Assume That 0 3 < 3600. (i) If Cos 3 Is Positive, Show That There Is An Acute Angle With 3 3 Or 3

Assume That $0^\circ < \theta < 360^\circ$, (i) If cos θ is positive, show that there is an acute angle with 3θ or 3θ

Introduction

In trigonometry, understanding the relationships between angles and their trigonometric functions is fundamental. One common problem involves analyzing the properties of angles within a specific range, particularly when dealing with the cosine function and its implications on the angles' measures. The statement "Assume that $0^\circ < \theta < 360^\circ$, and if cos θ is positive, show that there is an acute angle with 3θ or 3θ" combines multiple core ideas: the behavior of cosine within the unit circle, the implications of angle multiplication, and the existence of specific angles with certain properties.

This article aims to explore this problem comprehensively. We will delve into the geometric and algebraic foundations of the cosine function, examine how angles behave under multiplication, and demonstrate the existence of an acute angle related to 3θ when cos θ is positive. Throughout, the discussion will be structured to ensure clarity, depth, and SEO optimization to aid learners and enthusiasts in mastering this aspect of trigonometry.

Understanding the Range of θ and the Behavior of Cos θ

The Range $0^\circ < \theta < 360^\circ$

In the unit circle, angles are measured from 0° to 360°, covering all four quadrants:

- Quadrant I: $0^\circ < \theta < 90^\circ$
- Quadrant II: $90^\circ < \theta < 180^\circ$
- Quadrant III: $180^\circ < \theta < 270^\circ$
- Quadrant IV: $270^\circ < \theta < 360^\circ$

Understanding this helps determine the sign of cosine:

- Cosine is positive in Quadrant I and Quadrant IV.
- Cosine is negative in Quadrant II and Quadrant III.

Why is the positivity of cos θ important?

Since cosine relates to the x-coordinate of a point on the unit circle, $\cos \theta > 0$ indicates that the point lies to the right of the y-axis. This geometric insight is crucial for understanding the behavior of 3θ and its relationship with θ.

The Significance of Cos θ Being Positive

Geometric Interpretation

When $\cos \theta > 0$, the point corresponding to angle θ on the unit circle is located in Quadrant I or Quadrant IV. This implies:

- The x-coordinate of the point is positive.
- The angle θ is less than 90° (Quadrant I) or between 270° and 360° (Quadrant IV).

Algebraic Implications

Since $\cos \theta > 0$, the value of $\cos 3\theta$ can be expressed using the triple-angle formula, which is fundamental in this analysis:

$$
\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta
$$

This formula allows us to determine the sign of $\cos 3\theta$ based on the value of $\cos \theta$.

Exploring the Triple-Angle Formula and Its Consequences

The Triple-Angle Formula

The triple-angle identity for cosine states:

$$
\boxed{
\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta
}
$$

This identity is central to analyzing how the cosine of a multiple of an angle relates to the cosine of the original angle.

Sign of cos 3θ When cos θ Is Positive

Given $\cos \theta > 0$, analyze the sign of $\cos 3\theta$:

\[
\cos 3θ = 4 \cos^3 θ - 3 \cos θ
\]

Factor out cos θ:

\[
\cos 3θ = \cos θ (4 \cos^2 θ - 3)
\]

Since cos θ > 0, the sign of cos 3θ depends on the term \(4 \cos^2 θ - 3\):

- If \(4 \cos^2 θ - 3 > 0\), then cos 3θ > 0.
- If \(4 \cos^2 θ - 3 < 0\), then cos 3θ < 0.

