{-10}}{1.7708 \times 10^{-11}} \approx 44.58
$$

Calculate the square root:

$$
\sqrt{0.1225 + 0.01} = \sqrt{0.1325} \approx 0.364
$$

Calculate the fraction:

$$
\frac{0.10}{0.364} \approx 0.275
$$

Finally, compute E_z:

$$
E_z \approx 44.58 \times (1 - 0.275) = 44.58 \times 0.725 \approx 32.37\, \text{N/C}
$$

Thus, the electric field at 10 cm along the axis is approximately 32.4 N/C directed along the axis away from the disk (assuming the positive sign indicates outward).

Step 4: Effect of Distance and Practical Implications

The calculated electric field diminishes with increasing \(z \), following the relation:

\[

E_z(z) = \frac{\sigma}{2 \

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