Assuming Ideal Op Amps, Find The Voltage Gain Vo/vi And Input Resistance Rin Of Each Of The Circuits

Assuming Ideal Op Amps, Find The Voltage Gain Vo/vi And Input Resistance Rin Of Each Of The Circuits

Operational amplifiers (op amps) are fundamental building blocks in analog electronic circuits, renowned for their versatility and high gain characteristics. When analyzing op amp circuits, the assumption of an ideal op amp simplifies calculations significantly. An ideal op amp is characterized by infinite open-loop gain, infinite input impedance, zero output impedance, and zero input bias current. These idealizations allow engineers and students to focus on the fundamental behavior of the circuits without the complexities introduced by real-world non-idealities. In this article, we will explore how to determine the voltage gain (Vo/vi) and input resistance (Rin) of various op amp configurations under the assumption of ideal op amps. We will cover common configurations, including the inverting amplifier, non-inverting amplifier, voltage follower, and difference amplifier, providing detailed derivations and explanations for each case.

Fundamental Assumptions for Ideal Op Amps

Characteristics of an Ideal Op Amp

- Infinite open-loop voltage gain ($A_{OL} \to \infty$)
- Infinite input impedance ($R_{in} \to \infty$)
- Zero output impedance ($R_{out} \to 0$)
- Zero input bias current ($I_{in+} = I_{in-} = 0$)
- Zero offset voltage

These assumptions lead to the following key implications when analyzing circuits:

- The voltage difference between the inverting and non-inverting terminals is zero when negative feedback is used (virtual short concept).

- The input terminals draw no current, simplifying resistor and load calculations.

Analysis of Common Op Amp Circuits

1. Inverting Amplifier

Configuration Overview

The inverting amplifier configuration involves an input resistor R_{in} connected between the input voltage source (vi) and the inverting terminal of the op amp. The non-inverting terminal is grounded, and a feedback resistor R_f connects the output (Vo) to the inverting terminal.

Deriving Voltage Gain Vo/vi

1. Since the op amp is ideal and negative feedback is used, the voltage at the inverting terminal (V_-) is approximately equal to the non-inverting terminal voltage, which is ground (0 V). This is known as the virtual ground concept.

2. The input current through R_{in} is:

   $i_{in} = (vi - V_-) / R_{in} = vi / R_{in}$ (because $V_- \approx 0$ V)

3. Similarly, the current through R_f is:

   $i_f = (V_- - Vo) / R_f = -Vo / R_f$

4. Since the input bias current is zero and the input terminal draws no current, the current entering R_{in} must equal the current leaving R_f:

   $i_{in} = i_f$

5. Therefore,

   $vi / R_{in} = -Vo / R_f$

6. Rearranging the equation gives the voltage gain:

   $\mathbf{Vo / vi = - R_f / R_{in}}$

This negative sign indicates a 180° phase shift between input and output, characteristic of the inverting amplifier.

Input Resistance Rin

From the perspective of the input source, the input resistance is simply the resistor R_{in} since the ideal op amp draws no current into its terminals.

- **R_{in} (input resistance) = R_{in}**

2. Non-Inverting Amplifier

Configuration Overview

The non-inverting amplifier features the input voltage (vi) applied directly to the non-inverting terminal. The feedback resistor R_f connects the output Vo to the inverting terminal, which is also connected to ground through resistor R_g (or R_f if configured accordingly).

Deriving Voltage Gain Vo/vi

1. Since the op amp is ideal, the voltage at the inverting terminal (V_-) is equal to the non-inverting terminal (vi), due to the virtual short:

   $V_- \approx vi$

2. The inverting terminal is connected to the output through R_f and to ground through R_g, forming a voltage divider:

   $V_- = Vo\ (R_g / (R_f + R_g))$

3. Rearranged to find Vo:

   $Vo = V_-\ (1 + R_f / R_g)$

4. Substituting $V_- \approx vi$:

   $Vo = vi\ (1 + R_f / R_g)$

5. Thus, the voltage gain is:

   $Vo / vi = 1 + R_f / R_g$

Input Resistance Rin

The input resistance seen by the source is essentially the input impedance at the non-inverting terminal, which, for an ideal op amp, is infinite. Practically, the input resistance is dominated by the source impedance and any resistor connected to the input terminal:

- $\mathbf{R_{in} = R_{source}}$

3. Voltage Follower (Buffer)

Configuration Overview

The voltage follower involves connecting the output directly to the inverting terminal, with the input voltage applied to the non-inverting terminal. No resistors are used in the feedback path.

