An Acute Triangle Has Sides Measuring 10 Cm And 16 Cm. The Length Of The Third Side Is Unknown.Which

An Acute Triangle Has Sides Measuring 10 Cm And 16 Cm. The Length Of The Third Side Is Unknown. Which is a common problem in geometry that involves understanding the properties of triangles, specifically acute triangles, and how to determine the possible length of the third side given certain measurements. This article provides a comprehensive overview of the concepts involved, the mathematical principles used, and step-by-step methods to find the unknown side length, ensuring clarity for students, teachers, and geometry enthusiasts.

Understanding the Basics of Triangles

Before diving into the specifics of the problem, it is essential to understand the fundamental properties of triangles, particularly the types of triangles based on their angles and side lengths.

Types of Triangles Based on Sides

- Equilateral Triangle: All three sides are equal.
- Isosceles Triangle: Two sides are equal.
- Scalene Triangle: All sides are different in length.

Types of Triangles Based on Angles

- Acute Triangle: All three angles are less than 90°.
- Right Triangle: One angle is exactly 90°.
- Obtuse Triangle: One angle is greater than 90°.

In this context, since the triangle is acute, all angles are less than 90°, which influences the range of the third side's possible lengths.

Given Data and the Problem Statement

- Side AB = 10 cm
- Side AC = 16 cm
- Third side BC = ? (unknown)

The goal is to determine the possible length of side BC that would make the triangle ABC an acute triangle.

Mathematical Principles Involved

To solve this problem, we need to understand and apply the following fundamental geometric and algebraic principles:

The Triangle Inequality Theorem

This theorem states that:

- The sum of the lengths of any two sides of a triangle must be greater than the length of the remaining side.

Applying this to our problem:

$$
|AB| + |AC| > |BC| \\
|AB| + |BC| > |AC| \\
|AC| + |BC| > |AB|
$$

Plugging in the known values:

$$
10 + 16 > |BC| \Rightarrow 26 > |BC| \Rightarrow |BC| < 26
$$
$$
10 + |BC| > 16 \Rightarrow |BC| > 6
$$
$$
16 + |BC| > 10 \Rightarrow |BC| > -6 \quad (\text{which is always true since length is positive})
$$

Thus, from the triangle inequality:

$$
6 < |BC| < 26
$$

This range indicates the possible lengths of the third side for any triangle, but to ensure the triangle is acute, more specific conditions are needed.

Conditions for an Acute Triangle

In any triangle with sides a, b, and c, the following condition must hold for the triangle to be acute:

$$
a^2 + b^2 > c^2
$$

and similarly for the other sides:

$$
a^2 + c^2 > b^2 \\
b^2 + c^2 > a^2
$$

In our scenario, since sides AB and AC are known, and BC is unknown, we analyze the conditions based on the possible lengths of BC to maintain acuteness.

Determining the Range of the Third Side for an Acute Triangle

Let's denote:

- $AB = 10\,cm$
- $AC = 16\,cm$
- $BC = x\,cm$

Our goal:

- Find the range of x such that triangle ABC is acute.

Step 1: Establish the Triangle Inequality Range

As established, for a valid triangle:

$$
6 < x < 26
$$

Step 2: Apply the Acuteness Condition

Since the triangle is acutely angled, the following inequalities must be satisfied:

1. $AB^2 + AC^2 > BC^2$:
$$
(10)^2 + (16)^2 > x^2 \\
100 + 256 > x^2 \\
356 > x^2 \\
x < \sqrt{356} \approx 18.87
$$

2. $AB^2 + BC^2 > AC^2$:
$$
100 + x^2 > 256 \\
x^2 > 156 \\
x > \sqrt{156} \approx 12.49
$$

3. $AC^2 + BC^2 > AB^2$:
$$
256 + x^2 > 100 \\
x^2 > -156 \quad (\text{always true for positive } x)
$$

Summary of inequalities:

$$
12.49 < x < 18.87
$$

Considering the triangle inequality, the range for x that makes the triangle both valid and acute is:

$$
\boxed{12.49\,cm < x < 18.87\,cm}
$$

Visualizing and Verifying the Range

To better understand these bounds, consider the following:

- When x is less than approximately 12.49 cm, the triangle becomes obtuse or degenerate.
- When x exceeds approximately 18.87 cm, the triangle ceases to be acute.
- Within the range $12.49\,cm < x < 18.87\,cm$, the triangle remains acute.

