A Weight Of 50.0 N Is Attached To The Free End Of A Light String Wrapped Around A Reel With A Radius

A Weight Of 50.0 N Is Attached To The Free End Of A Light String Wrapped Around A Reel With A Radius is a classic physics problem that involves understanding concepts such as torque, rotational equilibrium, and the relationship between force and motion. This scenario provides a practical illustration of Newton's laws of motion, especially in rotational dynamics, and serves as a foundation for understanding how forces interact when applied to circular objects like reels or pulleys. Whether you're a student studying physics, an engineer designing machinery, or simply curious about the principles governing everyday objects, analyzing this problem offers valuable insights into the mechanics of rotational systems.

Understanding the Basic Setup

Scenario Description

In this problem, a weight of 50.0 N is attached to the end of a light (massless) string that is wound around a reel or pulley with a known radius. As the weight hangs freely, gravity exerts a downward force, causing the reel to rotate or move accordingly. The key questions usually revolve around the torque produced, the angular acceleration, and whether the system reaches equilibrium or continues to accelerate.

Key Components

- The weight (50.0 N): The force due to gravity acting downward.
- The string: Assumed to be massless and inextensible, transmitting force without slipping.
- The reel (or pulley): Rotates about its axis; characterized by its radius, which influences the torque.
- The radius of the reel: Denoted as r, it determines how the force translates into torque.

Fundamental Concepts in Rotational Dynamics

Force and Torque Relationship

Torque (\(\tau\)) is a measure of the tendency of a force to cause rotation about a point or axis. It is given by:

$$
\tau = r \times F
$$
where:
- r is the radius (lever arm length)
- F is the force applied perpendicular to the lever arm

In this scenario, the force is the weight ($W = 50.0\, \text{N}$), and it acts vertically downward at the edge of the reel.

Moment of Inertia and Angular Acceleration

The rotational equivalent of Newton's second law states:
$$
\tau = I \alpha
$$
where:
- I is the moment of inertia of the reel
- α is the angular acceleration

For a reel or pulley, the moment of inertia depends on its shape and mass distribution. For example, a solid disk has:
$$
I = \frac{1}{2} M R^2
$$
where M is the mass and R is the radius.

Analyzing the System: Key Questions and Calculations

1. Is the System in Equilibrium?

If the weight is stationary, the forces are balanced, and the reel does not rotate. In this case:
$$
\text{Tension } T = W = 50.0\, \text{N}
$$
and the net torque is zero.

2. What is the Tension in the String?

If the system is accelerating or descending, the tension in the string may differ from the weight's force. Using Newton's second law for the mass:
$$
W - T = m a
$$

where m is the mass ($m = W/g = 50.0\, \text{N} / 9.8\, \text{m/s}^2 \approx 5.10\, \text{kg}$), and a is the acceleration.

3. Determining the Angular Acceleration

The torque exerted on the reel:

$$
\tau = T \times r
$$

From the rotational equation:

$$
T r = I \alpha
$$

Since $\alpha = a / r$, the relationship between linear and angular acceleration links the two quantities.

Calculations and Example Problems

Example 1: System in Equilibrium

If the weight is just hanging without moving:
- The tension equals the weight: $T = 50.0\, \text{N}$.
- The reel remains stationary; no angular acceleration occurs.
- No torque is generated, and the system stays at rest.

Example 2: System Accelerating

Suppose the reel has a radius $r = 0.2\, \text{m}$, and the mass of the reel is negligible (light reel assumption). To find the acceleration a:

1. Calculate the mass:

$$
m = \frac{50.0\, \text{N}}{9.8\, \text{m/s}^2} \approx 5.10\, \text{kg}
$$

2. Use Newton's second law:

$$
W - T = m a
$$

3. Express tension in terms of torque:

$$
T = \frac{I \alpha}{r}
$$

But since the reel is light:

$$

$$
I \approx 0
$$
and tension equals weight, implying acceleration is negligible.

