Answer The Following Questions For The Function F(x)=x(x^2+16) Defined On The Interval -6 4. F(x) Is

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Understanding the behavior of a mathematical function is essential for analyzing its properties, including its critical points, intervals of increase or decrease, concavity, and overall shape. The function under consideration, $F(x) = x(x^2 + 16)$, defined on the interval $[-6, 4]$, provides a rich example for such an analysis. This article delves into the detailed examination of this function, exploring its domain, derivative, critical points, increasing/decreasing intervals, concavity, and points of inflection, ultimately offering a comprehensive understanding of its characteristics.

1. Definition and Basic Properties of the Function

1.1 Function Expression

The given function is expressed as:

$$F(x) = x(x^2 + 16)$$

which simplifies to:

$$F(x) = x^3 + 16x$$

1.2 Domain

The domain of $F(x)$ is the interval $[-6, 4]$. Since the function is a polynomial, it is continuous and differentiable everywhere, but our focus is on the specified interval.

2. Basic Analysis of F(x)

2.1 Endpoints of the Interval

- The function values at the endpoints:
- $F(-6) = (-6)^3 + 16 \times (-6) = -216 - 96 = -312$
- $F(4) = 4^3 + 16 \times 4 = 64 + 64 = 128$

These points serve as reference points for analyzing maximums and minimums within the interval.

2.2 Symmetry and Behavior at Large |x|

- As $x \to \pm \infty$, $F(x) \to \pm \infty$, but since our interval is bounded, the behavior within $[-6, 4]$ is more relevant.

3. Derivative and Critical Points

3.1 First Derivative

To analyze increasing/decreasing behavior, find the first derivative:

$$
F'(x) = \frac{d}{dx}(x^3 + 16x) = 3x^2 + 16
$$

3.2 Critical Points

Critical points occur where $F'(x) = 0$ or where $F'(x)$ is undefined. Since $F'(x) = 3x^2 + 16$, which is always positive (since $3x^2 \geq 0$ and $16 > 0$), the derivative never equals zero:

$$
3x^2 + 16 > 0 \quad \text{for all real } x
$$

Thus, there are no critical points within the interval $[-6, 4]$ or elsewhere.

3.3 Implication of the Derivative

Since $F'(x) > 0$ everywhere, the function is strictly increasing over its entire domain, including $[-6, 4]$.

4. Monotonicity and Extrema

4.1 Increasing/Decreasing Intervals

- Because $F'(x) > 0$ for all x, the function is strictly increasing throughout the interval $[-6, 4]$.

4.2 Maximum and Minimum Values

- The minimum value occurs at the left endpoint $x = -6$:

$$
F(-6) = -312
$$

- The maximum value occurs at the right endpoint $x = 4$:

$$
F(4) = 128
$$

Therefore, within $[-6, 4]$, $F(x)$ increases monotonically from -312 to 128.

5. Concavity and Inflection Points

5.1 Second Derivative

To analyze the concavity, find the second derivative:

$$
F''(x) = \frac{d}{dx} (3x^2 + 16) = 6x
$$

5.2 Concavity Analysis

- The second derivative $F''(x) = 6x$ determines concavity:
- When $F''(x) > 0$, the graph is concave upward.
- When $F''(x) < 0$, the graph is concave downward.

- Within the interval $[-6, 4]$:
- For $x > 0$, $F''(x) > 0$ (concave upward).
- For $x < 0$, $F''(x) < 0$ (concave downward).

5.3 Inflection Point

- The inflection point occurs where $F''(x) = 0$:

$$
6x = 0 \Rightarrow x = 0
$$

- At $x = 0$, the concavity changes:

$$

$$
F(0) = 0^3 + 16 \times 0 = 0
$$

Thus, the inflection point is at $(0, 0)$.

6. Summary of the Function’s Graphical Behavior

6.1 Overall Shape

- The function $F(x) = x^3 + 16x$ is a cubic polynomial with a monotonic increasing trend over the interval $[-6, 4]$.
- It features an inflection point at $(0, 0)$, where the concavity switches from downward to upward.

6.2 Key Features

- No local maxima or minima within the interval.
- Function values increase from -312 at $x = -6$ to 128 at $x = 4$.
- The inflection point at $(0, 0)$ indicates a change in the curvature of the graph.

7. Final Conclusions

7.1 Nature of the Function on $[-6, 4]$

- The function $F(x)$ is strictly increasing throughout the interval.
- It has no critical points in the interval, meaning no local maxima or minima within $[-6, 4]$.
- The behavior is characterized by an inflection point at $x=0$, where the curvature changes.

7.2 Practical Implications

- Since $F(x)$ increases monotonically, the function is invertible on this interval.
- The function's graph on $[-6, 4]$ smoothly transitions from its minimum value at $x = -6$ to its maximum at $x=4$.

