An Equivalent Equation Has Been Written By Multiplying The Second Equation By 2. 4x 9y = 7 4x 9y = 7

An Equivalent Equation Has Been Written By Multiplying The Second Equation By 2. 4x 9y = 7 4x 9y = 7 is a statement that often appears in algebra lessons and exercises aimed at understanding the concept of equivalent equations. This process, which involves multiplying an entire equation by a constant, is fundamental in solving systems of equations and simplifying algebraic expressions. In this article, we will explore what it means to write equivalent equations, how multiplying an equation by a number affects its solutions, and the importance of this technique in algebra.

Understanding Equivalent Equations

What Are Equivalent Equations?

Equivalent equations are equations that have the same set of solutions. In other words, even if they look different on the surface, they represent the same relationship between variables. For example, the equations:

- $2x + 3y = 6$
- $4x + 6y = 12$

are equivalent because multiplying the first equation by 2 yields the second. Both equations describe the same line on a graph, and any solution that satisfies one will satisfy the other.

Why Are Equivalent Equations Important?

Equivalent equations are crucial in algebra for several reasons:

- Simplification: They allow us to manipulate equations into forms that are easier to solve.
- Consistency: They help verify solutions by showing that different forms represent the same relationship.
- Solving Systems: When solving systems of equations, generating equivalent equations can facilitate elimination or substitution methods.

Multiplying Equations by a Constant

The Process of Multiplying Equations

Multiplying an entire equation by a non-zero constant is a common technique to create an equivalent equation. The key points include:

- The multiplication applies to all terms in the equation.
- The solution set remains unchanged because the equation is effectively scaled, not altered in its fundamental relationship.
- This process is an example of an algebraic operation that preserves equivalence.

Example: Multiplying a Equation by 2

Let's consider the original equation:

$$4x + 9y = 7$$

Multiplying both sides by 2:

$$2 \quad (4x + 9y) = 2 \quad 7$$

which simplifies to:

$$8x + 18y = 14$$

This new equation is equivalent to the original because it has the same solutions.

Implications of Multiplying Equations

Effect on Solutions

Since multiplying both sides of an equation by a non-zero constant scales the entire equation equally, the solutions remain unchanged. This means:

- Any (x, y) that satisfies the original equation also satisfies the

multiplied version.
- The solution set is preserved, making the equations equivalent.

Graphical Representation

On a Cartesian plane, the original and the multiplied equations represent the same line. The slope and intercepts are scaled, but the line itself remains unchanged in position and shape.

Application in Solving Systems of Equations

Using Equivalent Equations for Elimination

When solving systems using elimination, generating equivalent equations is often a key step:

- Multiply one or both equations by suitable constants to align coefficients.
- Add or subtract the equations to eliminate a variable.
- Solve for the remaining variable.

Example

Suppose we have the system:

$$4x + 9y = 7$$
$$2x - 3y = 4$$

To eliminate x, multiply the second equation by 2:

$$2 \quad (2x - 3y) = 2 \quad 4$$

which gives:

$$4x - 6y = 8$$

Now, subtract the first equation from this:

$$(4x - 6y) - (4x + 9y) = 8 - 7$$

Simplifies to:

$0x - 15y = 1$

which simplifies to:

$-15y = 1$

Solution:

$$y = -\frac{1}{15}$$

Substitute back into one of the original equations to find x.

Common Mistakes and Misconceptions

Multiplying by Zero

- Multiplying an equation by zero results in 0=0, which is always true but does not preserve the original solutions. It essentially destroys the information contained in the original equation.
- Always multiply by non-zero constants to maintain equivalence.

Misinterpreting the Effect of Multiplication

- Remember, multiplying both sides by a constant does not change the set of solutions.
- However, it changes the coefficients and constants, which can affect the way the equation looks but not its solutions.