Alternatively, if the reel has mass M, then:
$$
I = \frac{1}{2} M R^2
$$
and the equations become more complex, requiring calculations based on the actual mass.

The Role of Radius in Rotational Dynamics

Impact of Radius on Torque and Acceleration

The radius r of the reel plays a crucial role in determining how force translates into rotational motion:

- Larger radius: Increases torque for the same tension, resulting in greater angular acceleration.
- Smaller radius: Reduces torque, possibly leading to slower acceleration.

Mathematically:
$$
\alpha = \frac{T r}{I}
$$
So, increasing r (assuming constant T and I) enhances α.

Design Implications

Understanding the relationship between radius and rotational motion is essential in designing systems such as:

- Pulleys and Reels: To optimize force transmission and motion.
- Elevators: Where pulleys of specific radii control lifting speed.
- Clutches and Brakes: Where torque transmission depends on radius.

Real-World Applications of Rotational Mechanics

Engineering and Machinery

Engineers utilize principles from this problem to design efficient:

- Winches and Hoists: To lift heavy loads with controlled acceleration.
- Rotary Tools: Ensuring optimal torque for different sizes.
- Automotive Systems: Using pulleys and belts to transfer power.

Everyday Devices

Many everyday objects operate based on similar principles:

- Doorbells with pulleys
- Clotheslines with pulleys
- Gym equipment with rotating discs

Advanced Topics and Considerations

Friction and Air Resistance

In real systems, factors like friction in the reel's axle and air resistance affect motion, often reducing acceleration and torque effectiveness.

Mass Distribution and Moment of Inertia

The shape and mass distribution of the reel influence the moment of inertia I, thereby affecting how torque translates into angular acceleration.

Multiple Pulleys and Mechanical Advantage

Complex systems often involve multiple pulleys to distribute forces and gain mechanical advantage, reducing the effort needed to lift weights.

Summary and Key Takeaways

- The force of 50.0 N represents the weight attached to the reel, influencing the system's torque.
- The radius of the reel directly impacts the torque generated and the resulting angular acceleration.
- Understanding the relationship between force, torque, and moment of inertia is essential in analyzing rotational systems.
- Practical applications range from simple pulleys to complex machinery, all relying on principles

illustrated by this scenario.
- Factors like friction, mass distribution, and system design considerations significantly influence real-world performance.

Conclusion

Analyzing a weight attached to a string wrapped around a reel with a specific radius provides fundamental insights into rotational dynamics. By understanding how force translates into torque and how the radius influences the system's motion, engineers and students alike can design and analyze a variety of mechanical systems. Whether in theoretical physics or practical engineering, mastering these concepts is crucial for advancing technological innovations and solving real-world problems involving rotational motion.

Keywords for SEO Optimization:
- Rotational dynamics
- Torque and radius relationship
- Pulley systems
- Moment of inertia
- Angular acceleration
- Physics of pulleys
- Mechanical advantage
- Rotational motion calculations
- Engineering applications of torque
- Light string reel physics

Frequently Asked Questions

How does the weight of 50.0 N affect the tension in the string wrapped around the reel?

The weight of 50.0 N creates a tension in the string equal to its weight, assuming the system is in equilibrium or moving at constant velocity, thus the tension in the string is approximately 50.0 N.

What is the relationship between the weight attached to the string and the torque on the reel?

The torque on the reel is calculated by multiplying the tension in the string (which equals the weight if static) by the radius of the reel: $\tau = T \times r$. Therefore, a larger radius increases the torque for the same weight.

If the reel has a radius 'r', how can we determine the angular acceleration when the weight causes the reel to spin?

The angular acceleration α can be found using $\tau = I\alpha$, where τ is the torque ($T \times r$), and I is the moment of inertia of the reel. Rearranged, $\alpha = (T \times r) / I$.

What factors influence the rotational motion of the reel when a 50.0 N weight is attached?