Deriving Voltage Gain Vo/vi

1. Since the op amp is ideal, the voltage at the inverting terminal (V_-) is approximately equal to the non-inverting terminal voltage (vi):
   $V_- \approx vi$

2. With direct feedback, the output adjusts itself to maintain $V_- = vi$, resulting in:

   $Vo = V_- = vi$

3. The voltage gain:

   $\mathbf{Vo / vi = 1}$

Input Resistance Rin

The input resistance seen by the source is effectively infinite, as the ideal op amp input draws no current, and the circuit functions as a buffer.

- $\mathbf{R_{in} = \infty}$

4. Difference Amplifier

Configuration Overview

The difference amplifier compares two voltages, v_1 and v_2. It uses a combination of resistors R_1, R_2, R_3, and R_4 to produce an output proportional to their difference.

Deriving Voltage Gain $Vo/(v_1 - v_2)$

1. Assuming resistor ratios are matched appropriately ($R_1/R_2 = R_3/R_4$), the output voltage is:

   $Vo = (R_2/R_1)\ (v_1$

Frequently Asked Questions

What is the voltage gain (Vo/vi) in an ideal op-amp in a non-inverting configuration?

In a non-inverting configuration with an ideal op-amp, the voltage gain is given by 1 + (Rf/Rin), where Rf is the feedback resistor and Rin is the input resistor. Since the op-amp is ideal, the gain can be very high depending on resistor values.

How do you determine the input resistance (Rin) of an op-amp circuit assuming the op-amp is ideal?

For an ideal op-amp, the input resistance Rin is considered infinite because the input terminals draw no current. Therefore, Rin effectively appears as an open circuit at the input terminal.

What is the voltage gain (Vo/vi) for an inverting amplifier with an ideal op-amp?

The voltage gain for an inverting amplifier with an ideal op-amp is given by -Rf/Rin, where Rf is the feedback resistor and Rin is the input resistor. The negative sign indicates phase inversion.

In a differential amplifier using an ideal op-amp, how is the voltage gain calculated?

For a differential amplifier with an ideal op-amp, the voltage gain depends on the resistor ratios and is typically given as Vo/vi = (Rf/Rin) for the difference between the input voltages, assuming matched resistor pairs.

Why can the input resistance Rin be considered infinite in an ideal op-amp circuit?

Because an ideal op-amp draws no current at its input terminals, the input resistance Rin is considered infinite, preventing any loading effect on the source.

How does the assumption of an ideal op-amp simplify the analysis of amplifier circuits?

Assuming an ideal op-amp simplifies circuit analysis by considering infinite input resistance, zero output resistance, infinite open-loop gain, and zero input bias currents, which allows straightforward calculation of gain and input/output relationships.

What is the effect of feedback resistor values on the voltage gain in an ideal op-amp circuit?

In an ideal op-amp circuit, the voltage gain is directly influenced by the ratio of the feedback resistor to the input resistor. Adjusting these resistor values allows precise control of the gain.

Can the input resistance Rin be calculated for a specific op-amp circuit assuming ideal conditions? How?

Yes, for an ideal op-amp, Rin is considered infinite and does not depend on circuit configuration. However, in practical analysis, Rin can be calculated based on the input network, but under ideal assumptions, it remains infinite.

What is a key limitation of assuming ideal op-amps when analyzing real-world circuits?

Assuming ideal op-amps ignores real characteristics like finite gain, input bias currents, input/output resistance, and bandwidth limitations, which can lead to discrepancies between analysis and actual circuit behavior.