Graphical interpretation:

- Plotting x against the inequality conditions can help visualize the feasible region.
- The bounds are derived from the Pythagorean inequalities that define the acuteness condition.

Practical Application and Problem-Solving Steps

Suppose you are given this problem in a test or real-world scenario, here's how you can approach it:

1. Identify the known side lengths and the type of triangle you need (acute).
2. Apply the triangle inequality to find the initial feasible range for the third side.
3. Use the Pythagorean inequality conditions to refine the range for acuteness:
    - Calculate the squares of known sides.
    - Set inequalities for the unknown side based on the sum of squares.
4. Combine the results to determine the precise interval for the third side length that guarantees an acute triangle.
5. Verify the results with practical examples or geometric constructions if necessary.

Additional Considerations

Special Cases

- When x equals the bounds ($12.49\,\text{cm}$ or $18.87\,\text{cm}$), the triangle becomes right-angled, not acute.
- For x less than 12.49 cm, the triangle is obtuse.
- For x greater than 18.87 cm but less than 26 cm, the triangle is obtuse or right-angled, depending on the exact value.

Real-World Applications

Understanding the range of side lengths for an acute triangle is crucial in various fields:

- Engineering: Designing structures that require specific angles.
- Navigation and Surveying: Calculating accurate land measurements.
- Computer Graphics: Rendering triangles with specific angle properties for realistic images.

Summary and Conclusion

In conclusion, when given two sides of an acute triangle as 10 cm and 16 cm with an unknown third side, the possible lengths of the third side are constrained by the triangle inequality and the acuteness condition. The key results are:

$$
\boxed{
\text{Length of third side } x \text{ must satisfy} \quad 12.49\,\text{cm} < x < 18.87\,\text{cm}
}
$$

This precise interval ensures that the triangle remains valid (satisfying the triangle inequality) and is acute (all angles less than 90°). Understanding these principles not only solves the specific problem but also enhances one's grasp of fundamental geometric concepts, applicable in various mathematical and practical contexts.

Further Resources and Practice

To strengthen your understanding of triangle properties and inequalities, consider exploring:

- Geometry textbooks on triangle classification.
- Interactive online tools for triangle construction.
- Practice problems involving triangle inequalities and angle conditions.
- Tutorials on the Law of Cosines and Law of Sines for more advanced triangle analysis.

By mastering these concepts, you'll be well-equipped to handle similar problems involving triangle side lengths and angle classifications confidently.

Frequently Asked Questions

What is the range of possible lengths for the third side of the triangle?

The third side must be greater than 6 cm and less than 26 cm, so 6 cm < third side < 26 cm.

Why does the triangle need to be acute given the side lengths of 10 cm and 16 cm?

Because the triangle is specified as acute, the third side length must satisfy conditions that make all angles less than 90°, which restricts the third side's length accordingly.

How can we determine if a triangle with sides 10 cm, 16 cm, and the unknown side is acute?

By checking if the squares of the sides satisfy the conditions: for all sides, the square of the longest side is less than the sum of the squares of the other two sides.

What is the maximum length the third side can be for the triangle to remain acute?

Less than 26 cm, specifically, it must be less than the sum of the two shorter sides (10 + 16 = 26 cm), but also satisfy the acuteness condition.

What is the minimum length the third side can be for the triangle to be valid and acute?

Greater than 6 cm, to satisfy the triangle inequality, and also ensuring the triangle remains acute based on the angle conditions.

If the third side is 20 cm, will the triangle be acute?

Yes, because the squares of the sides satisfy the condition: 20^2 = 400, which is less than 10^2 + 16^2 = 100 + 256 = 356? Actually, since 400 > 356, the triangle would be obtuse with a side of 20 cm, so in this case, it would not be acute.

