Assuming Friction Forces Are Negligible, Write An Expression, Using Only The Variables Provided, For

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In the realm of physics, understanding the motion of objects often involves analyzing forces acting upon them. When friction forces are deemed negligible, the problem simplifies significantly, allowing us to focus solely on other fundamental forces such as gravity, tension, normal force, and applied forces. This assumption is commonly made in introductory physics problems to facilitate the derivation of basic equations of motion without the complications introduced by frictional resistance.

In this article, we will explore how to derive a comprehensive expression for the acceleration or other key quantities of a system, using only the variables provided, under the assumption that friction forces are negligible. We aim to provide a detailed, step-by-step approach, organized with clear headings and subheadings, to help students and enthusiasts understand the underlying principles and apply them effectively.

Understanding the Basic Variables and Assumptions

Before deriving the expression, it is crucial to identify the common variables involved and clarify the assumptions made:

Common Variables in Mechanics Problems

- m: Mass of the object or system
- g: Acceleration due to gravity (usually 9.8 m/s^2)
- T: Tension force in a string or cable
- F: Applied force (external force acting on the object)
- a: Acceleration of the object
- x: Displacement or position variable
- θ: Angle of incline or applied force relative to the horizontal or vertical
- N: Normal force exerted by a surface

Assumption: Frictional forces are negligible, meaning that the force of kinetic or static friction is zero or so small that it can be ignored in calculations.

Fundamental Principles: Newton’s Second Law

At the heart of deriving expressions in mechanics lies Newton's Second Law:

$$ \sum F = m a $$

This law states that the net force acting on an object equals its mass times its acceleration. When friction is negligible, the net force includes only the other forces present in the system.

Deriving the Expression in Different Contexts

Depending on the specific scenario—such as an object sliding down an incline, a mass hanging from a pulley, or an object pulled across a surface—the variables and forces involved change. Below, we examine common situations and derive general expressions accordingly.

Case 1: Object Moving Along an Incline Without Friction

Scenario: A mass m is placed on an inclined plane at an angle θ. A force F is applied parallel to the incline, and gravity acts vertically downward.

Forces acting along the incline:
- Gravitational component: $m g \sin \theta$
- Applied force: F (if any)
- Tension T, if connected to a pulley system or cable (optional)

Equation of motion along the incline:

$$
\sum F_{\text{along incline}} = m a
$$

which expands to:

$$
F + m g \sin \theta = m a
$$

Expression for acceleration a:

$$
a = \frac{F + m g \sin \theta}{m}
$$

Simplified form:

$$
a = \frac{F}{m} + g \sin \theta
$$

This expression indicates that the acceleration depends only on the applied force F, the gravitational component, and the mass m, under the assumption of negligible friction.

Case 2: Masses Connected by a Rope Over a Pulley

Scenario: Two masses, m_1 and m_2, are connected via a massless, inextensible string passing over a frictionless pulley. Gravity acts on both, and an external force F may be applied to one of the masses.

Variables:
- m_1, m_2: masses
- g: acceleration due to gravity
- T: tension in the string
- a: acceleration of the system

Equations of motion:
For m_1:
$$
T = m_1 a
$$

For m_2:
$$
m_2 g - T = m_2 a
$$

Adding the two equations:
$$
m_2 g = (m_1 + m_2) a
$$

Expression for acceleration:
$$
a = \frac{m_2 g}{m_1 + m_2}
$$

If an external force F acts on m_1 or m_2, the equations modify accordingly. For example, if F acts on m_1:

$$
T = m_1 a + F
$$

and similarly for m_2. Solving these equations yields an expression for a involving only the variables F, m_1, m_2, g.

Case 3: Object Moving Horizontally Under Applied Force

Scenario: A block of mass m is pushed across a frictionless horizontal surface with an applied force F.

Forces:
- Applied force: F
- Normal force: N (equal to $m g$), but cancels out for horizontal motion)

Equation of motion:

$$
F = m a
$$

Expression for acceleration:

$$
a = \frac{F}{m}
$$

This simple relation illustrates the direct proportionality between the applied force and acceleration when friction is negligible.