Solve for when \(4 \cos^2 θ - 3 = 0\):

\[
4 \cos^2 θ = 3 \implies \cos^2 θ = \frac{3}{4} \implies |\cos θ| =
\frac{\sqrt{3}}{2}
\]

Since cos θ > 0, the critical points are:

\[
\cos θ = \frac{\sqrt{3}}{2}
\]

Corresponding angles:

\[
θ = 30^\circ \quad (\text{since } \cos 30^\circ = \sqrt{3}/2)
\]

and

\[
θ = 330^\circ \quad (\text{in Quadrant IV}).
\]

Range of θ for cos 3θ to be positive

- When \(|\cos θ| > \frac{\sqrt{3}}{2} \), that is, θ in (0°, 30°) or (330°, 360°), then:

\[
4 \cos^2 θ - 3 > 0 \implies \cos 3θ > 0
\]

- When \(|\cos θ| < \frac{\sqrt{3}}{2} \), that is, θ in (30°, 150°) or (210°, 330°), then:

$$
4 \cos^2 \theta - 3 < 0 \implies \cos 3\theta < 0
$$

The Existence of an Acute Angle with 3θ or 3θ

The Core Objective

The primary goal is to prove that if $\cos \theta > 0$, then there exists an acute angle (i.e., an angle less than 90°) that is either equal to 3θ or supplementary to 3θ.

Approach to the Proof

- Determine the range of θ where $\cos \theta > 0$.
- Analyze the values of 3θ in relation to the unit circle.
- Show that within these ranges, 3θ or its supplementary angle $(180° - 3\theta)$ is acute.

Step 1: Analyzing θ in $(0°, 90°)$

In Quadrant I, where $0° < \theta < 90°$, $\cos \theta > 0$.

Calculate 3θ:

- 3θ ranges from 0° to 270° (since $3\ 90° = 270°$).

Within this range:

- For θ in $(0°, 30°)$: 3θ in $(0°, 90°)$. Here, 3θ is acute.
- For θ in $(30°, 90°)$: 3θ in $(90°, 270°)$. Here, 3θ may be obtuse or reflex, but the key is that when 3θ is less than 90°, it is already acute.

Conclusion: When θ is in $(0°, 30°)$, 3θ is acute.

Step 2: Analyzing θ in $(270°, 360°)$

In Quadrant IV, where $270° < \theta < 360°$, $\cos \theta > 0$.

Calculate 3θ:

- 3θ ranges from 810° to 1080°, which modulo 360° gives:

$$
3\theta \equiv 810° - 2360° = 810° - 720° = 90° \quad \text{to} \quad 1080° - 3360° = 1080° - 1080° = 0°
$$

So, 3θ modulo 360°:

- Ranges from 0° to 90°, depending on θ.

Implication:

- For θ in $(270^\circ, 330^\circ)$:

$$
3\theta \in (810^\circ, 990^\circ) \implies 3\theta \equiv 810^\circ - 2360^\circ = 90^\circ \text{ to } 990^\circ - 2360^\circ = 990^\circ - 720^\circ = 270^\circ
$$

- For θ in $(330^\circ, 360^\circ)$:

$$
3\theta \in (990^\circ, 1080^\circ) \implies 3\theta \equiv 990^\circ - 2360^\circ = 270^\circ \text{ to } 1080^\circ - 3360^\circ = 0^\circ
$$

Thus, in Quadrant IV, 3θ can be congruent to angles in the first and third quadrants, but importantly, the value of 3θ modulo 360° can be less than 90°, making 3θ or its supplementary angle acute.

Step 3: Establishing the Existence of an Acute Angle

Based on the above analysis:

- When $\cos \theta > 0$, θ lies in $(0^\circ, 90^\circ)$ or $(270^\circ, 360^\circ)$.
- The corresponding 3θ (or its equivalent modulo 360°) falls into ranges that include acute angles (less than 90°).
- Therefore, either 3θ itself is acute, or the supplementary angle $(180^\circ - 3\theta)$ is acute.

Explicitly:

- If $3\theta < 90^\circ$, then 3θ is an acute angle.
- If $3\theta > 90^\circ$, then $180^\circ - 3\theta < 90^\circ$, making the supplementary angle acute.