How do the triangle inequality and the acuteness condition interact for determining the third side?

The triangle inequality ensures the sides can form a triangle (each side less than the sum of the other two), while the acuteness condition requires the square of the longest side to be less than the sum of the squares of the other two sides.

Can the third side be exactly 15 cm? Why or why not?

Yes, because it satisfies the triangle inequality (15 is between 6 and 26) and can produce an acute triangle if it meets the acuteness condition, specifically if 15^2 < 10^2 + 16^2, which is 225 < 356, so yes, 15 cm can produce an acute triangle.

What is the range of the third side length that guarantees the triangle is both valid and acute?

The third side must be greater than 6 cm and less than approximately 16.55 cm (since when the third side is about 16.55 cm, the triangle becomes right-angled), so roughly 6 cm < third side < 16.55 cm.

Additional Resources

An Acute Triangle Has Sides Measuring 10 Cm And 16 Cm. The Length Of The Third Side Is Unknown. Which

Understanding the properties and characteristics of triangles is fundamental in geometry, and when dealing with specific types such as acute triangles, the problem-solving process becomes both intriguing and educational. In this article, we delve into the scenario where an acute triangle has two known sides measuring 10 cm and 16 cm, while the third side remains unknown. Our objective is to analyze the possible length of the third side, explore the geometric constraints involved, and understand how to approach such problems systematically.

Understanding the Basics: Types of Triangles and Their Properties

Before we analyze the specific problem, it is essential to review the fundamental concepts related to triangles.

Types of Triangles Based on Angles

- Acute Triangle: All three angles are less than 90°.
- Right Triangle: One angle is exactly 90°.
- Obtuse Triangle: One angle is greater than 90°.

Properties of an Acute Triangle

- All sides satisfy the triangle inequality theorem.
- The largest angle is opposite the longest side.
- For a triangle with sides a, b, and c, the triangle is acute if and only if:

$$
c^2 < a^2 + b^2
$$

where c is the longest side.

Setting Up the Problem: Known Data and Unknowns

Given:
- Side $AB = 10\,cm$
- Side $AC = 16\,cm$
- Side $BC = x\,cm$ (unknown)

Objective:

- Determine the range of possible lengths for x that result in an acute triangle.

Applying the Triangle Inequality Theorem

The triangle inequality theorem states that for any triangle with sides a, b, and c:
- $a + b > c$
- $a + c > b$
- $b + c > a$

Applying this to our problem:
- $10 + 16 > x \Rightarrow 26 > x$
- $10 + x > 16 \Rightarrow x > 6$
- $16 + x > 10 \Rightarrow x > -6$ (which is always true for positive lengths)

Thus, the possible range for x is:
$$
6 < x < 26
$$

Any value of x within this interval can form a triangle with sides 10 and 16.

Conditions for the Triangle to Be Acute

To ensure the triangle is acute, we need to analyze the angles based on the side lengths.

Recall the Law of Cosines:
$$
c^2 = a^2 + b^2 - 2ab \cos C
$$

Where:
- a, b, c are the sides,
- C is the angle opposite side c.

In an acute triangle, all angles are less than 90°, meaning:

$$
\cos C > 0 \Rightarrow c^2 < a^2 + b^2
$$

Applying this to each angle:

- For angle A opposite side $a = BC = x$:

$$
x^2 < 10^2 + 16^2 = 100 + 256 = 356
$$

- For angle B opposite side $b = 16\,cm$:

$$
16^2 < 10^2 + x^2 \Rightarrow 256 < 100 + x^2 \Rightarrow x^2 > 156 \Rightarrow x > \sqrt{156} \approx 12.49
$$

- For angle C opposite side $c = 10\,\text{cm}$:

$$
10^2 < 16^2 + x^2 \Rightarrow 100 < 256 + x^2 \Rightarrow x^2 > -156
$$

This last inequality is always true for positive x.

