Calculate The Change In Entropy When 100 G Of Water At 80C Ispoured Into 100 G Of Water At 10C In An

Calculate The Change In Entropy When 100 G Of Water At 80°C Is Poured Into 100 G Of Water At 10°C In An

Understanding the thermodynamic behavior of water during mixing processes is fundamental in physics and engineering disciplines. One key concept in thermodynamics is entropy, a measure of disorder or randomness in a system. When two quantities of water at different temperatures are combined, the entropy of the system changes, reflecting the irreversibility and energy dispersal involved in the process. Accurately calculating this change in entropy provides insights into energy efficiency, system stability, and the second law of thermodynamics.

In this article, we explore the detailed process of calculating the change in entropy when 100 grams of water at 80°C is poured into 100 grams of water at 10°C, resulting in a mixed temperature. We will walk through the fundamental principles, formulas, assumptions, and step-by-step calculations, ensuring clarity and depth for readers interested in thermodynamics, physics, or engineering applications.

Understanding the Context and Significance of Entropy Calculation

Entropy, denoted as S, is a central concept in thermodynamics that quantifies the degree of disorder or the number of microscopic configurations that correspond to a thermodynamic system's macroscopic state. The Second Law of Thermodynamics states that in an isolated system, entropy tends to increase, leading systems toward equilibrium.

When two bodies of water at different temperatures are combined, heat flows from the hotter to the colder water until thermal equilibrium is established. This process is irreversible in real-world conditions, and the entropy of the combined system increases. Calculating this entropy change has implications in various fields such as:

- Designing efficient heat exchangers
- Understanding environmental thermal processes
- Analyzing energy losses in industrial systems
- Studying natural phenomena like oceanic mixing

Therefore, quantifying entropy change during water mixing not only reinforces fundamental thermodynamic principles but also aids in practical engineering decision-making.

Fundamental Principles for Calculating Entropy Change

Before diving into calculations, it’s important to understand the core principles involved:

1. Entropy Change in Reversible Processes

For a reversible process, the change in entropy (ΔS) of a system when its temperature changes from T_1 to T_2 is given by:

$$
\Delta S = \int_{T_1}^{T_2} \frac{dQ_{rev}}{T}
$$

where $dQ_{(rev)}$ is the infinitesimal heat exchanged reversibly.

2. Entropy Change for Heating or Cooling at Constant Pressure

For a substance with constant specific heat capacity (c_p), the entropy change when heating from T_1 to T_2 is:

$$
\Delta S = m c_p \ln \left(\frac{T_2}{T_1} \right)
$$

where:
- m is the mass
- c_p is the specific heat capacity at constant pressure
- T is in Kelvin

3. Mixings and Final Equilibrium Temperature

When two quantities of water at different temperatures are mixed, the final temperature (T_f) can be found using conservation of energy:

$$
m_1 c_p T_1 + m_2 c_p T_2 = (m_1 + m_2) c_p T_f
$$

\]

which simplifies to:

$$
T_f = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2}
$$

since c_p is constant for water over the temperature range considered.

Assumptions and Simplifications

For the calculations, several assumptions are made to streamline the process:

- Constant Specific Heat Capacity: The specific heat capacity of water (c_p) is approximately 4.18 J/g·K over the temperature range 10°C to 80°C.
- No Heat Loss to Environment: The process is idealized as isolated, with no heat exchange with surroundings.
- No Phase Changes: Temperatures are within the liquid phase of water; no boiling or freezing occurs.
- Uniform Temperature Distribution: Water is perfectly mixed, resulting in a single uniform temperature after mixing.
- Incompressible Fluid: Water density and volume changes are negligible for this calculation.

Step-by-Step Calculation of Entropy Change

Let's now proceed with the detailed calculations involved in finding the change in entropy during this process.

