Anu's Age Exceeds Sumbo's Age By 15 The Sum Of The Square Of Their Ages Is 725. What Are Their Ages?

Introduction

Anu's Age Exceeds Sumbo's Age By 15 The Sum Of The Square Of Their Ages Is 725. What Are Their Ages? This problem presents a classic algebraic challenge involving age relationships and quadratic equations. Such problems are common in mathematics, especially in algebra, where they test one's ability to translate verbal statements into equations and then solve for unknowns. Understanding how to approach this problem involves a step-by-step process of setting variables, forming equations, and solving them systematically.

Understanding the Problem

Breaking Down the Information

The problem provides two key pieces of information:

- Anu's age exceeds Sumbo's age by 15.
- The sum of the squares of their ages is 725.

Our goal is to determine their current ages based on these details. To do this, we need to convert these statements into algebraic equations and then solve for the unknowns.

Setting Up Variables

Assigning Variables

Let:

- S = Sumbo's age
- A = Anu's age

From the first statement, "Anu's age exceeds Sumbo's age by 15," we can

write:

A = S + 15

Formulating the Equations

Equation from the Age Difference

As established, the first equation is:

A = S + 15

Equation from the Sum of Squares

The second statement says, "The sum of the squares of their ages is 725," which translates to:

A^2 + S^2 = 725

Substituting and Simplifying

Substitute A in the Second Equation

Replacing A with S + 15 in the second equation yields:

(S + 15)^2 + S^2 = 725

Expanding the Equation

Expand (S + 15)^2:

S^2 + 30S + 225 + S^2 = 725

Combine like terms:

2S^2 + 30S + 225 = 725

Bring all terms to one side to form a quadratic equation:

2S^2 + 30S + 225 - 725 = 0

2S^2 + 30S - 500 = 0

Divide through by 2 for simplicity:

S^2 + 15S - 250 = 0

Solving the Quadratic Equation

Applying the Quadratic Formula

The quadratic is in standard form: S^2 + 15S - 250 = 0. The quadratic formula is:

S = [-b ± √(b^2 - 4ac)] / 2a

Where:

- a = 1
- b = 15
- c = -250

Calculating the Discriminant

The discriminant D is:

D = b^2 - 4ac = 15^2 - 4(1)(-250) = 225 + 1000 = 1225

Finding the Square Root of the Discriminant

√D = √1225 = 35

Calculating the Roots

S = [-15 ± 35] / 2

First root:

S = (-15 + 35) / 2 = 20 / 2 = 10

Second root:

S = (-15 - 35) / 2 = -50 / 2 = -25

Interpreting the Solutions

Since age cannot be negative, we discard S = -25. Therefore, Sumbo's age is:

S = 10

Now, find Anu's age:

A = S + 15 = 10 + 15 = 25

Final Answer and Verification

Summary of Their Ages

- Sumbo's age: 10 years
- Anu's age: 25 years

Verification of the Solution

To ensure the solution is correct, verify the sum of the squares:

A^2 + S^2 = 25^2 + 10^2 = 625 + 100 = 725

This matches the given condition, confirming the correctness of the solution.

Additional Insights

Understanding the Context

Such algebraic problems are relevant in various real-life contexts, such as age-related puzzles, financial calculations, and scientific measurements, where relationships between quantities are expressed mathematically. Recognizing patterns and translating words into equations is a fundamental skill in problem-solving.

Alternative Methods

While the quadratic formula is the most straightforward approach here, other methods such as factoring (if applicable) or completing the square can also be used, especially for simpler quadratics. In this case, the quadratic formula was efficient due to the discriminant being a perfect square.

Conclusion

The problem of determining Anu's and Sumbo's ages based on the given conditions demonstrates the power of algebra in solving real-world problems. By carefully defining variables, translating verbal statements into equations, and applying systematic solving techniques, we arrived at the solution: Sumbo is 10 years old, and Anu is 25 years old. These ages satisfy both conditions provided in the problem, confirming the accuracy of our solution.

Frequently Asked Questions

What are Anu's and Sumbo's ages if Anu's age exceeds Sumbo's age by 15 and the sum of the squares of their ages is 725?

Anu is 25 years old, and Sumbo is 10 years old.

How can we set up the equations to find Anu's and Sumbo's ages based on the given information?

Let Sumbo's age be S; then Anu's age is S + 15. The sum of their squares gives the equation (S + 15)^2 + S^2 = 725.

What algebraic steps are involved in solving for Sumbo's age in this problem?

Expand (S + 15)^2 to get S^2 + 30S + 225, then add S^2 to get 2S^2 + 30S + 225 = 725. Simplify to 2S^2 + 30S - 500 = 0, then solve the quadratic.

What quadratic equation do we obtain when solving for Sumbo's age?

The quadratic equation is 2S^2 + 30S - 500 = 0.

How do you solve the quadratic equation 2S^2 + 30S - 500 = 0?

Divide through by 2 to get S^2 + 15S - 250 = 0, then use the quadratic formula S = [-15 ± √(15^2 - 41(-250))]/2.

What are the solutions for Sumbo's age after solving

the quadratic equation?

S = 10 or S = -25. Since age cannot be negative, Sumbo is 10 years old, and Anu is 25.

What is the significance of the negative root in the quadratic solution?

The negative root (-25) is discarded because age cannot be negative; only the positive root is valid.

How does the given problem illustrate the use of algebra in real-life scenarios?

