A Very Long Straight Wire Has Charge Per Unit Length 1.451010 C/m. At What Distance From The Wire Is

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Understanding the electrostatic properties of long charged wires is fundamental in physics and electrical engineering. When dealing with a very long straight wire carrying a uniform linear charge density, it becomes essential to determine how the electric field behaves at various distances from the wire. This knowledge helps in designing electrical systems, understanding electromagnetic phenomena, and solving various practical problems. In this article, we explore how to determine the electric field at a given distance from a long charged wire, derive the relevant formulas, and analyze the implications of the charge density provided.

Introduction to Electric Field of a Long Charged Wire

The electric field created by a charged wire depends primarily on its linear charge density and the distance from the wire. For an infinitely long wire with a uniform charge distribution, the field is symmetric and depends only on the perpendicular distance from the wire, simplifying the analysis significantly.

Linear Charge Density

- Defined as the amount of charge per unit length along the wire.
- Denoted as λ (lambda), typically in units of Coulombs per meter (C/m).
- In this case, $\lambda = 1.451010$ C/m, indicating a substantial charge per unit length.

Assumptions for Simplification

- The wire is infinitely long, which allows us to neglect edge effects.
- The charge distribution is uniform.
- The space around the wire is vacuum or air, with permittivity ε_0.

Electric Field Due to a Long Charged Wire

The electric field at a distance r from a long charged wire is given by a well-known formula derived from Gauss's law:

$$E(r) = (\lambda) / (2 \pi \varepsilon_0 r)$$

where:
- $E(r)$ is the magnitude of the electric field at distance r.
- λ is the linear charge density.
- ε_0 is the permittivity of free space, approximately 8.854×10^{-12} F/m.
- r is the perpendicular distance from the wire.

This formula indicates that the electric field decreases inversely with distance from the wire.

Derivation of the Electric Field Formula

- Consider a cylindrical Gaussian surface coaxial with the charged wire.
- Applying Gauss's law: the total electric flux through the surface equals the enclosed charge divided by ε_0.
- Symmetry ensures the electric field magnitude is constant on the cylindrical surface and directed radially outward.
- Solving for E yields the formula above.

Calculating the Electric Field at a Specific Distance

Given the linear charge density $\lambda = 1.451010$ C/m, to find the electric field at a certain distance r, plug the values into the formula:

$$E(r) = (1.451010) / (2 \pi 8.854 \times 10^{-12} r)$$

Simplifying:

$$E(r) \approx (1.451010) / (55.599 \times 10^{-12} r)$$

or

$$E(r) \approx (2.612 \times 10^{10}) / r \ (V/m)$$

This means that the electric field magnitude in volts per meter at distance r (measured in meters) is approximately 2.612×10^{10} divided by r.

Determining the Distance for a Given Electric Field

Suppose you want to find the distance r from the wire where the electric field reaches a specific value, E_0 (in V/m). Rearranging the formula:

$$r = (\lambda) / (2 \ \pi \ \varepsilon_0 \ E_0)$$

Example Calculation:

If the electric field is known to be 10^6 V/m (which is extremely high and unlikely in practical scenarios but useful for illustration):

$$r = (1.451010) / (2 \ \pi \ 8.854 \times 10^{-12} \ 10^6)$$

Calculating the denominator:

$$2 \ \pi \ 8.854 \times 10^{-12} \ 10^6 \approx 55.599 \times 10^{-6}$$

Thus,

$$r \approx 1.451010 / (55.599 \times 10^{-6}) \approx 26{,}110 \text{ meters}$$

This means that at approximately 26.1 km from the wire, the electric field would be 10^6 V/m, assuming such a high field exists without breakdown.

Practical Implications and Applications

Understanding the electric field distribution around a long charged wire has multiple practical applications:

1. Electrical Safety and Insulation

- High electric fields near charged conductors can lead to dielectric breakdown or arcing.
- Engineers must ensure that the distance from the wire to any conductive or sensitive component exceeds the calculated safe distance for the desired electric field limit.

2. Electromagnetic Compatibility (EMC)

- Knowledge of field strengths at various distances helps in designing shielding and grounding strategies to prevent interference.

3. Design of Transmission Lines

- High-voltage power lines need to consider electric field effects to prevent corona discharge and ensure safety.

4. Scientific Experiments and Measurements

- Precise knowledge of electric field distributions aids in experimental setups involving charged wires or conductors.

Additional Factors Affecting Electric Field Calculations

While the basic formula provides a good approximation, real-world scenarios might require considering:

1. Environment and Medium

- Presence of dielectric materials can alter the permittivity, changing the electric field.

2. Finite Length of the Wire

- For wires not infinitely long, edge effects become significant, and the field distribution is more complex.

3. Charge Distribution Variations

- Non-uniform charge distributions lead to different field profiles.

4. External Fields and Influences

- External electric or magnetic fields can modify the local field environment.

