

Calculate The 90% Confidence Interval For The Following Sample Sample: 7.9, 8.3, 8.4, 9.6, 7.7, 8.1,

Calculate The 90% Confidence Interval For The Following Sample Sample: 7.9, 8.3, 8.4, 9.6, 7.7, 8.1, is a fundamental statistical task that helps researchers and analysts understand the range within which a population parameter, such as the mean, is likely to fall with a specified level of confidence. Confidence intervals are essential tools in inferential statistics, providing a measure of uncertainty around a sample estimate. This article will guide you through the process of calculating the 90% confidence interval for a given sample, explain the underlying concepts, and illustrate each step with the provided data set.

Understanding Confidence Intervals

What Is a Confidence Interval?

A confidence interval (CI) is a range of values, derived from sample data, that is believed to contain the true population parameter (e.g., the mean) with a certain probability, known as the confidence level. In this case, a 90% confidence level indicates that if the same sampling process were repeated numerous times, approximately 90% of the calculated intervals would contain the true population mean.

Why Use Confidence Intervals?

Confidence intervals offer several advantages:

- Quantify uncertainty in estimates
- Provide a range that is likely to contain the true parameter
- Assist in decision-making by understanding the precision of estimates

Given Data and Objective

The sample data provided is:

- 7.9

- 8.3
- 8.4
- 9.6
- 7.7
- 8.1

Our goal is to calculate the 90% confidence interval for the population mean based on this sample.

Step-by-Step Calculation of the 90% Confidence Interval

Step 1: Calculate the Sample Mean (Average)

The sample mean ($\bar{x}$) is the sum of all sample values divided by the number of observations.

- Sum of sample values:
 $7.9 + 8.3 + 8.4 + 9.6 + 7.7 + 8.1 = 49.0$
- Number of observations (n):
6
- Sample mean:

$$\bar{x} = \frac{49.0}{6} \approx 8.1667$$

Interpretation: The average of your sample data is approximately 8.17.

Step 2: Calculate the Sample Standard Deviation (s)

The sample standard deviation measures the variability or dispersion of the data points around the mean.

- Calculate each data point's deviation from the mean and square it:

Data Point	Deviation ($x_i - \bar{x}$)	Squared Deviation $(x_i - \bar{x})^2$
7.9	-0.2667	0.0711
8.3	0.1333	0.0178
8.4	0.2333	0.0544
9.6	1.4333	2.0544
7.7	-0.4667	0.2178
8.1	-0.0667	0.0044
Sum	0.0	2.4199

7.9	7.9 - 8.1667 = -0.2667	0.0711
8.3	8.3 - 8.1667 = 0.1333	0.0178
8.4	8.4 - 8.1667 = 0.2333	0.0544
9.6	9.6 - 8.1667 = 1.4333	2.0560
7.7	7.7 - 8.1667 = -0.4667	0.2178
8.1	8.1 - 8.1667 = -0.0667	0.0044

- Sum of squared deviations:

$$\begin{aligned} & \backslash [\\ & 0.0711 + 0.0178 + 0.0544 + 2.0560 + 0.2178 + 0.0044 = 2.4215 \\ & \backslash] \end{aligned}$$

- Sample variance ((s^2)):

$$\begin{aligned} & \backslash [\\ & s^2 = \frac{\text{Sum of squared deviations}}{n - 1} = \frac{2.4215}{5} \\ & \approx 0.4843 \\ & \backslash] \end{aligned}$$

- Standard deviation ((s)):

$$\begin{aligned} & \backslash [\\ & s = \sqrt{0.4843} \approx 0.6960 \\ & \backslash] \end{aligned}$$

Interpretation: The variability of the data points around the mean is approximately 0.696.

Step 3: Determine the Appropriate Critical t-Value

Since the sample size is small ($(n=6)$), and the population standard deviation is unknown, we use the Student's t-distribution.

- Degrees of freedom ((df)):

$$\begin{aligned} & \backslash [\\ & df = n - 1 = 5 \\ & \backslash] \end{aligned}$$

- Critical t-value for a 90% confidence level:

Using a t-distribution table or calculator, for $(df = 5)$ and a 90% confidence interval (which corresponds to $(\alpha=0.10)$, and thus 0.05 in each tail):

$$\begin{aligned} & \backslash [\\ & t_{\{0.95, 5\}} \approx 2.015 \\ & \backslash] \end{aligned}$$

Note: The t-value varies slightly depending on the table, but 2.015 is a common approximation.

Step 4: Calculate the Margin of Error (ME)

The margin of error quantifies the extent of the interval on either side of the sample mean.

$$ME = t_{\alpha/2, df} \times \frac{s}{\sqrt{n}}$$

- Standard error (SE):

$$SE = \frac{s}{\sqrt{n}} = \frac{0.6960}{\sqrt{6}} \approx \frac{0.6960}{2.449} \approx 0.284$$

- Margin of error:

$$ME = 2.015 \times 0.284 \approx 0.573$$

Interpretation: The margin of error is approximately 0.573.

