At What Point Do The Curves (t) = (t, 1 T, 3+ T) And (s) = (3 S, S 2, S) Intersect? Find Their Angle

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Understanding the intersection points and angles of curves in three-dimensional space is fundamental in advanced calculus and vector analysis. When dealing with parametric curves such as (t) = (t, 1 T, 3+ T) and (s) = (3 S, S 2, S), it becomes essential to determine precisely where these curves meet and how they intersect in terms of their angles. This article aims to explore these questions thoroughly, providing step-by-step calculations, explanations, and insights into the geometric and algebraic aspects of curve intersections.

Introduction to Parametric Curves in 3D Space

Parametric equations define curves in space by expressing the coordinates as functions of parameters, typically denoted as t or s. In three dimensions, each point on the curve is represented as a triplet (x, y, z), where each coordinate depends on the parameter.

For the curves in question:

- Curve (t): (x, y, z) = (t, 1 T, 3 + T)
- Curve (s): (x, y, z) = (3 S, S^2, S)

Understanding these curves requires interpreting their parametric forms and analyzing how their paths in space might intersect.

Understanding the Given Curves

Curve (t)

The parametric equations:

- x(t) = t
- y(t) = 1 T
- z(t) = 3 + T

Assuming "1 T" and "3 + T" are typographical representations, they likely mean:

- y(t) = T (which is the same as t)

- z(t) = 3 + T (or 3 + t)

Thus, the curve (t) simplifies to:

- $x(t) = t$
- $y(t) = t$
- $z(t) = 3 + t$

This describes a line in space where x, y, and z are linear functions of t, with the z-component offset by 3.

Curve (s)

The parametric equations:

- $x(s) = 3 S$
- $y(s) = S^2$
- $z(s) = S$

Here, the curve is defined with:

- $x = 3 S$
- $y = S^2$
- $z = S$

This describes a space curve where x and z are linear in S, and y is quadratic in S.

Finding the Intersection Point(s)

To determine where these two curves intersect, we need to find values of parameters t and s such that:

- $x(t) = x(s)$
- $y(t) = y(s)$
- $z(t) = z(s)$

This leads to a system of equations:

1. $t = 3 S$
2. $t = S$
3. $3 + t = S$

Let's analyze these step by step.

Step 1: Equate x-components

From the curves:

- $x(t) = t$
- $x(s) = 3 S$

Set equal:

$t = 3 S$

Step 2: Equate y-components

- $y(t) = t$
- $y(s) = S^2$

Set equal:

$t = S^2$

Step 3: Equate z-components

- $z(t) = 3 + t$
- $z(s) = S$

Set equal:

$3 + t = S$

Solving the System of Equations

From earlier, we have:

- $t = 3 S$ (from x-components)
- $t = S^2$ (from y-components)
- $3 + t = S$ (from z-components)

Substitute $t = 3 S$ into the second and third equations.

Express t in terms of S

- $t = 3 S$

- $t = S^2$

Set equal:

$3S = S^2$

Rearranged:

$S^2 - 3S = 0$

Factor:

$S(S - 3) = 0$

Solutions:

- $S = 0$
- $S = 3$

Now, find t for each S:

- For $S = 0$:

$t = 3\ 0 = 0$

- For $S = 3$:

$t = 3\ 3 = 9$

Next, verify the z-component equation:

$3 + t = S$

- For $S = 0$, $t = 0$:

Left side: $3 + 0 = 3$

Right side: 0

Not equal, so no intersection here.

- For $S = 3$, $t = 9$:

Left side: $3 + 9 = 12$

Right side: 3

Not equal, so no intersection here.

Conclusion: The derived values do not satisfy all three equations simultaneously, indicating no intersection points exist between these two curves.

Verifying the Absence of Intersection

The inconsistency suggests the curves do not intersect at any point in space. To confirm, we substitute the parametric values back into the original equations:

- For S = 0:

- $x = 0$
- $y = 0^2 = 0$
- $z = 0$

Corresponding t from earlier: $t = 0$

Check the values:

- $x(t) = 0$, matches $x(s)$
- $y(t) = 0$, matches $y(s)$
- $z(t) = 3 + 0 = 3$, but $z(s) = 0$

Since z-components don't match ($3 \neq 0$), no intersection here.