Conclusion

Understanding that an equivalent equation can be obtained by multiplying the original equation by a constant—such as in the case of 4x + 9y = 7 and its scaled version—is vital for mastering algebra. This technique allows for flexible manipulation of equations, simplifying the process of solving systems and analyzing relationships between variables. Remember that multiplying by any non-zero number preserves the solution set, making this a powerful and permissible operation in algebraic problem-solving. Whether you're working on systems of equations, simplifying expressions, or verifying solutions, recognizing and applying the concept of equivalent equations is an essential skill for students and professionals alike.

Frequently Asked Questions

Why is the second equation written as 8x 18y = 14 after multiplying the original equation by 2?

Multiplying both sides of the original equation $4x + 9y = 7$ by 2 results in $8x + 18y = 14$, which is an equivalent equation that maintains the equality.

Does multiplying an entire equation by a constant change its solution set?

No, multiplying both sides of an equation by a non-zero constant produces an equivalent equation with the same solution set.

What is the purpose of creating equivalent equations by multiplying through by a number?

Creating equivalent equations helps in simplifying, solving, or comparing equations without changing their solution sets.

Are the equations 4x + 9y = 7 and 8x + 18y = 14 considered equivalent?

Yes, because the second equation is obtained by multiplying the first by 2, so both have the same solutions.

Can multiplying an equation by 2 lead to mistakes in solving if not done carefully?

Yes, if you forget to multiply all terms correctly or make a calculation error, it can lead to incorrect solutions or misunderstandings.

Additional Resources

An Equivalent Equation Has Been Written By Multiplying The Second Equation By 2. 4x 9y = 7 4x 9y = 7

In the realm of algebra, the process of manipulating equations to uncover solutions or simplify expressions is fundamental. The phrase "An equivalent equation has been written by multiplying the second equation by 2" encapsulates a common technique used in solving systems of equations or transforming expressions without altering their solutions. This practice hinges on the principle that multiplying both sides of an equation by a non-zero number yields an equivalent equation—that is, an equation with the same solution set. The specific example involving the equations 4x + 9y = 7, followed by its modification through multiplication, exemplifies this concept. To fully grasp the significance, implications, and methods behind such transformations, a comprehensive examination is warranted.

Understanding Equations and Their Equivalence

What Is an Equation?

An equation is a mathematical statement asserting the equality of two expressions. It often contains variables, constants, and operations. For example, in the equation 4x + 9y = 7, the expressions on both sides are set equal, establishing a relationship between the variables x and y.

Defining Equivalent Equations

Two equations are considered equivalent if they have identical solution sets—that is, the same values of variables satisfy both equations. Transformations such as adding or subtracting the same quantity from both sides, or multiplying both sides by a non-zero constant, preserve equivalence. These transformations are fundamental tools in algebra for simplifying equations and solving systems.

The Significance of Equivalence in Algebraic Manipulation

Maintaining the solution set during transformations is critical. If an operation alters the solutions, the resulting equation would no longer represent the original problem. Multiplying both sides of an equation by a non-zero number, for instance, scales the entire expression but does not change the solutions because the balance of the equation remains intact.

Breaking Down the Transformation: Multiplying

by 2

The Original Equation

The initial equation provided is:

$$
4x + 9y = 7
$$

This linear equation relates variables x and y within a two-dimensional coordinate system.

The Modified Equation

The statement indicates that the second equation is multiplied by 2, leading to:

$$
2 \times (4x + 9y) = 2 \times 7
$$

which simplifies to:

$$
8x + 18y = 14
$$

This new equation is equivalent to the original, as it maintains the same solution set.

Why Multiply by 2?

Multiplying both sides of an equation by 2 serves multiple purposes:
- Simplification: It can create coefficients that are easier to work with, especially for elimination methods.
- Alignment: When solving systems, aligning coefficients can streamline the process.
- Demonstration of Equivalence: It exemplifies that such multiplication preserves the solution set, reinforcing the principle of equivalence.

Implications and Applications of Multiplying Equations

Solving Systems of Equations

In systems with multiple equations, multiplying an entire equation by a scalar is a strategic step to facilitate elimination or substitution methods.