Additional Resources

Assuming Ideal Op Amps, Find The Voltage Gain Vo/vi And Input Resistance Rin Of Each Of The Circuits

In the realm of electronic circuit analysis, operational amplifiers (op amps) are fundamental components that enable a wide array of signal processing functions. When analyzing op amp circuits, simplifying assumptions—particularly the notion of an ideal op amp—are frequently employed to facilitate understanding and calculation. These assumptions typically include infinite open-loop gain, infinite input impedance, zero output impedance, infinite bandwidth, and zero input bias current. By assuming ideal conditions, engineers and students can derive analytical expressions for key parameters such as voltage gain (Vo/vi) and input resistance (Rin), which are critical for designing and analyzing electronic systems.

This article aims to provide a comprehensive review of how to determine the voltage gain and input resistance of various op amp circuits under the assumption of ideal op amps. We will systematically explore common circuit configurations, elucidate the underlying principles, and present detailed derivations, complemented by practical insights and implications.

Fundamentals of Ideal Op Amp Assumptions

Before delving into specific circuit configurations, it is essential to understand the foundational assumptions that define an ideal op amp:

1. Infinite Open-Loop Gain ($A \to \infty$): The voltage difference between the inverting and non-inverting inputs results in an infinitely large output voltage, unless limited by the power supply.

2. Infinite Input Impedance ($R_{in} \to \infty$): No current flows into the input terminals; the input node draws negligible or zero current.

3. Zero Output Impedance ($R_{out} \to 0$): The output can source or sink any amount of current without internal voltage drop.

4. Zero Input Bias Current: No current flows into the input terminals, preventing loading effects.

5. Zero Input Offset Voltage: There is no voltage difference required at the inputs for the output to be zero.

Using these assumptions simplifies the analysis of op amp circuits, allowing us to focus on the fundamental relationships dictated by circuit topology rather than component non-idealities.

Analysis of Typical Op Amp Configurations

In practice, op amps are configured in various ways depending on the desired function. Here, we analyze common configurations such as the voltage follower, inverting amplifier, and non-inverting amplifier, emphasizing the derivation of voltage gain and input resistance under ideal assumptions.

---

1. Voltage Follower (Buffer Amplifier)

Circuit Description:

The voltage follower is a unity-gain buffer where the output is directly connected to the inverting input, and the input signal is applied to the non-inverting input.

Analysis:

- Since the op amp is ideal, the input bias current is zero, and the input terminals draw no current.
- The op amp output adjusts such that the voltage difference between the inputs is virtually zero; that is,

$$
v_{+} \approx v_{-}
$$

- With the non-inverting input connected to the input voltage v_i, the inverting input will also be at approximately v_i.

- The output voltage v_o is fed back directly to the inverting input, maintaining the condition $v_{-} \approx v_{+}$.

Voltage Gain V_o / V_i:

$$
V_o = v_{-} \quad \text{(since the inverting terminal is connected directly to the output)}
$$

Given the ideal assumptions, the voltage gain of a voltage follower is:

\[

\boxed{
\frac{V_o}{V_i} = 1
}
\]

Input Resistance \(R_{in} \):

Since the input is connected to the non-inverting terminal with infinite input impedance, and no current flows into the input terminal, the input resistance is:

\[
\boxed{
R_{in} \rightarrow \infty
}
\]

Implication:

The voltage follower provides high input impedance, low output impedance, and unity gain, making it ideal for buffering signals without loading the source.

---

2. Inverting Amplifier

Circuit Description:

An inverting amplifier configuration involves applying the input signal through a resistor \(R_{in} \) to the inverting input, with the non-inverting input grounded. The output is fed back to the inverting input through a resistor \(R_f \).

Analysis:

- Due to the ideal op amp assumptions, the inverting input node is at virtual ground (\(v_{-} \approx 0 \)), since the non-inverting input is grounded.