Summary and Key Takeaways

- The electric field due to an infinitely long charged wire with linear charge density λ is given by: $E(r) = \lambda / (2\pi \varepsilon_0 r)$.
- With λ = 1.451010 C/m, the electric field at a distance r (meters) is approximately $2.612 \times 10^{10} / r$ V/m.
- To find the distance at which the electric field reaches a specific value, rearrange the formula to solve for r.
- Understanding these principles is crucial for designing safe electrical systems, preventing breakdown, and analyzing electromagnetic phenomena.

Conclusion

The study of electric fields around long charged wires not only enriches our understanding of fundamental physics but also provides practical tools for engineers and scientists. Accurate calculations of electric field strength at various distances enable safer electrical system designs and help prevent hazards associated with high voltage and electric fields. By mastering these concepts, professionals can better analyze and utilize the properties of charged conductors in diverse applications.

Note: Always consider safety margins and environmental factors when applying these calculations in real-world scenarios.

Frequently Asked Questions

A very long straight wire has a charge per unit length of 1.45×10^{-10} C/m. At what distance from the wire is the electric field strength 1000 N/C?

Using the formula $E = \lambda / (2\pi\varepsilon_0 r)$, rearranged as $r = \lambda / (2\pi\varepsilon_0 E)$. Substituting $\lambda = 1.45 \times 10^{-10}$ C/m, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, and $E = 1000$ N/C, we get $r \approx 2.6$ meters.

How does the electric field vary with distance from a long charged wire with charge density 1.45×10^{-10} C/m?

The electric field varies inversely with the distance (r) from the wire, following $E = \lambda / (2\pi\varepsilon_0 r)$. As r increases, E decreases proportionally.

If the charge per unit length of a long wire is increased to 2.9×10^{-10} C/m, how does the electric field at a fixed distance change?

The electric field doubles since E is directly proportional to the charge density λ. So, at the same distance, the electric field would be twice as strong.

What is the significance of knowing the charge per unit length in a long wire for electrostatics calculations?

Knowing the charge per unit length allows us to calculate the electric field at any point around the wire using the relation $E = \lambda / (2\pi\varepsilon_0 r)$, which is essential for understanding electrostatic interactions.

Can the electric field around a long charged wire be considered uniform? Why or why not?

No, the electric field around a long charged wire is not uniform; it varies with distance from the wire, decreasing as 1/r, so it is stronger closer to the wire.

At what distance from the wire with charge density 1.45×10^{-10} C/m is the electric field 500 N/C?

Using $r = \lambda / (2\pi\varepsilon_0 E)$, substituting the values, $r \approx 5.2$ meters.

What are the units of the charge per unit length in the context of this problem?

The units are Coulombs per meter (C/m).

How would you experimentally verify the electric field strength at a certain distance from the wire?

You could use an electric field probe or test charge method to measure the force experienced by a known small charge placed at that distance and calculate the field from the force.

If the charge per unit length is doubled, what happens to the electric field at a fixed distance from the wire?

The electric field doubles because it is directly proportional to the charge per unit length (λ).

Additional Resources

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Introduction

The physics of electric fields generated by charged conductors is fundamental in understanding electrostatics. Among the simplest models is that of an infinitely long, straight wire uniformly charged along its length. Such a wire creates an electric field that diminishes with distance, and understanding this behavior is crucial for applications in electrical engineering, telecommunications, and physics education.

This analysis aims to explore the detailed calculations, principles, and implications when given a specific charge density: a charge per unit length of 1.45×10^{-10} C/m. The core question addressed is: At what distance from this long, charged wire does the electric field reach a certain value?

While the initial problem statement is incomplete regarding the electric field's magnitude, this review will consider common scenarios, including calculating the electric field at specific distances for the given charge density, and understanding how the electric field varies with distance.

Fundamental Principles of Electric Fields Around a Long Charged Wire

Coulomb’s Law and Infinite Line Charges

The electric field produced by a point charge diminishes with the square of the distance, as described by Coulomb’s law:

$$
E = \frac{k Q}{r^2}
$$

where k is Coulomb’s constant, Q is the charge, and r is the distance from the charge.

However, for an infinitely long charged wire, the symmetry simplifies the problem. The field depends only on the perpendicular distance from the wire and follows an inverse proportionality:

$$
E(r) = \frac{\lambda}{2\pi \varepsilon_0 r}
$$

where:

- λ = charge per unit length (C/m)
- ε_0 = permittivity of free space ($8.854 \times 10^{-12} \, \mathrm{F/m}$)
- r = perpendicular distance from the wire

This formula indicates that the electric field decreases linearly with increasing distance from the wire.

Step-by-Step Calculation of Electric Field at a Given Distance

1. Identify the Given Data:

- Charge per unit length, $\lambda = 1.45 \times 10^{-10} \, \mathrm{C/m}$
- Permittivity of free space, $\varepsilon_0 = 8.854 \times 10^{-12} \, \mathrm{F/m}$

2. Determine the Electric Field Expression:

$$
E(r) = \frac{\lambda}{2 \pi \varepsilon_0 r}
$$

This is a standard formula derived from Gauss's law for an infinitely long uniformly charged wire.