Formal Proof of the Statement

Frequently Asked Questions

Given that 0 < 3 < 3600 and cos 3 is positive, how can we determine the possible values of angle 3?

Since cos 3 is positive, angle 3 must lie within the first or fourth quadrants, specifically between 0° and 90° or between 270° and 360°, indicating the presence of an acute angle.

What is the significance of the condition 0 < 3 < 3600 in analyzing angle 3?

The condition specifies that angle 3 is within a certain range of degrees (possibly in a multiple of degrees or radians), ensuring that the trigonometric functions are evaluated within standard periodic intervals, which helps in determining the positivity of cos 3.

How can we show that there exists an acute angle related to 3 based on the given conditions?

Since cos 3 is positive, we can find an angle between 0° and 90° (an acute angle) that has the same cosine value, demonstrating the existence of an acute angle associated with 3.

Is it possible for angle 3 to be in the second or third quadrants under the given conditions? Why or why not?

No, because in the second and third quadrants, cosine values are negative. Since the problem states cos 3 is positive, angle 3 cannot be in these quadrants.

How does the periodicity of cosine function help in establishing the existence of an acute angle related to 3?

The cosine function is periodic with period 360°, and since cos 3 is positive, there exists an angle in the first quadrant (0° to 90°) with the same cosine value, confirming the presence of an acute angle related to 3.

Additional Resources

Understanding the Trigonometric Relationship Between Angles: A Deep Dive into Cos 3 and Its Implications

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Introduction

In the world of mathematics, especially in trigonometry, understanding the relationships between angles and their trigonometric functions is fundamental. One intriguing problem involves exploring the nature of an angle θ when $0 < \theta < 360^\circ$, particularly focusing on the behavior of its cosine value. Specifically, the question arises: If $\cos 3\theta$ is positive, how can we demonstrate that there exists an acute angle related to θ that satisfies certain conditions?

This exploration is not only a testament to the elegance of trigonometric identities but also a practical example of how mathematical reasoning can reveal intrinsic properties of angles within a specified range. This article aims to unpack this problem thoroughly, providing clear explanations, detailed proofs, and insightful interpretations along the way.

Setting the Stage: The Problem Statement

Let's restate the problem in a precise manner:

> Assume that $0 < \theta < 360^\circ$.
> (i) If $\cos 3\theta$ is positive, show that there exists an acute angle α such that either $\alpha = 3\theta$ or $\alpha = \theta$, satisfying specific trigonometric conditions.

At its core, the problem revolves around understanding the implications of $\cos 3\theta > 0$ and the existence of an acute angle (i.e., an angle less than 90°) that relates to θ in a meaningful way. To approach this comprehensively, we'll break down the problem into smaller, manageable parts, starting with the properties of the cosine function and the behavior of multiple angles.

Fundamental Concepts in Trigonometry

Before delving into the proof, it's essential to revisit some foundational concepts:

1. Range of θ
- $0 < \theta < 360^\circ$ implies that θ is within the standard circle, covering all four quadrants.

2. Periodic Nature of Cosine
- $\cos$ is periodic with period 360°; this means $\cos(\theta + 360^\circ) = \cos \theta$.

3. Cosine of Multiple Angles
- Key identities such as:
$$
\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta
$$
are instrumental in relating $\cos 3\theta$ to $\cos \theta$.

4. Sign of $\cos 3\theta$

- Since $\cos 3\theta$ can be positive or negative depending on θ, analyzing its sign helps identify the corresponding intervals for θ.

Analyzing the Sign of $\cos 3\theta$

To understand the condition $\cos 3\theta > 0$, let's analyze the behavior of $\cos 3\theta$:

1. Range of 3θ
- Since $0 < \theta < 360^\circ$, then $0 < 3\theta < 1080^\circ$.

2. Intervals for $\cos 3\theta > 0$
- Recall that $\cos x > 0$ in the intervals:

$$
-90^\circ < x < 90^\circ \quad (\text{or equivalently } 270^\circ < x < 450^\circ)
$$

when considering the principal cycle, and similarly for other cycles shifted by 360°.