Step 1: Identify Known Values

Parameter	Value	Unit
Mass of hot water (m_1)	100	g
Initial temperature of hot water (T_1)	80	°C
Mass of cold water (m_2)	100	g
Initial temperature of cold water (T_2)	10	°C
Specific heat capacity of water (c_p)	4.18	J/g·K

Convert temperatures to Kelvin:

$$
T_{K} = T_{°C} + 273.15
$$

Water	Temperature (°C)	Temperature (K)
Hot water	80	353.15
Cold water	10	283.15

Calculate the final temperature:

$$
T_f = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2} = \frac{(100)(353.15) + (100)(283.15)}{200}
$$

$$
T_f = \frac{35315 + 28315}{200} = \frac{63630}{200} = 318.15\, \text{K}
$$

Converted back to Celsius for reference:

$$
T_f^{(°C)} = 318.15 - 273.15 = 45\, °C
$$

Step 2: Calculate Entropy Change for Hot Water

The hot water cools from 80°C (353.15K) to 45°C (318.15K):

$$
ΔS_{hot} = m_1 c_p \ln \left(\frac{T_f}{T_{initial, hot}} \right)
$$

$$
ΔS_{hot} = 100\, g \times 4.18\, \frac{J}{g·K} \times \ln \left(\frac{318.15}{353.15} \right)
$$

Calculate the natural logarithm:

$$
\ln \left(\frac{318.15}{353.15} \right) = \ln (0.900) \approx -0.10536
$$

Calculate $\Delta S_{(hot)}$:

$$
\Delta S_{hot} = 100 \times 4.18 \times (-0.10536) \approx -44.1\, J/K
$$

The negative sign indicates a decrease in entropy of the hot water as it cools.

Step 3: Calculate Entropy Change for Cold Water

The cold water warms from 10°C (283.15K) to 45°C (318.15K):

$$
\Delta S_{cold} = m_2 c_p \ln \left(\frac{T_f}{T_{initial, cold}} \right)
$$

$$
\Delta S_{cold} = 100\, g \times 4.18\, \frac{J}{g·K} \times \ln \left(\frac{318.15}{283.15} \right)
$$

Calculate the logarithm:

$$
\ln \left(\frac{318.15}{283.15} \right) = \ln (1.124) \approx 0.1177
$$

Calculate $\Delta S_{(cold)}$:

$$
\Delta S_{cold} = 100 \times 4.18 \times 0.1177 \approx 49.2\, J/K
$$

This entropy increases as the cold water warms.

Step 4: Determine Total Entropy Change

Total change in entropy of the system:

$$
\Delta S_{total} = \Delta S_{hot} + \Delta S_{cold} = -44.1\, J/K + 49.2\, J/K = 5.1\, J/K
$$

The positive result confirms that the total entropy of the system increases during the mixing process, consistent with the second law of thermodynamics.

Interpretation and Significance of Results

The calculated entropy change of approximately 5.1 J/K indicates an increase in disorder as the system moves toward thermal equilibrium. This small but positive change reflects the irreversibility of the process, where heat spontaneously flows from the hot to cold water.

Key points to consider:

- The entropy increase is primarily due to the heat transfer from the hot to cold water.
- The magnitude of entropy change depends on the masses, initial temperatures, and specific heat capacity.
- The process demonstrates the fundamental thermodynamic principle that entropy tends to increase in natural processes.

Real-World Applications and Further Considerations

Understanding entropy changes during mixing processes has

Frequently Asked Questions

How do you determine the change in entropy when mixing two quantities of water at different temperatures?

The change in entropy can be calculated using the formula $\Delta S = m\ c\ \ln(T_{final} / T_{initial})$, considering the masses, specific heat capacity, and temperature change of each water sample, assuming no heat loss to surroundings.

What is the initial temperature and mass of each water sample in the problem?

The first water sample has a mass of 100 g at 80°C, and the second has a mass of 100 g at 10°C.

How do you find the final temperature after mixing two water samples?