- No current flows into the inverting input terminal (infinite input impedance), so the current through \(R_{in} \) is:

\[
i_{in} = \frac{v_i - v_{-}}{R_{in}} = \frac{v_i - 0}{R_{in}} = \frac{v_i}{R_{in}}
\]

- This current must flow through the feedback resistor \(R_f \), as no current enters the op amp input:

\[
i_{f} = \frac{v_{-} - v_o}{R_f} = \frac{0 - v_o}{R_f} = - \frac{v_o}{R_f}

\]

- Equating the currents:

$$
i_{in} = i_{f} \Rightarrow \frac{v_i}{R_{in}} = - \frac{v_o}{R_f}
$$

- Solving for v_o:

$$
v_o = - \frac{R_f}{R_{in}} v_i
$$

Voltage Gain V_o / V_i:

$$
\boxed{
\frac{V_o}{V_i} = - \frac{R_f}{R_{in}}
}
$$

Input Resistance R_{in}:

The input resistance seen by the source is simply R_{in}, since no current enters the op amp input:

$$
\boxed{
R_{in} = R_{in}
}
$$

Implication:

The inverting amplifier provides a voltage gain set by the resistor ratio, with high input resistance determined by R_{in}. The negative sign indicates phase inversion.

---

3. Non-Inverting Amplifier

Circuit Description:

In this configuration, the input signal is applied directly to the non-inverting terminal. The inverting terminal is connected to ground through resistor R_{1} and to the output via resistor R_{2}.

Analysis:

- The non-inverting input is at v_i.

- Due to the ideal op amp, the voltage at the inverting input v_{-} is equal to v_{+}:

$$
v_{-} \approx v_i
$$

- The inverting input node forms a voltage divider between the output and ground:

$$
v_{-} = v_i = \frac{R_{1}}{R_{1} + R_{2}} v_o
$$

- Rearranging to find v_o:

$$
v_o = v_i \left(1 + \frac{R_{2}}{R_{1}} \right)
$$

Voltage Gain V_o / V_i:

$$
\boxed{
\frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}
}
$$

Input Resistance R_{in}:

Since the input is connected directly to the non-inverting terminal with high impedance, the input resistance is:

$$
\boxed{
R_{in} \rightarrow \infty
}
$$

Implication:

The non-inverting amplifier provides gain greater than or equal to one, with the gain determined solely by resistor ratios, and maintains high input impedance.

Deriving Input Resistance R_{in} in General

While the previous configurations illustrate ideal assumptions, analyzing the input resistance in real or more complex circuits involves considering the actual circuit topology and the effect of feedback. Under ideal assumptions, the input resistance seen from the source is generally:

- Infinite in voltage follower and non-inverting configurations.

- Equal to the resistor connected to the source (e.g., R_{in}) in inverting configurations.

In more complex circuits, or when feedback is involved, the input resistance can be significantly affected by feedback, often described by the concept of effective input resistance:

$$
R_{in, \text{effective}} = R_{in} \times (1 + A \beta)
$$

where β is the feedback factor, and A is the open-loop gain. Under ideal assumptions, $A \rightarrow \infty$, leading to:

$$
R_{in, \text{effective}} \rightarrow \infty
$$

This phenomenon, known as feedback-induced impedance transformation, explains why negative feedback circuits can significantly increase input impedance.

Implications for Circuit Design and Practical Considerations

Although the assumption of ideal op amps simplifies analysis, real-world devices have finite gain, non-zero input bias currents, and other non-idealities. Nevertheless, the ideal analysis provides valuable bounds and qualitative understanding.

Design Advantages:

- High Input Impedance

Assuming Ideal Op Amps Find The Voltage Gain Vo Vi And Input Resistance Rin Of Each Of The Circuits

Assuming Ideal Op Amps Find The Voltage Gain Vo Vi And Input Resistance Rin Of Each Of The Circuits

Related Articles

- According To The Segment, How Are Businesses Classified?A) Number Of EmployeesB) Work Activities The
- Cell G: We Observe The Devenant Em This Branch Emitting Periodic Flashes Of Light. What Is This Does
- Ben Owns A Shoe Store. He Has A New Line Of Limited-edition Running Shoes That He Will Be Selling In

Back to Home