At What Point Do The Curves R(t) = T, 1 T, 3 + T And R2(s) = 3 S, S 2, S Intersect? Find Their Angle

At What Point Do The Curves R(t) = T, 1 T, 3 + T And R2(s) = 3 S, S 2, S Intersect? Find Their Angle

Understanding the intersection points and angles between curves is a fundamental aspect of vector calculus and differential geometry. In this article, we analyze two parametric curves: $R(t) = (t, 1, 3 + t)$ and $R2(s) = (3s, s^2, s)$. Our goal is to determine the point(s) where these curves intersect and calculate the angle at the intersection point if they do intersect.

Introduction to Parametric Curves

Parametric curves are a way to represent a three-dimensional (or two-dimensional) path using parameters. Each point on the curve is described by a set of equations involving parameters such as t or s.

- $R(t) = (x(t), y(t), z(t))$: A parametric curve in space, with t as the parameter.
- $R2(s) = (x(s), y(s), z(s))$: Another parametric curve, with s as the parameter.

The intersection of two curves occurs when there exist values of t and s such that:

$$
R(t) = R2(s)
$$

Once the intersection point(s) are identified, the next step is to calculate the angle between the curves at the intersection point. This involves computing the tangent vectors at that point and evaluating the angle between these vectors.

Analyzing the Curves

Let's examine the given curves in detail.

Curve 1: $R(t) = (t, 1, 3 + t)$

- $x(t) = t$
- $y(t) = 1$
- $z(t) = 3 + t$

This is a straight line in three-dimensional space, parametrized by t.

Curve 2: $R2(s) = (3s, s^2, s)$

- $x(s) = 3s$
- $y(s) = s^2$
- $z(s) = s$

This curve is a space curve involving quadratic and linear components.

Finding the Intersection Point(s)

To find the intersection point(s), set R(t) equal to R2(s):

$$
(t, 1, 3 + t) = (3s, s^2, s)
$$

This gives a system of three equations:

1. $t = 3s$
2. $1 = s^2$
3. $3 + t = s$

Let's analyze these equations step by step.

Step 1: Solve for s using the second equation

From the second equation:

$$
1 = s^2 \implies s = \pm 1

\]

So, the potential intersection points correspond to \(s = 1\) and \(s = -1\).

Step 2: Determine t using the first and third equations

Recall from the first equation:

\[
t = 3s
\]

From the third equation:

\[
3 + t = s
\]

Substitute \(t = 3s \) into the third equation:

\[
3 + 3s = s
\]

Simplify:

\[
3 + 3s = s
\]

\[
3 = s - 3s
\]

\[
3 = -2s
\]

\[
s = -\frac{3}{2}

\]

But earlier, from the quadratic, $s = \pm 1$. Since these do not match, the system is inconsistent for the same s value. Let's verify the two possibilities:

- For $s = 1$:

- $t = 3(1) = 3$

- Check the third equation:

$$
3 + t = s \implies 3 + 3 = 1 \implies 6 = 1
$$

False.

- For $s = -1$:

- $t = 3(-1) = -3$

- Check the third equation:

$$
3 + t = s \implies 3 + (-3) = -1 \implies 0 = -1
$$

False.

Since neither of these satisfy the third equation, the curves do not intersect.

Conclusion on Intersection

Based on the calculations, the two parametric curves do not intersect in space because there are no values of t and s satisfying all three equations simultaneously.

However, for completeness, we can note that the only potential intersection points would require the equations to be consistent, which they are not. Therefore, the curves are skew and do not intersect.

Finding the Angle Between the Curves

Even though the curves do not intersect, for educational purposes, let's explore how to compute the angle between their tangent vectors at potential points of intersection or at specific points.

Step 1: Determine the tangent vectors

The tangent vector to a parametric curve at a point is given by the derivative of the position vector with respect to its parameter.

- For $R(t)$:

$$
R'(t) = \frac{d}{dt} (t, 1, 3 + t) = (1, 0, 1)
$$

- For $R2(s)$:

$$
R2'(s) = \frac{d}{ds} (3s, s^2, s) = (3, 2s, 1)
$$

Step 2: Evaluate tangent vectors at intersection points

Since there are no intersection points, we can evaluate the tangent vectors at any arbitrary points or at the candidate points $s = \pm 1$.

- At $s = 1$:

$$
R2'(1) = (3, 2(1), 1) = (3, 2, 1)
$$

- For $R(t)$, pick t corresponding to $s = 1$:

$$
t = 3(1) = 3
$$

\[
R'(3) = (1, 0, 1)
\]

Step 3: Calculate the angle between the tangent vectors

The angle \(\theta\) between two vectors \(\mathbf{A} \) and \(\mathbf{B} \) is given by:

\[
\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|}
\]

where:

- \(\mathbf{A} \cdot \mathbf{B} \) is the dot product
- \(\|\mathbf{A}\| \) and \(\|\mathbf{B}\| \) are the magnitudes

Calculate:

\[
\mathbf{A} = R'(t) = (1, 0, 1)
\]
\[
\mathbf{B} = R2'(s) = (3, 2, 1)
\]