Now, to ensure all angles are less than 90°, the largest side must satisfy the inequality:

$$
\text{Largest side}^2 < \text{sum of squares of other two sides}
$$

Given that, the maximum side length x can be less than 26, but to keep the triangle acute, it must also satisfy:

$$
x^2 < 356
$$
and
$$
x > 12.49
$$

Because:

- If x becomes the longest side (i.e., $x > 16\,\text{cm}$), then for the triangle to be acute:

$$
x^2 < 10^2 + 16^2 = 356
$$

$$
x < \sqrt{356} \approx 18.87
$$

- If $x < 16\,\text{cm}$, then the longest side is 16 cm, and the condition for acuteness is:

$$
16^2 < 10^2 + x^2 \Rightarrow 256 < 100 + x^2 \Rightarrow x^2 > 156 \Rightarrow x > 12.49
$$

Summary of the conditions:

- For $6 < x < 16$, the triangle is acute if $x > 12.49$, i.e.,

$$
x \in (12.49, 16)
$$

- For $16 < x < 18.87$, the triangle remains acute if:

\[

$$
x < 18.87
$$

Combining these:

$$
x \in (12.49, 18.87)
$$

- For $x > 18.87$, the triangle becomes obtuse because:

$$
x^2 > 356
$$

which violates the acuteness condition.

Final Range for the Third Side Length

Based on the above analysis, the length of the third side x must satisfy:

$$
\boxed{
\text{Range of } x: \quad 12.49\,cm < x < 18.87\,cm
}
$$

Within this interval, the triangle formed with sides 10 cm, 16 cm, and x is acute.

Visualizing the Triangle: Geometric Intuition

Visualizing the possible triangles helps cement understanding:

- When x is close to 6 cm (but greater than 6), the triangle is very "sharp" with a small third side.
- As x increases toward 16 cm, the triangle becomes more "equilateral" in shape, approaching a more balanced form.
- Once x surpasses approximately 18.87 cm, the triangle transitions into an obtuse shape because the largest angle exceeds 90°.

This geometric intuition complements the algebraic inequalities, providing a more holistic understanding.

Practical Applications and Examples

Understanding the ranges of side lengths for specific triangle types has practical significance in fields such as engineering, architecture, and design.

Example 1:
Suppose a construction project requires an acute triangular support with two sides fixed at 10 cm and 16 cm. The third side must be between approximately 12.5 cm and 18.9 cm to ensure the support maintains the desired shape.

Example 2:
In a design context, if an artist or engineer needs an acute triangle with sides close to 10 cm and 16 cm, they now know the precise length of the third side to achieve the aesthetic or structural property.

Challenges and Limitations

- Approximate Values: The critical bounds are approximate due to square root calculations, which could be refined for more precision.
- Assumption of Euclidean Geometry: These conclusions assume a flat, Euclidean plane. Different geometries might alter the conditions.
- Real-world Constraints: Practical factors like material flexibility and constraints could influence feasible side lengths beyond mathematical bounds.

Conclusion

The problem of determining the third side of an acute triangle with two known sides of 10 cm and 16 cm reveals the intricate relationship between side lengths and angles. By applying the triangle inequality theorem and the Law of Cosines, we deduce that the third side must lie within approximately 12.5 cm and 18.9 cm to ensure the triangle remains acute. This analysis underscores the importance of combining algebraic and geometric reasoning to solve real-world geometric problems effectively.

Understanding these principles enables students, engineers, architects, and designers to make informed decisions when working with specific triangle configurations, ensuring structural integrity and aesthetic appeal. The core takeaway is that the properties of triangles are deeply interconnected, and careful analysis allows us to predict and control their shape and characteristics within given constraints.

An Acute Triangle Has Sides Measuring 10 Cm And 16 Cm The Length Of The Third Side Is Unknown Which

An Acute Triangle Has Sides Measuring 10 Cm And 16 Cm The Length Of The Third Side Is Unknown Which

Related Articles

- An Arrangement For One Container Needs To Have At Least 4 Flowers In It And The Total Wholesale Cost
- After Deciding To Buy A New Car, You Can Either Lease The Car Or Purchase It On A Two-year Loan. The
- As A Supervisor Of A Production Department, You Must Decide The Daily Production Totals Of A Certain

Back to Home