8. Additional Remarks

8.1 Sign of the Derivative and Concavity

- The positive derivative indicates a consistently increasing function.
- The change in concavity at $x=0$ affects the shape but not the monotonicity.

8.2 Relevance to Applications

- Understanding the increasing nature and inflection point can inform applications involving the modeling of growth or change, where the monotonic trend and curvature influence predictions or interpretations.

Conclusion

The detailed analysis demonstrates that the function $F(x) = x^3 + 16x$, defined on $[-6, 4]$, is a strictly increasing cubic function with an inflection point at the origin. It exhibits a change in concavity at $x=0$, transitioning from concave downward to concave upward. Its behavior is fully characterized by its monotonic increase from -312 to 128, with the critical and inflection points providing insight into its geometric and analytical properties. This comprehensive understanding allows for effective use of the function in various mathematical, scientific, or engineering contexts where such properties are relevant.

Frequently Asked Questions

What is the function F(x) = x(x^2 + 16) defined on the interval [-6, 4]?

$F(x) = x(x^2 + 16)$, defined for x in the interval [-6, 4], is a cubic function that combines a linear term with a quadratic expression.

Is the function F(x) = x(x^2 + 16) increasing or decreasing on the interval [-6, 4]?

The function is decreasing on parts of the interval where its derivative is negative and increasing where the derivative is positive. Analyzing $F'(x)$ helps determine the exact intervals of increase and decrease.

What are the critical points of F(x) = x(x^2 + 16) on the interval [-6, 4]?

Critical points occur where $F'(x) = 0$ or is undefined. For this function, critical points are at $x = 0$ and $x = -4\sqrt{3}$, within the interval [-6, 4].

What is the value of F(x) at the endpoints of the interval?

At x = -6, F(-6) = -6((-6)^2 + 16) = -6(36 + 16) = -6(52) = -312. At x = 4, F(4) = 4(16 + 16) = 4(32) = 128.

Does the function F(x) = x(x^2 + 16) have any local maxima or minima on the interval?

Yes, by analyzing critical points, the function has a local maximum at certain points and a local minimum at others within the interval, depending on the sign of the second derivative.

What is the domain and range of F(x) = x(x^2 + 16) on [-6, 4]?

The domain is the interval [-6, 4]. The range can be found by evaluating the function at critical points and endpoints, resulting in a range from approximately -312 to 128.

Is the function F(x) = x(x^2 + 16) continuous and differentiable on the interval [-6, 4]?

Yes, since it is a polynomial function, F(x) is continuous and differentiable everywhere, including on the interval [-6, 4].

How does the function F(x) behave as x approaches the endpoints of the interval?

As x approaches -6, F(x) decreases to -312; as it approaches 4, F(x) increases to 128. The function's behavior is smooth and continuous within the interval.

Can the function F(x) = x(x^2 + 16) be used to model real-world phenomena?

Yes, cubic functions like this can model various phenomena such as physics trajectories, economic models, and other scenarios involving polynomial relationships within specific domains.

Additional Resources

Answer The Following Questions For The Function F(x)=x(x^2+16) Defined On The Interval -6 to 4: An In-Depth Analytical Review

In the realm of calculus and mathematical analysis, understanding the behavior of functions over specific intervals is fundamental. The function \(F(x) = x(x^2 + 16)\), defined on the interval \([-6, 4]\), presents an interesting case for examination due to its polynomial nature and the potential insights it offers into the concepts of critical points, increasing and decreasing intervals, concavity, and overall behavior. This comprehensive review aims to thoroughly investigate the properties of $F(x)$, answer pertinent questions about its characteristics, and provide an insightful analysis suitable for academic or educational review.

Overview of the Function $F(x) = x(x^2 + 16)$

Before diving into detailed properties, it is essential to understand the structure of the function:

- Type: Polynomial function of degree 3 (cubic).
- Expression: $F(x) = x^3 + 16x$.
- Domain: $[-6, 4]$.

This function combines a cubic term with a linear component, resulting in a graph that exhibits both increasing and decreasing behavior, as well as potential points of inflection.

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1. Simplification and Basic Properties

1.1. Simplified Form:

Expressed explicitly, the function is:

$$
F(x) = x^3 + 16x
$$

This form is helpful for derivative calculations and analyzing the function's behavior.

1.2. Endpoints Evaluation:

Calculating $F(x)$ at the interval endpoints:

- At $x = -6$:

$$
F(-6) = (-6)^3 + 16(-6) = -216 - 96 = -312
$$

- At $x = 4$:

$$
F(4) = 4^3 + 16 \times 4 = 64 + 64 = 128
$$

These values establish the overall range of the function over the interval.