- For S = 3:

- $x = 3\ 3 = 9$
- $y = 9$
- $z = 3$

Corresponding t: $t = 9$

Check z:

- $z(t) = 3 + 9 = 12$, but $z(s) = 3$

Again, z-components don't match, confirming no point of intersection.

Therefore, the curves do not intersect in space.

Calculating the Angle of Intersection (If Any)

Since the curves do not intersect, the question of their intersection angle is moot. However, for educational purposes, we can analyze their orientations and determine the angle between their tangent vectors at hypothetical points.

Deriving Tangent Vectors

The tangent vector of a parametric curve is given by the derivative of its position vector with respect to its parameter.

- For curve (t):

$\mathbf{r}_t(t) = \frac{d}{dt}(t, t, 3 + t) = (1, 1, 1)$

- For curve (s):

$\mathbf{r}_s(s) = \frac{d}{ds}(3 S, S^2, S) = (3, 2S, 1)$

Finding the Angle Between the Tangent Vectors

The angle θ between two vectors $\mathbf{A}$ and $\mathbf{B}$ is given by:

$$
\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|}
$$

Where:

- $\mathbf{A} = (1, 1, 1)$
- $\mathbf{B} = (3, 2S, 1)$

Calculate the dot product:

$$
\mathbf{A} \cdot \mathbf{B} = (1)(3) + (1)(2S) + (1)(1) = 3 + 2S + 1 = 4 + 2S
$$

Magnitudes:

$$
\|\mathbf{A}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}
$$

$$
\|\mathbf{B}\| = \sqrt{(3)^2 + (2S)^2 + 1^2} = \sqrt{9 + 4S^2 + 1} = \sqrt{10 + 4S^2}
$$

The angle θ at parameter S:

$$
\cos \theta = \frac{4 + 2S}{\sqrt{3} \times \sqrt{10 + 4S^2}}
$$

Because the curves do not intersect, this angle varies along the parameter S, illustrating the relative orientation of their tangent vectors in space.

Summary and Final Remarks

- The parametric curves given by:

$$
\mathbf{r}_t(t) = (t, t, 3 + t)
$$

and

$$
\mathbf{r}_s(s) = (3 S, S^2, S)
$$

do not intersect in space, as confirmed through system analysis.

- Their tangent vectors are constant for $\mathbf{r}_t(t)$ and variable for $\mathbf{r}_s(s)$, with the angle between tangents depending on

Frequently Asked Questions

How do we find the intersection point of the curves t = (t, 1 + t, 3 + t) and s = (3s, s^2, s)?

To find their intersection, set the components equal: $t = 3s$, $1 + t = s^2$, and $3 + t = s$. Solve these equations simultaneously to find the values of t and s where the curves intersect.

What is the first step in determining where the curves (t) = (t, 1 + t, 3 + t) and (s) = (3s, s^2, s) intersect?

The first step is to express s in terms of t or vice versa using the component equations, for example from $t = 3s$, then substitute into the other equations to find consistent solutions.

How do you calculate the angle between the two curves at their point of intersection?

Calculate the tangent vectors of each curve at the intersection point, then use the dot product formula: angle = arccos[(v1 · v2) / (|v1| |v2|)] to find their angle.

What are the tangent vectors for the curves t and s?

For $t(t) = (t, 1 + t, 3 + t)$, the tangent vector is $(1, 1, 1)$. For $s(s) = (3s, s^2, s)$, the tangent vector is $(3, 2s, 1)$.

How do you determine the value of s and t at the intersection point?

Use the equations t = 3s and 3 + t = s to find s and t. From t = 3s, substitute into 3 + t = s to get 3 + 3s = s, then solve for s and t accordingly.

What is the intersection point of the two curves?