Factors include the magnitude of the weight (which determines tension), the radius of the reel, the moment of inertia of the reel, and whether the system is accelerating or in equilibrium.

How does the mass of the reel impact the system's behavior when a 50.0 N weight is attached?

The mass of the reel affects its moment of inertia; a heavier reel has a larger I, which resists angular acceleration, potentially reducing the angular acceleration for a given torque created by the weight.

Additional Resources

A Weight Of 50.0 N Is Attached To The Free End Of A Light String Wrapped Around A Reel With A Radius

---

Introduction

Understanding the dynamics of a weight attached to a string wrapped around a reel is fundamental in classical mechanics, especially in the study of rotational motion and tension in cords. When a 50.0 N weight is tied to a light (massless) string wrapped around a reel, the system exemplifies key principles like tension, torque, angular acceleration, and equilibrium. Exploring this scenario in depth provides insights into how forces and motion interact in rotational systems, which are foundational for engineering applications such as pulleys, cranes, and gear mechanisms.

Fundamental Concepts Involved

Before diving into the specifics of the problem, it’s crucial to understand the core principles at play:

1. Tension in the String

- The tension is the force transmitted through the string due to the weight hanging from it.
- Since the string is assumed to be massless and inextensible, the tension throughout its length is constant, provided there is no acceleration.

2. Gravitational Force (Weight)

- The weight (W) acts vertically downward and is given as 50.0 N.
- The weight is related to the mass (m) of the object via $W = mg$, where g is the acceleration due to gravity (~9.81 m/s^2).

3. Rotational Motion of the Reel

- The reel experiences torque due to the tension in the string.
- The reel's moment of inertia (I) characterizes its resistance to angular acceleration.
- The angular acceleration (α) of the reel relates to the linear acceleration (a) of the falling weight.

4. Equilibrium and Dynamics

- When the system is in motion, the balance of forces and torques dictates the acceleration.
- The analysis involves Newton's second law both in linear form (for the falling mass) and rotational form (for the reel).

Setting Up the Problem

To analyze the system, we need to define the parameters and assumptions:

- Mass of the hanging weight: $m = \frac{W}{g} = \frac{50.0\,N}{9.81\,m/s^2} \approx 5.10\,kg$
- Radius of the reel: R (given or to be specified)
- Moment of inertia of the reel: I (depends on the reel's geometry and mass distribution)
- Assumption: The string is massless and does not slip on the reel.
- Scenario: The weight is released from rest and allowed to fall, causing the reel to rotate.

Analyzing the Forces and Torques

1. Linear Dynamics of the Falling Weight

- The forces acting on the weight:
- Gravitational force downward: $W = 50.0\,N$
- Tension in the string upward: T

- Equation of motion:
$$
m a = m g - T
$$
where:
- a = linear acceleration of the weight downward.

2. Rotational Dynamics of the Reel

- The tension provides a torque (τ) that causes the reel to rotate:
$$
\tau = T R
$$
- R is the radius of the reel.
- Newton's second law for rotation:
$$
\tau = I \alpha
$$
where:
- α = angular acceleration.
- The relation between linear acceleration and angular acceleration:
$$
a = R \alpha
$$

3. Combining the Equations

- Substituting $\alpha = \frac{a}{R}$:
$$
T R = I \frac{a}{R} \Rightarrow T = \frac{I a}{R^2}
$$
- From the linear motion equation:
$$
m g - T = m a
$$
- Substitute T:
$$
m g - \frac{I a}{R^2} = m a
$$
- Rearranged to solve for a:
$$
m g = m a + \frac{I a}{R^2}
$$

$$
a \left(m + \frac{I}{R^2} \right) = m g
$$

$$
a = \frac{m g}{m + \frac{I}{R^2}}
$$

This expression gives the acceleration of the falling weight, accounting for the rotational inertia of the reel.