Generalized Expression for Systems with Multiple Forces

In more complex systems involving multiple forces, the key is to sum all forces acting along the direction of motion and divide by the mass to find the acceleration:

$$
a = \frac{\sum \text{forces along the direction of motion}}{m}
$$

Important notes:
- When multiple forces are involved, only include the variables provided, such as F, T, m, g, and angles θ.
- For inclined planes, resolve forces into components using $\sin \theta$ and $\cos \theta$.
- For systems with pulleys, consider tension and gravity carefully, but remember that friction is neglected, simplifying tension calculations.

Summary of Key Expressions

Scenario	Expression for Acceleration a	Notes
Inclined plane with applied force F	$a = \frac{F + m g \sin \theta}{m}$	No friction
Masses connected via pulley	$a = \frac{m_2 g}{m_1 + m_2}$ (if only gravity)	Adjust for external forces as needed
Horizontal push with force F	$a = \frac{F}{m}$	No friction

Conclusion

Assuming friction forces are negligible simplifies the derivation of expressions for motion significantly. By carefully considering the forces involved and applying Newton's second law, you can derive a variety of expressions for acceleration or other quantities using only the provided variables. These expressions are foundational to understanding more complex systems and serve as stepping stones toward mastering dynamics in physics.

Remember, always identify the forces acting along the direction of motion, resolve components where necessary, and keep track of variables to develop accurate and meaningful formulas. This approach not only enhances problem-solving skills but also deepens your understanding of fundamental physics principles.

Frequently Asked Questions

Assuming friction forces are negligible, write an expression for the acceleration of an object of mass m pulled by a force F.

$a = F / m$

Assuming friction forces are negligible, what is the expression for the tension T in a massless string connecting two masses m_1 and m_2 when a force F is applied to m_1?

$T = m_2 a$, where $a = F / (m_1 + m_2)$

Assuming friction forces are negligible, derive the expression for the work done W by a constant force F over a displacement d.

$W = F d$

Assuming friction forces are negligible, write an expression for the gravitational potential energy U of an object of mass m at height h.

$U = m g h$

Assuming friction forces are negligible, what is the expression for the kinetic energy KE of an object moving with velocity v?

$KE = (1/2) m v^2$

Assuming friction forces are negligible, write an expression for the velocity v of an object after falling from height h under gravity g, starting from rest.

$v = \sqrt{2 g h}$

Assuming friction forces are negligible, express the total mechanical energy E of an object of mass m at height h with velocity v.

$E = m g h + (1/2) m v^2$

Assuming friction forces are negligible, write an expression for the period T of a simple pendulum with length L under gravity g.

$T = 2\pi \sqrt{L / g}$

Additional Resources

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An Investigation into Deriving Kinetic Expressions under the Assumption of Negligible Friction

In the realm of classical mechanics, the simplification of systems by neglecting certain forces often leads to more tractable mathematical models. One such common approximation is assuming friction forces are negligible, which allows for a clearer focus on the primary dynamics dictated by applied forces and inertial effects. This article seeks to methodically derive an expression—using only the variables provided—under this key assumption, and to analyze its implications within physical systems.

Introduction and Contextual Framework

Friction, a resistive force arising from contact interactions, often complicates the analysis of mechanical systems. Ignoring friction simplifies equations of motion, enabling the derivation of closed-form solutions that can serve as foundational models or idealized benchmarks.

In many classical problems, variables such as mass (m), displacement (x), velocity (v), acceleration (a), and applied force (F) are the primary quantities of interest. When friction forces are assumed negligible, the equations governing the system reduce to their ideal forms, often revealing fundamental relationships.

The goal of this investigation is to derive an explicit expression that describes the system's behavior—such as acceleration or velocity—using only the provided variables: m, x, v, a, F—while explicitly stating the assumptions and limitations involved.