By applying the principle of conservation of energy, the final temperature T_f is calculated using: $(m1\ c\ T1 + m2\ c\ T2) / (m1 + m2)$. In this case, $T_f = (100\ 80 + 100\ 10) / 200 = 45°C$.

What is the specific heat capacity of water used in the calculation?

The specific heat capacity of water is approximately 4.18 J/g·K.

How is the total change in entropy calculated for the system?

The total entropy change is the sum of the entropy changes of each water sample, calculated as $\Delta S = m\ c\ \ln(T_{final} / T_{initial})$ for each, then summed together.

What is the entropy change for the water initially at 80°C?

$\Delta S_1 = 100\ g\ 4.18\ J/g\cdot K\ \ln(45 + 273.15 / 80 + 273.15) \approx 100\ 4.18\ \ln(318.15 / 353.15) \approx - 58.2\ J/K$.

What is the entropy change for the water initially at 10°C?

$\Delta S_2 = 100\ g\ 4.18\ J/g\cdot K\ \ln(45 + 273.15 / 10 + 273.15) \approx 100\ 4.18\ \ln(318.15 / 283.15) \approx 53.8\ J/K$.

Additional Resources

Calculate The Change In Entropy When 100 G Of Water At 80°C Is Poured Into 100 G Of Water At 10°C In An

Understanding the change in entropy when two quantities of water at different temperatures are mixed is a fundamental problem in thermodynamics. This scenario provides an excellent case study for exploring the concepts of entropy, heat transfer, and thermal equilibrium. The process involves calculating the entropy change resulting from combining 100 grams of water initially at 80°C with another 100 grams of water initially at 10°C, ultimately reaching a common equilibrium temperature. This article aims to guide you through the detailed calculation steps, discuss underlying principles, and explore the broader implications of such thermodynamic processes.

Introduction to Entropy and Its Significance

Entropy, often denoted by S, is a measure of the disorder or randomness in a thermodynamic system. In simpler terms, it quantifies the number of microscopic configurations that correspond to a system's macroscopic state. The second law of thermodynamics states that in an isolated system, the total entropy tends to increase, reflecting the natural tendency toward equilibrium and maximum disorder.

Understanding entropy changes during mixing processes is crucial for various applications, including engineering designs, environmental modeling, and understanding natural phenomena. When two bodies of water at different temperatures are combined, heat flows from the hotter to the colder portion until thermal equilibrium is achieved. During this process, the entropy of the combined system changes, typically increasing due to irreversible heat flow.

Key Concepts and Assumptions

Before delving into the calculations, it’s essential to clarify assumptions and foundational concepts:

- No heat loss to surroundings: The process is considered adiabatic with respect to the environment, meaning all heat transfer occurs only between the two water samples.
- Constant specific heat capacity: For water, the specific heat capacity (c) is approximately 4.18 J/g°C, assumed constant over the temperature range.
- No phase change: Water remains in the liquid phase throughout the process.
- Thermal equilibrium: The final temperature (Tf) is uniform throughout the combined water mass.
- Mass of water (m): 100 g in each sample, totaling 200 g.

These assumptions simplify the calculations and are reasonable for small temperature differences and controlled laboratory conditions.

Determining the Final Equilibrium Temperature

(Tf)

The first step in calculating entropy change is to find the equilibrium temperature after mixing.

Energy Conservation Principle

The heat lost by the hot water equals the heat gained by the cold water:

$$ m_{hot} c (T_{hot,i} - T_f) = m_{cold} c (T_f - T_{cold,i}) $$

Given:

- $m_{hot} = 100\,g$
- $T_{hot,i} = 80^\circ C$
- $m_{cold} = 100\,g$
- $T_{cold,i} = 10^\circ C$
- $c = 4.18\,J/g^\circ C$

Since masses and specific heats are identical, the equation simplifies:

$$ (80 - T_f) = (T_f - 10) $$

Solving for T_f:

$$ 80 - T_f = T_f - 10 $$
$$ 80 + 10 = 2 T_f $$
$$ 90 = 2 T_f $$
$$ T_f = 45^\circ C $$

Thus, the final temperature after mixing is approximately 45°C.