- Since 3θ is within 0° to 1080°, the positive regions for $\cos 3\theta$ are within intervals:

$$
0^\circ < 3\theta < 90^\circ, \quad 270^\circ < 3\theta < 360^\circ, \quad 540^\circ < 3\theta < 630^\circ, \quad 810^\circ < 3\theta < 900^\circ
$$

- Dividing these by 3 gives the corresponding intervals for θ:

$$
0^\circ < \theta < 30^\circ, \quad 90^\circ < \theta < 120^\circ, \quad 180^\circ < \theta < 210^\circ, \quad 270^\circ < \theta < 300^\circ
$$

Key Observation:
In these intervals, $\cos 3\theta$ is positive.

Establishing the Connection: The Existence of an Acute Angle α

The core of the problem asks us to demonstrate the existence of an acute angle α such that either:

- $\alpha = 3\theta$, which implies $0^\circ < \alpha < 90^\circ$, or
- $\alpha = \theta$, with some relationship to the positivity of $\cos 3\theta$.

Given the analysis above, the primary focus is on the interval where \(

3θ is within 0° and 90°.

Case 1: 3θ is in $(0^\circ, 90^\circ)$
- Here, $\cos 3\theta > 0$ by the property of cosine.
- Since 3θ is an acute angle itself, we can directly set:
$$
\alpha = 3\theta
$$
which is acute by assumption, satisfying the condition.

Conclusion for Case 1:
If 3θ is within $(0^\circ, 90^\circ)$, then $\alpha = 3\theta$ is an acute angle, fulfilling the requirement.

Case 2: 3θ is in other positive intervals

Consider the other intervals where $\cos 3\theta > 0$, such as:

- $270^\circ < 3\theta < 360^\circ$
- $540^\circ < 3\theta < 630^\circ$
- $810^\circ < 3\theta < 900^\circ$

In these cases, 3θ is not acute, but perhaps related to an acute angle through certain identities or transformations.

Connecting θ and 3θ

To deepen our understanding, let's analyze the relationship between θ and 3θ, especially considering the multiple angle identities.

Identity: $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$

This cubic expression relates $\cos 3\theta$ to $\cos \theta$. The sign of $\cos 3\theta$ depends on the value of $\cos \theta$:

$$
\cos 3\theta > 0 \Leftrightarrow 4 \cos^3 \theta - 3 \cos \theta > 0
$$

Dividing through by $\cos \theta$, considering $\cos \theta \neq 0$:

$$
4 \cos^2 \theta - 3 > 0
$$

which simplifies to:

\[
\cos^2 \theta > \frac{3}{4}
\]

or

\[
|\cos \theta| > \frac{\sqrt{3}}{2}
\]

Since \(|\cos \theta| > \frac{\sqrt{3}}{2} \) implies \(\cos \theta > \frac{\sqrt{3}}{2} \) or \(\cos \theta < -\frac{\sqrt{3}}{2} \).

- When \(\cos \theta > \frac{\sqrt{3}}{2} \), \(\theta \) is in the first quadrant, specifically \(0^\circ < \theta < 30^\circ \), where \(\theta \) is acute.
- When \(\cos \theta < -\frac{\sqrt{3}}{2} \), \(\theta \) is in the second quadrant, specifically \(150^\circ < \theta < 180^\circ \).

Implication: The Existence of an Acute \(\alpha \)

From the above, the key takeaway is:

- If \(\cos 3\theta > 0 \), then either \(\cos \theta > \frac{\sqrt{3}}{2} \) (implying \(\theta \) is acute), or \(\cos \theta < -\frac{\sqrt{3}}{2} \).

In the case where \(\cos \theta > \frac{\sqrt{3}}{2} \), then:

\[
0^\circ < \theta < 30^\circ
\]

which is acute

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