Solve the equations: t = 3s and 3 + t = s. Substituting t = 3s into 3 + t = s gives 3 + 3s = s, leading to s = -1. Then t = 3(-1) = -3. The intersection point is (-3, 1 - 3, 3 - 3) = (-3, -2, 0).

What is the angle between the curves at their intersection point?

At the intersection point, the tangent vectors are v1 = (1, 1, 1) and v2 = (3, 2s, 1) with s = -1, so v2 = (3, -2, 1). The angle is found using the dot product: arccos[(v1 · v2) / (|v1| |v2|)] = arccos[(13 + 1(-2) + 11) / (√3 √14)] = arccos[(3 - 2 + 1) / (√3 √14)] = arccos[2 / (√3 √14)].

How do you compute the numerical value of the angle between the curves?

Calculate the dot product numerator: 2. Compute the magnitudes: |v1| = √3 ≈ 1.732, |v2| = √14 ≈ 3.741. Then, angle = arccos(2 / (1.732 3.741)) ≈ arccos(2 / 6.481) ≈ arccos(0.308) ≈ 72.0 degrees.

Why is understanding the intersection point and angle important in analyzing curves?

Knowing where curves intersect and the angle between their tangents helps understand their geometric relationship, such as whether they are crossing, tangent, or skew, which is essential in fields like differential geometry and physics.

Additional Resources

At What Point Do The Curves (t) = (t, 1 T, 3+ T) And (s) = (3 S, S 2, S) Intersect? Find Their Angle is a classic problem in multivariable calculus and vector analysis, combining the concepts of parametric curves, intersection points, and angles between curves. Understanding where two curves intersect and determining the angle at their point of intersection is essential in fields ranging from physics and engineering to computer graphics and geometric modeling. This guide will walk you through the detailed process of solving this problem, including how to find the intersection point, verify the point lies on both curves, and compute the angle between the two curves at that point.

Introduction to the Problem

Imagine two parameterized curves, each described by their respective vector functions:

- Curve (t): $\mathbf{r}_1(t) = (t, 1T, 3 + T)$
- Curve (s): $\mathbf{r}_2(s) = (3S, S^2, S)$

The question posed is: At what point do these curves intersect? Once the intersection point is identified, the next step is to find the angle between the curves at the point of intersection.

Understanding the Curves

Parametric Equations

Let's analyze the given curves:

Curve (t):

$$
\mathbf{r}_1(t) = (t, 1T, 3 + T)
$$

Note that the notation suggests T is a parameter, so we can write:

$$
\mathbf{r}_1(t) = (t, T, 3 + T)
$$

But to avoid confusion, let's clarify that:

- The first component is t, which is the parameter.
- The second component is $1 \times T$, which simplifies to T.

Given the notation, it seems consistent to assume T is the parameter corresponding to the second component. Therefore, for the curve (t), the parameter is t, but the second component is expressed in terms of T. To clarify, perhaps the notation intends:

$$
\mathbf{r}_1(T) = (T, T, 3 + T)
$$

Similarly, the second curve:

$$
\mathbf{r}_2(S) = (3S, S^2, S)
$$

Step 1: Clarify the Parametric Equations

Based on the notation and typical conventions, it's reasonable to interpret:

- Curve (t):

$$
\boxed{
\mathbf{r}_1(T) = (T, T, 3 + T)
}
$$

- Curve (s):

$$
\boxed{
\mathbf{r}_2(S) = (3S, S^2, S)
}
$$

This interpretation makes sense because:

- The first curve's components depend on a single parameter T.
- The second curve's components depend on a parameter S.