3. Calculate the Electric Field at a Specific Distance:

Suppose the question is: At what distance r from the wire is the electric field intensity E?

Rearranged,

$$
r = \frac{\lambda}{2 \pi \varepsilon_0 E}
$$

To find r, the value of E must be specified. For illustration, let’s consider some common electric field magnitudes:

- $E = 1 \, \mathrm{kV/m} = 1000 \, \mathrm{V/m}$
- $E = 10 \, \mathrm{kV/m} = 10,000 \, \mathrm{V/m}$

Example Calculations

Scenario 1: Electric Field of 1000 V/m

$$
r = \frac{1.45 \times 10^{-10}}{2 \pi \times 8.854 \times 10^{-12} \times 1000}
$$

Calculating denominator:

$$
2 \pi \times 8.854 \times 10^{-12} \times 10^3 \approx 2 \times 3.1416 \times 8.854 \times 10^{-9}
$$

$$
\approx 6.2832 \times 8.854 \times 10^{-9} \approx 55.6 \times 10^{-9} = 5.56 \times 10^{-8}
$$

Now, the numerator:

$$
1.45 \times 10^{-10}
$$

Therefore,

$$
r = \frac{1.45 \times 10^{-10}}{5.56 \times 10^{-8}} \approx 0.00261 \, \mathrm{m} = 2.61\, \mathrm{mm}
$$

Interpretation:
At a distance of approximately 2.61 mm from the wire, the electric field is about 1000 V/m.

Scenario 2: Electric Field of 10,000 V/m

Similarly,

$$
r = \frac{1.45 \times 10^{-10}}{2 \pi \times 8.854 \times 10^{-12} \times 10^4}
$$

Calculating denominator:

$$
6.2832 \times 8.854 \times 10^{-8} \approx 556 \times 10^{-9} = 5.56 \times 10^{-7}
$$

Now,

$$
r = \frac{1.45 \times 10^{-10}}{5.56 \times 10^{-7}} \approx 0.000261 \, \mathrm{m} = 0.261\, \mathrm{mm}
$$

Interpretation:
At approximately 0.26 mm from the wire, the electric field reaches 10,000 V/m.

Physical Significance and Practical Implications

Understanding the relationship between electric field and distance is vital in designing electrical systems. For example:

- Insulation Design:
The proximity of conductors must be such that electric fields do not exceed dielectric breakdown thresholds.

- Electrostatic Shielding:
The innermost regions in devices can be shielded based on the known decay of the electric field.

- High-Voltage Transmission:
Engineers carefully evaluate the electric field at the surface of high-voltage conductors to prevent corona discharge or arcing.

Additional Considerations

1. Limitations of the Model

- The formula assumes an idealized infinite wire, which in real-life applications is finite, and edge effects can alter the field distribution.
- The surrounding environment (air, insulation) affects the actual electric field and breakdown conditions.
- The charge distribution is assumed uniform; real wires may have non-uniform charge densities due to surface imperfections.

2. Units and Conversions

- When performing calculations, ensure units are consistent.
- Electric field E is in V/m, distance r in meters.
- Charge density λ is in C/m.

3. Electrostatic Potential

The potential $V(r)$ relative to the wire can be derived by integrating the electric field:

$$
V(r) = \frac{\lambda}{2\pi \varepsilon_0} \ln \left(\frac{r}{r_0} \right)
$$

where r_0 is a reference point. Understanding potential is crucial for voltage insulation considerations.

Broader Context and Applications

Electromagnetic Compatibility (EMC)

Knowledge of electric fields around wires informs EMC practices, minimizing interference and ensuring safety.

Design of Sensors and Detectors

Sensors that rely on electric field detection, such as electric field mills, need precise understanding of field strength at various distances.

Electrostatics in Microelectronics

In micro- and nano-scale devices, even minor charge densities create significant local electric fields, affecting device operation.

Summary

- The electric field around a very long, straight wire with a uniform charge density $\lambda = 1.45 \times 10^{-10} \, \mathrm{C/m}$ follows the inverse relationship with distance:

$$
E(r) = \frac{\lambda}{2 \pi \varepsilon_0 r}
$$

- At specific electric field magnitudes, the distance from the wire can be calculated directly:

$$
r = \frac{\lambda}{2 \pi \varepsilon_0 E}
$$

- For common electric field levels (e.g., 1 kV/m, 10 kV/m), the distances are on the order of millimeters to sub-millimeters, emphasizing the importance of close proximity control in high-voltage environments.

- Real-world applications require careful consideration of environmental factors, wire length, and non-uniformities, which can modify the idealized models.

Final Remarks

Understanding the interplay between charge density, electric field, and distance is fundamental in electrostatics. These principles underpin many technological advancements, from electrical transmission to microelectronic device design. Mastery of these calculations enables engineers and physicists to ensure safety, optimize performance, and innovate within the realm of electric and magnetic phenomena.

Note: If the original question specifies a particular electric field magnitude at which to determine the distance, simply substitute that value into the formula:

$$
r = \frac{\lambda}{2 \pi \varepsilon_0 E}
$$

and proceed with the calculation accordingly.

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