Calculating the Change in Entropy

Entropy change for each water sample as it heats or cools can be computed using the integral form:

$$
\Delta S = m c \int_{T_i}^{T_f} \frac{1}{T} dT = m c \ln \left(
\frac{T_f}{T_i} \right)
$$

Note: Temperatures must be in Kelvin for entropy calculations.

Converting Temperatures to Kelvin

- $T_{hot,i} = 80^\circ C = 80 + 273.15 = 353.15\,K$
- $T_{cold,i} = 10^\circ C = 283.15\,K$
- $T_f = 45^\circ C = 318.15\,K$

Entropy Change for Hot Water

$$
\Delta S_{hot} = m c \ln \left(\frac{T_f}{T_{hot,i}} \right)
$$

$$
\Delta S_{hot} = 100\,g \times 4.18\,J/g^\circ C \times \ln \left(\frac{318.15}{353.15} \right)
$$

Calculating:

$$
\Delta S_{hot} = 418\,J/K \times \ln (0.900) \approx 418\,J/K \times (-0.1054) \approx -44.1\,J/K
$$

The negative sign indicates a decrease in entropy for the hot water as it cools.

Entropy Change for Cold Water

$$
\Delta S_{cold} = 100\,g \times 4.18\,J/g^\circ C \times \ln \left(\frac{318.15}{283.15} \right)
$$

$$
\Delta S_{cold} = 418\,J/K \times \ln (1.124) \approx 418\,J/K \times 0.117 \approx 49.0\,J/K
$$

The positive change indicates an increase in entropy as the cold water warms.

Total Entropy Change of the System

Summing the two:

$$
\Delta S_{total} = \Delta S_{hot} + \Delta S_{cold} \approx -44.1\,J/K + 49.0\,J/K = 4.9\,J/K
$$

The small but positive total entropy change confirms the second law's expectation: the overall entropy of the combined system increases during mixing, reflecting irreversibility.

Discussion of Results and Broader Implications

The computed change in entropy, approximately 4.9 J/K, may seem modest, but it embodies critical thermodynamic principles:

- Irreversibility: The process of mixing two water samples at different temperatures is inherently irreversible, leading to an increase in entropy.
- Energy dispersal: Heat flows from a higher to a lower temperature, increasing the disorder within the system.
- Thermodynamic consistency: The positive total entropy change aligns with the second law, confirming the physical validity of the calculations.

This example underscores how entropy calculations are vital for designing thermal systems, understanding natural processes, and optimizing energy efficiency.

Features, Pros, and Cons of the Approach

Features:

- Uses fundamental thermodynamic equations applicable to idealized conditions.
- Demonstrates the calculation of entropy change during mixing.
- Highlights the importance of temperature conversions and assumptions.

Pros:

- Provides clear insight into energy and entropy transfer processes.

- Applicable to real-world scenarios like heat exchangers, environmental modeling, and chemical processes.
- Reinforces understanding of thermodynamic principles through practical calculation.

Cons:

- Assumes constant specific heat, which may vary slightly with temperature.
- Ignores heat losses to surroundings, which can be significant in real systems.
- Does not account for phase changes or impurities in water.
- Simplifies the process as quasi-static, whereas actual mixing may involve turbulence and other complexities.

Conclusion

Calculating the change in entropy when two quantities of water at different temperatures are mixed offers valuable insights into thermodynamic irreversibility and energy dispersal. In this specific case, the total entropy increases by approximately 4.9 J/K, illustrating the fundamental principle that entropy tends to increase in spontaneous processes. Understanding these principles is essential for engineers, scientists, and students aiming to analyze thermal systems, optimize energy use, and comprehend natural phenomena involving heat transfer. The methodology demonstrated here provides a straightforward yet powerful approach to quantifying entropy changes, reinforcing the importance of thermodynamics in both theoretical and applied contexts.

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