Dot product:

\[
\mathbf{A} \cdot \mathbf{B} = (1)(3) + (0)(2) + (1)(1) = 3 + 0 + 1 = 4
\]

Magnitudes:

\[
\|\mathbf{A}\| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}
\]
\[
\|\mathbf{B}\| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}
\]

Therefore:

$$
\cos \theta = \frac{4}{\sqrt{2} \times \sqrt{14}} = \frac{4}{\sqrt{28}} = \frac{4}{2 \sqrt{7}} = \frac{2}{\sqrt{7}}
$$

Finally:

$$
\theta = \arccos \left(\frac{2}{\sqrt{7}} \right)
$$

This angle is approximately:

$$
\theta \approx \arccos \left(\frac{2}{2.6458} \right) = \arccos (0.7559) \approx 41.4^\circ
$$

Summary and Final Remarks

- The parametric curves $R(t) = (t, 1, 3 + t)$ and $R2(s) = (3s, s^2, s)$ do not intersect, as their equations do not yield a common point satisfying all three equations simultaneously.
- Despite the lack of intersection, the tangent vectors at specific parameter values can be computed, and the angle between these tangent vectors at corresponding points is approximately 41.4 degrees.
- The method demonstrated here is applicable to analyzing intersections and angles between any parametric curves in space.

Additional Tips for Analyzing Curves and Intersections

- Always verify the system of equations carefully to determine intersection points.
- When curves do not intersect, the concept of an intersection angle does not apply,

Frequently Asked Questions

How do you determine the point of intersection between the curves R(t) = (T, 1 T, 3 + T) and R2(s) = (3 S, S^2, S)?

To find their intersection, set the corresponding components equal: $T = 3S$, $1T = S^2$, and $3 + T = S$. Solve these equations simultaneously to find the values of T and S where the curves intersect.

What is the first step in finding the intersection point of the given curves?

The first step is to express T and S in terms of each other using the equations $T = 3S$ and $3 + T = S$, then substitute into the second equation to solve for one variable.

How do you verify if the curves intersect at a specific point?

Verify by substituting the calculated T and S values into both parametric equations and checking if all components match, confirming the point of intersection.

How do you find the angle between the two curves at their intersection point?

Calculate the tangent vectors (derivatives) of each curve at the intersection point and then find the angle between these vectors using the dot product formula: angle = $\arccos[(v1 \cdot v2) / (|v1||v2|)]$.

What are the derivatives of the given curves R(t) and R2(s) for finding the tangent vectors?

For $R(t) = (T, 1T, 3 + T)$, the derivative is $R'(t) = (1, 1, 1)$. For $R2(s) = (3S, S^2, S)$, the derivative is $R2'(s) = (3, 2S, 1)$.

How do you compute the angle between the curves once the tangent vectors are known?

Use the dot product: $\cos(\theta) = (R'(t) \cdot R2'(s)) / (|R'(t)|\,|R2'(s)|)$, then find θ = arccos of that value.

Are there special considerations when the tangent vectors are perpendicular or parallel?

Yes, if vectors are perpendicular, the angle is 90 degrees; if parallel, the angle is 0 degrees. This impacts the interpretation of how the curves intersect and their geometric relationship.

Can the curves intersect at multiple points, and how is that determined?

Yes, if multiple solutions satisfy the intersection equations, the curves intersect at multiple points. Solve the equations for different values to identify all intersection points.

What is the significance of finding the intersection point and the angle between the curves?

Understanding the intersection point provides geometric insight into where the curves meet, and the angle indicates how they intersect—whether tangentially, perpendicularly, or at an oblique angle—important in applications like physics and engineering.

Additional Resources

Curves Intersection and Angle Analysis in $R(t)$ and $R_2(s)$: An Expert Examination

Understanding the intersection points and the angles at which curves meet is fundamental in advanced calculus, differential geometry, and various engineering applications. Today, we delve into the intriguing problem of determining the point(s) where the parametric curves $R(t) = (T, 1, T)$ and $R_2(s) = (3s, s^2, s)$ intersect, and further, how to compute the angle at their intersection. This analysis combines algebraic solving, differential calculus, and vector operations, providing a comprehensive look into the geometric relationship between these two curves.

Overview of the Curves

Before diving into the calculations, it's crucial to understand the nature and parametric definitions of the given curves.

Curve $R(t)$:
- Defined as $R(t) = (T, 1, T)$
- This is a parametric line in three-dimensional space
- The x and z components are both T, and y is constantly 1
- Geometrically, this is a straight line parallel to the x-z plane at y=1, passing through points where x and z are equal

Curve $R_2(s)$:
- Defined as $R_2(s) = (3s, s^2, s)$
- This is a parabola in 3D space

- The x component scales linearly with s, y is quadratic in s, and z is linear in s
- The curve exhibits a parabolic shape in the y-direction while moving linearly in x and z

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Finding the Intersection Points

The core question: At what point(s) do these two curves cross? To find the intersection, we need to identify values of t and s such that:

$$ R(t) = R_2(s) $$

which expands into the system of equations:

$$
\begin{cases}
T = 3s \\
1 = s^2 \\
T = s
\end{cases}
$$

This set of equations can be solved step-by-step.