Example:
Suppose we have a system:

$$
\begin{cases}
4x + 9y = 7 \quad (1) \\
3x - 2y = 4 \quad (2)
\end{cases}
$$

To eliminate one variable, we might multiply equation (2) by 4:

$$
4 \times (3x - 2y) = 4 \times 4 \Rightarrow 12x - 8y = 16
$$

which can then be combined with (1) after suitable adjustments to solve for either variable.

Key Point: Multiplying equations by scalars preserves solutions but changes coefficients, enabling more straightforward elimination strategies.

Maintaining Equation Integrity

It's essential to remember that multiplying or dividing an equation by zero is invalid because it would eliminate information or produce undefined expressions. The operation must involve a non-zero scalar to ensure the equation remains equivalent.

Real-World Applications

The concept extends beyond academic exercises:
- Engineering: Adjusting equations to match units or scales.
- Economics: Comparing proportional relationships.
- Physics: Scaling formulas to fit different measurement units or conditions.

Analyzing the Specific Example: 4x + 9y = 7 and Its Equivalent

Original Equation: 4x + 9y = 7

This linear equation can be visualized graphically as a straight line in the xy-plane. Its solutions form a line where the sum of 4 times x and 9 times y equals 7.

Multiplying by 2: $8x + 18y = 14$

This new equation, though scaled, describes the same set of points as the original line. Graphically, it is the same line, just represented with different coefficients that are proportional to the original.

Why Is This Important? Insights and Clarifications

- Preservation of Solutions: Both equations define the same line.
- Coefficient Ratios: The ratios between coefficients remain constant:

$$
\frac{8}{4} = 2, \quad \frac{18}{9} = 2, \quad \frac{14}{7} = 2
$$

- Implication for Graphs: The two lines coincide, confirming their equivalence.

Practical Usage

In solving such equations, multiplying by 2 can be advantageous for elimination or substitution:
- To eliminate y, for example, multiply the second equation by a factor that aligns with the coefficient of y in the first.
- To compare ratios or find intercepts, scaled equations can simplify calculations.

Mathematical Principles Underpinning the Transformation

Properties of Equality

The core property allowing multiplication is:
> If $a = b$ and $k \neq 0$, then $ka = kb$.

This principle guarantees that multiplying both sides by a non-zero scalar yields an equivalent equation.

Implications for Solution Sets

Because the solutions are unaffected, the solution set of the original equation and its scaled version are identical. This is fundamental in linear algebra, where systems are often manipulated to reach a form suitable for solution extraction (e.g., row echelon form).

Limitations and Cautions

- Zero Multipliers: Multiplying both sides by zero invalidates the process because it reduces the equation to 0=0, which provides no useful information.
- Sign Changes: Multiplying by a negative number reverses inequalities in inequalities but does not affect equalities in the same way; the solution set remains the same.

Broader Context and Educational Significance

Learning Algebraic Manipulation

Understanding that multiplying an entire equation by a scalar yields an equivalent equation is foundational for students learning algebra. It emphasizes the importance of maintaining balance and recognizing the invariance of solutions.

Pedagogical Strategies

- Visualization: Graphing both equations helps students see their equivalence.
- Step-by-Step Exercises: Practice multiplying equations by different scalars to become comfortable with the process.
- Conceptual Discussions: Clarify why such operations do not alter the solution set to prevent misconceptions.

Common Mistakes to Avoid

- Multiplying only one side of the equation.
- Using zero as a multiplier.
- Forgetting to multiply all terms, leading to invalid transformations.

Conclusion: Significance of the Transformation

The statement "An equivalent equation has been written by multiplying the second equation by 2" encapsulates a fundamental principle in algebra: that multiplying both sides of an equation by a non-zero scalar preserves its solutions. This technique is not merely a mathematical formality but a practical tool that facilitates solving equations, analyzing systems, and understanding relationships between variables. The specific example involving

$4x + 9y = 7$ and its scaled version to $8x + 18y = 14$ underscores the importance of recognizing proportionality and the invariance of solutions under such transformations. Mastery of these concepts empowers learners and practitioners to manipulate equations confidently, ultimately enhancing problem-solving skills across mathematics and its numerous applications in science, engineering, economics, and beyond.

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