Fundamental Principles and Assumptions

Assumptions

1. Friction forces are negligible: No resistive force opposes the motion.
2. System is subjected to known, external forces: The only significant external influence is F.
3. Mass remains constant: No mass loss or gain occurs during motion.
4. Motion occurs in a single dimension: Simplifies vector considerations to scalar quantities.
5. No other dissipative or external forces: Air resistance or other minor effects are ignored.

Variables Involved

Variable	Description
m	Mass of the object
x	Displacement from a reference point
v	Velocity of the object
a	Acceleration of the object
F	External applied force

Derivation of the Expression

Step 1: Starting from Newton’s Second Law

The core principle applicable when friction is negligible is Newton's second law:

$$
F = m a
$$

Given that F is the only significant force, this relation directly links the force to the acceleration.

Step 2: Expressing Acceleration

Rearranged, the acceleration a is:

$$
a = \frac{F}{m}
$$

This is the fundamental expression under the assumption of negligible friction, directly relating the applied force, the mass, and the acceleration.

Step 3: Connecting Velocity and Displacement

Assuming initial conditions are known, and using basic kinematic relations, we relate velocity v, displacement x, and acceleration a:

$$
v^2 = v_0^2 + 2 a (x - x_0)
$$

where v_0 and x_0 are initial velocity and initial position, respectively.

In many cases, initial conditions are specified or can be set to zero for simplicity:

$$
v^2 = 2 a x
$$

assuming $v_0 = 0$ and initial position $x_0 = 0$.

Step 4: Expressing Velocity in Terms of Variables

Substituting a = F/m, we get:

$$
v^2 = 2 \frac{F}{m} x
$$

which simplifies to:

$$
v = \sqrt{2 \frac{F}{m} x}
$$

This expression relates velocity, displacement, force, and mass directly, assuming initial conditions are zero.

Step 5: Deriving an Expression for Displacement or Other Quantities

If initial velocity and position are known, and the goal is to express displacement in terms of v, F, and m, rearranged as:

$$
x = \frac{m v^2}{2 F}
$$

Similarly, if the velocity v is known at a certain point, one can find the displacement x using the above.

Final Expression and Its Domain of Validity

Overall Formula

Assuming friction forces are negligible, the primary expression relating the variables is:

$$
\boxed{
a = \frac{F}{m}
}
$$

which leads to kinematic relations such as:

$$
v = \sqrt{2 \frac{F}{m} x}
$$

and

$$
x = \frac{m v^2}{2 F}
$$

Interpretation

- These equations hold true only in the absence of friction.
- The initial conditions (initial velocity and position) are critical for precise calculations.
- For systems where initial velocity is non-zero, the general form is:

$$
v^2 = v_0^2 + 2 a (x - x_0)
$$

which, upon substitution, becomes:

$$
v^2 = v_0^2 + 2 \frac{F}{m} (x - x_0)
$$

Implications and Applications

Practical Relevance

- Idealized physics problems often assume negligible friction to focus on core dynamics.
- Engineering applications such as space vehicle motion, where gravitational and resistive forces are minimal, rely on these simplified relations.
- The derived expressions serve as benchmarks or initial approximations before including resistive effects.

Limitations and Further Considerations

- Real systems rarely have zero friction; thus, these models are approximations.
- Inclusion of friction would require additional terms, such as a friction coefficient μ and normal force, complicating the equations.
- Dynamic systems with variable forces or mass would necessitate more complex models beyond the scope of this derivation.

Conclusion

Neglecting friction forces simplifies the equations of motion to their fundamental forms, enabling straightforward expressions that relate force, mass, velocity, and displacement. The key result—$a = F/m$—serves as the cornerstone for analyzing such idealized systems. From this, classical kinematic equations allow us to relate these variables explicitly, providing a clear framework for understanding the dynamics under the assumption of negligible friction.

This investigation highlights the importance of assumptions in physical modeling and illustrates how fundamental principles can be combined with basic mathematics to derive meaningful, usable formulas. While real-world applications often require more complex models, these simplified expressions form the essential foundation upon which more comprehensive analyses are built.

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