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2. Derivative Analysis: Critical Points and Monotonicity

Understanding where the function increases or decreases requires examining its first derivative.

2.1. First Derivative:

$$
F'(x) = \frac{d}{dx}(x^3 + 16x) = 3x^2 + 16
$$

2.2. Critical Points:

Critical points occur where $F'(x) = 0$:

$$
3x^2 + 16 = 0
$$

$$
x^2 = -\frac{16}{3}
$$

Since x^2 cannot be negative for real x, the equation has no real solutions. Therefore,

Critical Point Conclusion:

- No real critical points exist within the domain.

- The derivative $F'(x) = 3x^2 + 16$ is always positive because the quadratic term is always positive, and 16 is positive.

2.3. Monotonicity:

Because $F'(x) > 0$ for all x, the function:

- Is strictly increasing over the entire domain $[-6, 4]$.

Implication:

- No local maxima or minima within the interval.
- The function steadily increases from $F(-6) = -312$ to $F(4) = 128$.

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3. Concavity and Inflection Points

Analyzing concavity involves the second derivative.

3.1. Second Derivative:

$$
F''(x) = \frac{d}{dx}(3x^2 + 16) = 6x
$$

3.2. Inflection Point:

An inflection point occurs where $F''(x) = 0$:

$$
6x = 0 \Rightarrow x = 0
$$

- At $x=0$, the concavity changes.

3.3. Concavity Analysis:

- For $x < 0$:

\[

F''(x) = 6x < 0
\]

- The function is concave downward on \([-6, 0)\).

- For \(x > 0\):

\[
F''(x) = 6x > 0
\]

- The function is concave upward on \((0, 4]\).

3.4. Corresponding \(F(x)\) at \(x=0\):

\[
F(0) = 0^3 + 16 \times 0 = 0
\]

Thus, the inflection point is at \((0, 0)\).

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4. Behavior of the Function on the Interval \([-6, 4]\)

4.1. Overall Increasing Trend:

- Since \(F'(x) > 0\) everywhere, the graph shows a monotonically increasing trend on \([-6, 4]\).
- The function moves from \(-312\) at \(x=-6\) to \(128\) at \(x=4\).

4.2. Concavity Changes and Inflection Point:

- The inflection point at \((0, 0)\) signifies a shift from concave downward to concave upward.
- The graph is "S-shaped" with a smooth transition at \(x=0\).

4.3. Local Extrema:

- No local maxima or minima exist within the domain because the derivative never equals zero.

4.4. Summary of behavior:

Interval	Monotonicity	Concavity	Notable Point
$[-6, 0)$	Increasing	Concave downward	Approaching 0 at $x=0$
$(0, 4]$	Increasing	Concave upward	Passing through inflection at $x=0$

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5. Graphical Interpretation

The function's graph over the interval $[-6, 4]$ would depict:

- A smooth, steadily increasing curve starting at $(-6, -312)$,
- A gentle inflection at $(0, 0)$, where the curvature changes,
- Continuing upward to $(4, 128)$.

The graph's shape resembles a cubic with a subtle "S" form, but with no peaks or valleys due to the absence of critical points.

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6. Summary of Key Findings

- Function form: $F(x) = x^3 + 16x$.
- Domain: $[-6, 4]$.
- Endpoints: $F(-6) = -312$, $F(4) = 128$.
- Critical points: None within the domain; the function is strictly increasing.
- Monotonicity: Increasing over entire interval.
- Concavity: Concave downward on $[-6, 0)$, inflection at $x=0$, then concave upward on $(0, 4]$.
- Inflection Point: At $(0, 0)$.

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7. Broader Implications and Applications

The analysis of $F(x) = x(x^2 + 16)$ exemplifies fundamental calculus concepts. Its strictly increasing nature simplifies certain analyses, such as solving inequalities or optimization problems within the domain. The presence of an inflection point indicates a change in the curvature, which could be relevant in contexts where the second derivative signifies acceleration or concavity, such as physics or economics.

Understanding these properties provides a foundation for more complex functions and can serve as an educational tool for illustrating the relationship between derivatives, critical points, and the shape of graphs.

Conclusion

The detailed investigation of \(F(x) = x(x^2 + 16)\) over \([-6, 4]\) reveals a function characterized by continuous increase, a single inflection point at the origin, and a change in concavity but no local extrema. Its behavior is straightforward yet rich enough to exemplify several core concepts in calculus, making it a valuable case study for students and professionals alike. This thorough analysis not only answers specific questions about the function but also underscores the importance of derivative tests and concavity in understanding the geometry of polynomial functions.

In essence, \(F(x)\) is a well-behaved, strictly increasing cubic function with a single inflection point, offering clear insights into the interplay between derivatives, concavity, and graph shape.

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