Final Step: Calculate the Confidence Interval

- Lower bound:

$$\bar{x} - ME = 8.1667 - 0.573 \approx 7.593$$

- Upper bound:

$$\bar{x} + ME = 8.1667 + 0.573 \approx 8.740$$

Conclusion: The 90% confidence interval for the population mean based on this sample is approximately (7.593, 8.740).

Interpreting the Confidence Interval

This interval suggests that we can be 90% confident that the true population mean falls between 7.593 and 8.740. It provides a range of plausible values for the mean, accounting for sampling variability.

Importance of Confidence Intervals in Statistical Analysis

Confidence intervals are invaluable in various fields such as medical research, economics, manufacturing, and social sciences. They help in:

- Assessing the reliability of sample estimates
- Making informed decisions based on data
- Communicating uncertainty effectively

Summary of the Calculation Process

To recap, calculating a confidence interval involves the following steps:

1. Compute the sample mean ($\bar{x}$)
2. Calculate the sample standard deviation (s)
3. Identify the appropriate critical t-value based on degrees of freedom and confidence level
4. Calculate the standard error (SE)
5. Determine the margin of error (ME)
6. Construct the interval by adding and subtracting ME from the sample mean

By following these steps, you can accurately estimate the interval within which the true population parameter is likely to lie with a specified confidence level.

Final Thoughts

Calculating the 90% confidence interval for a sample like 7.9, 8.3, 8.4, 9.6, 7.7, 8.1 provides valuable insights into the data's underlying population.

Whether you're conducting scientific research, quality control, or business analysis, understanding how to compute and interpret confidence intervals enhances the reliability and clarity of your conclusions. Remember, the confidence interval is not a guarantee but a probabilistic estimate, reflecting the inherent variability of sampling processes.

If you want to learn more about confidence intervals, statistical inference, or data analysis techniques, consider exploring additional resources or consulting with a statistician to deepen your understanding of these essential tools.

Frequently Asked Questions

What is the first step to calculate the 90% confidence interval for the given sample?

The first step is to compute the sample mean and sample standard deviation of the data points: 7.9, 8.3, 8.4, 9.6, 7.7, and 8.1.

How do you calculate the sample mean for the sample data provided?

Add all the sample values together and divide by the number of observations: $(7.9 + 8.3 + 8.4 + 9.6 + 7.7 + 8.1) / 6$.

What is the formula for the sample standard deviation, and how is it applied here?

The sample standard deviation is calculated using $s = \sqrt{[\sum (x_i - \bar{x})^2 / (n - 1)]}$, where x_i are data points, $\bar{x}$ is the mean, and n is the sample size.

How do you determine the t-value for a 90% confidence interval with 5 degrees of freedom?

Use a t-distribution table or calculator to find the t-value for a 90% confidence level with $df = n - 1 = 5$, which is approximately 2.015.

What is the formula for the confidence interval in this context?

The 90% confidence interval is given by: $\bar{x} \pm t (s / \sqrt{n})$, where $\bar{x}$ is the mean, s is the standard deviation, n is the sample size, and t is the t-value.

Can you compute the sample mean for the sample data?

Yes. The mean $\bar{x} = (7.9 + 8.3 + 8.4 + 9.6 + 7.7 + 8.1) / 6 = 49.0 / 6 \approx 8.167$.

What is the calculated standard deviation for the sample?

First, find each deviation squared, sum them, divide by 5, then take the square root: $s \approx 0.736$.

What is the final 90% confidence interval based on these calculations?

Using the mean 8.167, standard deviation 0.736, t-value 2.015, and $n=6$: $CI \approx 8.167 \pm 2.015 (0.736 / \sqrt{6}) \approx 8.167 \pm 2.015 \cdot 0.300 \approx (7.557, 8.777)$.

Why is it important to use the t-distribution instead of the normal distribution in this calculation?

Because the sample size is small ($n < 30$), the t-distribution accounts for additional uncertainty in estimating the population mean, providing a more accurate confidence interval.

Additional Resources

Confidence Interval Calculation

Introduction

In statistical analysis, understanding the reliability and variability of a sample is as crucial as measuring the sample itself. When we analyze a sample, such as a set of measurements or observations, one of the key concepts we rely on is the confidence interval (CI). The confidence interval provides a range within which we are statistically confident that the true population parameter (such as the mean) lies.

Today, we'll focus on calculating a 90% confidence interval for a specific sample: 7.9, 8.3, 8.4, 9.6, 7.7, 8.1. This process involves a detailed exploration of the sample data, the assumptions underlying the calculations, and step-by-step procedures to arrive at an accurate and meaningful interval estimate.

Understanding the Sample Data

Let's start by examining the data points:

Data Point	Value
1	7.9
2	8.3
3	8.4
4	9.6
5	7.7
6	8.1

Total observations, $n = 6$.

Significance of Confidence Intervals

Before diving into calculations, it's important to grasp why confidence intervals matter:

- Estimate of Population Mean: CI provides a range that likely contains the true population mean based on the sample.
- Quantifies Uncertainty: Unlike a single point estimate, CI accounts for variability and sample size.
- Decision-Making Tool: Helps in informed decision-making, especially in quality control, scientific research, and product evaluations.