Step 1: Equate the Components

From the definitions:

1. x-components:
$$
T = 3s
$$
2. y-components:
$$
1 = s^2
$$
3. z-components:

$$

$$
T = s
$$

Step 2: Solve for s

From the y-component:

$$
s^2 = 1 \Rightarrow s = \pm 1
$$

Case 1: $s = 1$

- From the z-component: $T = s = 1$
- From the x-component: $T = 3s = 3$

But here, T cannot be both 1 and 3 simultaneously. Therefore, this case is inconsistent, and no intersection occurs for $s=1$.

Case 2: $s = -1$

- From the z-component: $T = s = -1$
- From the x-component: $T = 3s = -3$

Again, T cannot be both -1 and -3 simultaneously. Hence, no intersection occurs at $s = -1$ either.

Conclusion on Intersection

Since neither $s=1$ nor $s=-1$ satisfies all three equations simultaneously, the curves do not intersect in real space at any point.

However, in some contexts, such as considering complex solutions or extending the domain, intersections could exist, but within the real domain, these curves do not meet.

Understanding the Geometric Relationship: Do They Ever Cross?

Despite the algebraic evidence indicating no real intersection, it's insightful to analyze their spatial relationship further:

- The line R(t) is fixed at y=1, with x and z equal to T.
- The parabola $R_2(s)$ passes through points where $y = s^2$, which is always ≥ 0, and at y=1, $s = \pm 1$.
- At s=1, $R_2(1) = (3,1,1)$, which has y=1, matching the y-value of R(t) at T=1.
- But the x and z components don't align: for R(t) at T=1, the point is (1,1,1); for $R_2(1)$, the point is (3,1,1).

Since the x-values differ (1 vs 3), the curves are not crossing at s=1. Similarly, at s=-1, the point is (-3,1,-1), which is not on the line R(t).

Thus, the curves are skew—they do not intersect nor are they parallel, but they are close in some regions, which leads us to the next question: what is the angle between the curves at the point of closest approach?

Calculating the Angle Between the Curves

When two curves do not intersect, a common problem is to determine the angle between their tangent vectors at the points closest to each other, or if they intersect, the angle at the intersection point.

Since these curves do not intersect, we focus on the angle between their tangent vectors at the points of closest approach.

2.1 Tangent Vectors of the Curves

- Tangent vector of R(t):

$$
R'(t) = \frac{d}{dt} (T, 1, T) = (1, 0, 1)
$$

This is a constant vector, indicating the direction of the line is always (1, 0, 1).

- Tangent vector of $R_2(s)$:

$$
R'_2(s) = \frac{d}{ds} (3s, s^2, s) = (3, 2s, 1)

\]

This vector varies with s.

2.2 Finding the Points of Closest Approach

Although the curves do not intersect, the minimal distance between them occurs at specific points. To find these points, we minimize the squared distance \(D^2(s,t)\):

\[
D^2(s,t) = \| R(t) - R_2(s) \|^2
\]

which expands to:

\[
D^2(s,t) = (T - 3s)^2 + (1 - s^2)^2 + (T - s)^2
\]

Since T and s are independent variables, and the line R(t) is parametrized by T, and the parabola by s, the minimal distance involves solving the coupled minimization problem:

\[
\frac{\partial D^2}{\partial T} = 0 \quad \text{and} \quad \frac{\partial D^2}{\partial s} = 0
\]

2.3 Minimization Procedure

Step 1: Express T in terms of s

From the x-components:

\[
T = 3s
\]

and from the z-components:

$$
T = s
$$

As before, these are incompatible unless T = s and T=3s imply:

$$
s = T = 3s \Rightarrow 3s = s \Rightarrow 2s=0 \Rightarrow s=0
$$

At s=0, T=0. Let's verify the distance at these points:

$$
T=0, \quad s=0
$$

Calculate the points:

- $R(0) = (0, 1, 0)$
- $R_2(0) = (0, 0, 0)$

Distance squared:

$$
D^2 = (0-0)^2 + (1-0)^2 + (0-0)^2= 1
$$

This suggests minimal or near-minimal distance occurs at s=0, T=0, but to be precise, we need to check derivatives.

Step 2: Compute derivatives

The minimal distance occurs where the vector connecting the two curves is orthogonal to both tangent vectors, i.e., the vector $R(t) - R_2(s)$ is perpendicular to both $R'(t)$ and $R'_2(s)$.

Set:

$$
\vec{C} = R(t) - R_2(s) = (T - 3s, 1 - s^2, T - s)
$$

The orthogonality conditions:

$$
\begin{cases}
\vec{C} \cdot R'(t) = 0 \\
\vec{C} \cdot R'_2(s) = 0
\end{cases}
$$

Calculate:

$$
\vec{C} \cdot R'(t) = (T - 3s)(1) + (1 - s^2)(0) + (T - s)(1) = T - 3s + T - s = 2T - 4s
$$

Set to zero:

$$
2T - 4s = 0 \Rightarrow T = 2s
$$

Similarly,

$$
\vec{C} \cdot R'_2(s) = (T - 3s)(3) + (1 - s^2)(2s)
$$

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