Step 2: Find the Intersection Point(s)

To find the intersection point(s), set the two vector functions equal:

$$
\mathbf{r}_1(T) = \mathbf{r}_2(S)
$$

which leads to the system of equations:

$$
\begin{cases}
T = 3S \\
T = S^2 \\
3 + T = S
\end{cases}
$$

Step 3: Solve the System of Equations

Let's analyze each equation:

1. From the first and third equations:

$$
T = 3S \quad \text{and} \quad 3 + T = S

$$

Substitute $T = 3S$ into the third:

$$
3 + 3S = S \Rightarrow 3S - S = -3 \Rightarrow 2S = -3 \Rightarrow S = -\frac{3}{2}
$$

Now, find T:

$$
T = 3S = 3 \times \left(-\frac{3}{2}\right) = -\frac{9}{2}
$$

2. Check the second equation:

$$
T = S^2
$$

Calculate S^2:

$$
\left(-\frac{3}{2}\right)^2 = \frac{9}{4}
$$

Compare with $T = -\frac{9}{2}$:

$$
-\frac{9}{2} \neq \frac{9}{4}
$$

Thus, the second equation is not satisfied with the values obtained from the first and third equations.

Conclusion:

The system is inconsistent; therefore, the curves do not intersect in real space.

Step 4: Re-evaluate and Confirm

Since the previous step indicates no intersection with our current assumptions, let's double-check the equations:

- The first and third equations are consistent if:

$$
T = 3S \quad \text{and} \quad S = 3 + T
$$

Substitute $T = 3S$ into the second:

$$
T = S^2
$$
and into the third:

$$
S = 3 + T
$$

Express T from the third:

$$
T = S - 3
$$

Now, substitute into the first:

$$
T = 3S \Rightarrow S - 3 = 3S \Rightarrow S - 3 = 3S \Rightarrow -3 = 2S \Rightarrow S = -\frac{3}{2}
$$

Calculate T:

$$
T = S -3 = -\frac{3}{2} - 3 = -\frac{3}{2} - \frac{6}{2} = -\frac{9}{2}
$$

Check the second equation:

$$
T = S^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}
$$

Compare with $T = -\frac{9}{2}$:

$$
-\frac{9}{2} \neq \frac{9}{4}
$$

Again, inconsistency arises.

Final Conclusion:

The parametric equations do not intersect because the system of equations derived from equating the curves has no common solution.

However, if the question is about the theoretical point of intersection or the closest approach, or if there's a typo in the original equations, further clarification would be necessary.

But, assuming a corrected set of equations or if the problem intends to analyze the angle at the closest point of approach, we can explore the following:

Calculating the Angle Between the Curves

Step 1: Find the derivatives (tangent vectors) of the curves

- For curve (T):

$$
\mathbf{r}_1(T) = (T, T, 3 + T)
$$
$$
\frac{d\mathbf{r}_1}{dT} = (1, 1, 1)
$$

- For curve (S):

$$
\mathbf{r}_2(S) = (3S, S^2, S)
$$
$$
\frac{d\mathbf{r}_2}{dS} = (3, 2S, 1)
$$

Step 2: Find the tangent vectors at the intersection point(s)

Since the curves do not intersect, the tangent vectors at any points are independent. But, for the purpose of understanding the angle between the curves, select arbitrary parameters T and S.

Step 3: Compute the angle θ between the tangent vectors

The formula for the angle between two vectors $\mathbf{u}$ and $\mathbf{v}$:

$$
\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}
$$

Suppose we pick parameters:

- For the first curve: $T = T_0$
- For the second curve: $S = S_0$

Then,

\[
\mathbf{u} = (1, 1, 1)
\]
\[
\mathbf{v} = (3, 2S_0, 1)
\]

Compute the dot product:

\[
\mathbf{u} \cdot \mathbf{v} = 1 \times 3 + 1 \times 2S_0 + 1 \times 1 = 3 + 2S_0 + 1 = 4 + 2S_0
\]

Compute the magnitudes:

\[
\|\mathbf{u}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}
\]
\[
\|\mathbf{v}\| = \sqrt{(3)^2 + (2S_0)^2 + 1^2} = \sqrt{9 + 4S_0^2 + 1} = \sqrt{10 + 4S_0^2}
\]

Thus,

\[
\cos \theta = \frac{4 + 2S_0}{\sqrt{3} \times \sqrt{10 + 4S_0^2

[At What Point Do The Curves T T 1 T 3 T And S 3 S S 2 S Intersect Find Their Angle]

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