It demonstrates how to translate word problems into equations and solve for unknowns using algebraic methods.

Can this method be applied to similar problems involving ages and squares?

Yes, this approach of setting up equations and solving quadratics can be applied to various age-related problems and other real-world situations involving quadratic relationships.

Additional Resources

Anu's Age Exceeds Sumbo's Age By 15 The Sum Of The Square Of Their Ages Is 725. What Are Their Ages? – this intriguing problem combines basic algebra with real-world applications, illustrating how mathematical reasoning can help us uncover hidden information about ages. Whether you're a student working through algebraic equations or simply someone curious about problem-solving strategies, this guide will walk you through a comprehensive, step-by-step approach to finding Anu and Sumbo's ages. By the end, you'll understand how to set up equations, manipulate them, and interpret your solutions with confidence.

Understanding the Problem

Before diving into formulas and calculations, it's crucial to understand the problem's details and what it asks us to find.

- Given:
- Anu's age exceeds Sumbo's age by 15 years.
- The sum of the squares of their ages is 725.

- To find:
- The current ages of Anu and Sumbo.

This is a classic algebraic problem involving two unknowns—Anu's age and Sumbo's age—and two key pieces of information: their age difference and a sum of squares condition. Recognizing these clues enables us to translate the problem into mathematical equations.

Setting Up Variables

The first step in solving any algebraic problem is to define variables that represent the unknowns.

- Let S = Sumbo's age
- Let A = Anu's age

With these variables, the problem's conditions can be expressed mathematically.

Translating the Conditions into Equations

1. Age Difference

Anu's age exceeds Sumbo's age by 15

Mathematically, this translates to:

$A = S + 15$

This simple equation captures the direct relationship between their ages.

2. Sum of the Squares of Ages

The sum of the squares of their ages is 725

Expressed mathematically:

$A^2 + S^2 = 725$

Now, with two equations:

- $A = S + 15$
- $A^2 + S^2 = 725$

we can proceed to solve for the variables.

Solving the System of Equations

Step 1: Substitute A in the second equation

Since A = S + 15, substitute this into the sum of squares:

$(S + 15)^2 + S^2 = 725$

Step 2: Expand the squared term

$(S + 15)^2$ expands to:

$S^2 + 2\ 15\ S + 15^2 = S^2 + 30S + 225$

So, the equation becomes:

$S^2 + 30S + 225 + S^2 = 725$

Step 3: Simplify the equation

Combine like terms:

$2S^2 + 30S + 225 = 725$

Subtract 725 from both sides:

$2S^2 + 30S + 225 - 725 = 0$

Simplify:

$2S^2 + 30S - 500 = 0$

Step 4: Divide through by 2 to simplify

$S^2 + 15S - 250 = 0$

Now, the quadratic equation is:

$S^2 + 15S - 250 = 0$

Solving the Quadratic Equation

The quadratic equation $S^2 + 15S - 250 = 0$ can be solved using the quadratic formula:

$S = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

where:

- a = 1
- b = 15
- c = -250

Step 1: Calculate the discriminant

$D = b^2 - 4ac = (15)^2 - 4\ 1\ (-250) = 225 + 1000 = 1225$

Step 2: Find the square root of the discriminant

$\sqrt{D} = \sqrt{1225} = 35$

Step 3: Apply the quadratic formula

$S = [-15 \pm 35] / 2$

This yields two solutions:

- $S = (-15 + 35) / 2 = 20 / 2 = 10$
- $S = (-15 - 35) / 2 = -50 / 2 = -25$

Step 4: Interpret the solutions

Since age cannot be negative, discard S = -25. Therefore:

Sumbo's age S = 10

Step 5: Find Anu's age

Recall that:

$A = S + 15$

Substitute S = 10:

$A = 10 + 15 = 25$

Final Answer and Verification

Anu's age = 25 years

Sumbo's age = 10 years

Verify the solution:

- Check the sum of squares:

$A^2 + S^2 = 25^2 + 10^2 = 625 + 100 = 725$

- Confirm the difference:

$A - S = 25 - 10 = 15$

Both conditions are satisfied, confirming the correctness of the solution.

Additional Insights

Real-world relevance:

- This type of problem demonstrates how algebra can model real-world scenarios involving ages, time, or measurements.
- The process emphasizes the importance of translating words into equations, which is a foundational skill in mathematics.

Variations of the problem:

- What if the sum of the ages is also given?
- How would the solution change if the sum of the ages were different?
- What if the sum of the squares was a different number?

Exploring these variations can deepen understanding and develop problem-solving flexibility.

Summary of the Solution Process

Here's a quick rundown of the steps taken:

1. Define variables for unknowns.
2. Translate the given conditions into algebraic equations.
3. Substitute one variable into the other to reduce to a quadratic.
4. Solve the quadratic using the quadratic formula.
5. Discard extraneous (negative) solutions.
6. Verify the solutions with original conditions.

Conclusion

The problem "Anu's age exceeds Sumbo's age by 15, and the sum of the squares of their ages is 725" is a classic example of how algebraic techniques can be applied to solve real-world problems. By carefully defining variables, translating words into equations, and systematically solving the resulting quadratic, we determine that Anu is 25 years old and Sumbo is 10 years old. This approach underscores the power of algebra to unravel puzzles and make sense of numerical relationships in everyday life.

Whether you're tackling similar age-related problems or exploring more complex algebraic scenarios, mastering these steps will enhance your problem-solving toolkit and mathematical confidence.

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