Special Cases and Simplifications

- Massless Reel: If the reel's moment of inertia is negligible ($I \rightarrow 0$), then:

$$
a = g
$$

- The weight falls freely under gravity with no rotational resistance.

- Massive Reel: For a reel with significant moment of inertia, the acceleration a will be less than g, as some of the gravitational potential energy goes into spinning the reel.

Calculating the Tension

Once a is known, the tension T can be calculated:

$$
T = m g - m a
$$

- Interpretation:
- If a is close to g, tension approaches zero, indicating the reel is spinning freely.
- If a is small, tension approaches $m g$, indicating the weight is accelerating slowly or is nearly stationary.

Determining the Moment of Inertia (I)

The moment of inertia depends on the geometry and mass distribution of the reel:

- Solid Cylinder or Disc:
$$
I = \frac{1}{2} M R^2
$$
- Hollow Cylinder (Thin-walled):
$$
I = M R^2
$$
- General case:
- I must be provided or measured. For typical reel systems, a disc approximation is common.

Impact of Radius on System Dynamics

The radius R plays a crucial role in the system:

- Influence on Angular acceleration:
$$
\alpha = \frac{a}{R}
$$
Larger R results in smaller angular acceleration for the same linear acceleration a.

- Effect on torque:
$$
\tau = T R
$$
For a given tension, increasing R increases torque, which could lead to different rotational behaviors.

- Energy considerations:
- The potential energy lost by the weight translates into both translational kinetic energy and rotational kinetic energy:
$$
m g h = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2
$$
where $v = a t$, and $\omega = \alpha t$.

Applications and Practical Considerations

Understanding this system is vital in many engineering contexts:

- Pulleys in Mechanical Systems:
- Designed to change force direction and magnitude.

- Considerations include friction, reel inertia, and string mass.

- Elevator and Crane Mechanisms:
- Rotational inertia of pulleys affects acceleration and energy efficiency.
- Tension control is critical for safety and operation.

- Educational Demonstrations:
- Visualizes rotational dynamics and Newtonian physics.
- Demonstrates the interplay between linear and angular quantities.

- Design Optimization:
- Minimizing reel inertia for faster acceleration.
- Selecting appropriate radii to balance torque and acceleration.

Additional Factors and Real-World Considerations

In practical situations, several factors influence the idealized analysis:

- Friction:
- Friction in the reel’s axle or between the string and reel can alter tension and acceleration.
- Often modeled as a torque opposing motion.

- String Mass and Stiffness:
- Real strings have mass, which affects tension distribution.
- Stiffness influences how the string responds to forces and motion.

- Elasticity and Damping:
- Material deformation and internal damping dissipate energy.
- These factors slow down acceleration and cause energy loss.

- Air Resistance:
- Especially relevant at high speeds or with large reel diameters.

- Dynamic Changes:
- As the weight falls, the system’s parameters change, especially if the reel or string properties are non-uniform.

Experimental and Numerical Analysis

- Experimental Setup:
- Use a reel of known mass and radius.
- Attach a 50.0 N weight.

- Measure acceleration using motion sensors or high-speed cameras.
- Record tension via force sensors.

- Numerical Simulation:
- Model the system using differential equations.
- Incorporate real-world factors like friction and damping.
- Use software (e.g., MATLAB, Python

A Weight Of 50 0 N Is Attached To The Free End Of A Light String Wrapped Around A Reel With A Radius

A Weight Of 50 0 N Is Attached To The Free End Of A Light String Wrapped Around A Reel With A Radius

Related Articles

- Amy Works For 36 Hours Per Week For 10 Weeks During The Summer, Making $\$3000$. If She Works For 30
- ANSWER ASAP DONT SEND A FILE. IS THIS SHAPE A ROTATION, REFLECTION,TRANSLATION, DILATION OR NONE????
- An AC Source Has A Maximum Voltage Of 170 V And A Frequency Of 60 Hz. A Capacitor Circuit Using This

Back to Home