Assumptions Underlying Confidence Interval Calculation

When calculating a confidence interval for the mean, certain assumptions are typically made:

1. Random Sampling: The sample should be randomly selected from the population.
2. Normality of Data: The data should approximately follow a normal distribution, especially for small samples.
3. Independence: Observations are independent of each other.
4. Known or Estimated Variance: Since the population variance is usually unknown, we use the sample variance and the t-distribution for small samples.

Given our small sample size ($n=6$), it's essential to employ the t-distribution rather than the normal distribution to account for increased variability.

Step 1: Calculating the Sample Mean

The sample mean ($\bar{x}$) is the average of all data points:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

Calculations:

$$\bar{x} = \frac{7.9 + 8.3 + 8.4 + 9.6 + 7.7 + 8.1}{6}$$

$$\bar{x} = \frac{49.9}{6} \approx 8.3167$$

So, the sample mean is approximately 8.317.

Step 2: Calculating the Sample Standard Deviation

The sample standard deviation (s) measures the dispersion of data points around the mean:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Calculations:

Data Point	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
7.9	7.9	-0.417	0.174
8.3	8.3	-0.017	0.0003
8.4	8.4	0.083	0.0069
9.6	9.6	1.283	1.646
7.7	7.7	-0.617	0.380
8.1	8.1	-0.217	0.047

Sum of squared deviations:

$$0.174 + 0.0003 + 0.0069 + 1.646 + 0.380 + 0.047 = 2.2542$$

Calculate s :

$$s = \sqrt{\frac{2.2542}{6 - 1}} = \sqrt{\frac{2.2542}{5}} = \sqrt{0.45084} \approx 0.672$$

\]

The sample standard deviation is approximately 0.672.

Step 3: Determining the t-Value

Since the sample size is small ($n=6$), and the population variance is unknown, we utilize the t-distribution.

Degrees of freedom (df): $(n - 1 = 5)$.

Using a t-distribution table or calculator, for a 90% confidence level:

- The alpha level: $(\alpha = 1 - 0.90 = 0.10)$.
- Since it's a two-tailed interval, divide alpha by 2: $(\alpha/2 = 0.05)$.

From t-distribution tables, the t-value for $df=5$ at 90% confidence:

\[
 $t_{\{0.05, 5\}} \approx 2.015$
\]

Step 4: Calculating the Margin of Error (ME)

The margin of error quantifies the extent of the interval around the sample mean:

\[
 $\text{ME} = t_{\{\alpha/2, df\}} \times \frac{s}{\sqrt{n}}$
\]

Calculations:

\[
 $\text{ME} = 2.015 \times \frac{0.672}{\sqrt{6}} = 2.015 \times \frac{0.672}{2.45} \approx 2.015 \times 0.274 \approx 0.552$
\]

Step 5: Constructing the Confidence Interval

The confidence interval is:

\[
 $\left(\bar{x} - \text{ME}, \quad \bar{x} + \text{ME} \right)$
\]

Plugging in the values:

```
\[
(8.317 - 0.552, \quad 8.317 + 0.552) \Rightarrow (7.765, 8.869)
\]
```

Final 90% Confidence Interval:

```
\[
\boxed{
\textbf{(7.77, 8.87)}
}
\]
```

(rounded to two decimal places)

Interpreting the Results

The 90% confidence interval from 7.77 to 8.87 suggests that, based on this sample, we are 90% confident that the true population mean falls within this range. It's important to note that this does not mean there's a 90% probability the true mean is within the interval—rather, that if we were to take many samples and compute a confidence interval for each, approximately 90% of those intervals would contain the true mean.

Additional Considerations

- Sample Size: With only 6 data points, the interval is relatively wide, reflecting greater uncertainty.
- Normality Assumption: Small samples require cautious interpretation; if the data are not approximately normal, the interval's validity may be compromised.
- Further Data Collection: Increasing the sample size can lead to a narrower confidence interval, providing more precise estimates.

Practical Applications

Calculating confidence intervals is vital in various fields:

- Quality Control: Assessing process means in manufacturing.
- Scientific Research: Estimating true effect sizes.
- Product Development: Evaluating consistency in measurements.
- Business Analytics: Forecasting and risk assessment.

In this context, whether you're measuring product dimensions, response times,

or other metrics, understanding how to accurately compute and interpret confidence intervals ensures informed decision-making backed by statistical rigor.

Conclusion

Calculating a 90% confidence interval for a sample involves a systematic approach:

1. Compute the sample mean.
2. Calculate the sample standard deviation.
3. Determine the appropriate t-value based on degrees of freedom and confidence level.
4. Calculate the margin of error.
5. Construct the interval.

For the sample data 7.9, 8.3, 8.4, 9.6, 7.7, 8.1, the resulting interval (7.77, 8.87) offers a statistically sound estimate of the true population mean. This process exemplifies how statistical tools can translate raw data into meaningful insights, guiding effective decisions in product analysis and beyond.

Calculate The 90 Confidence Interval For The Following Sample Sample 7 9 8 3 8 4 9 6 7 7 8 1

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