

Calculate The Force Of Gravity On The 1- Kg Mass If It Were 3.2106 M Above Earth's Surface (that Is,

Calculate The Force Of Gravity On The 1- Kg Mass If It Were 3.2106 M Above Earth's Surface (that Is, a fundamental problem in physics that bridges our understanding of Newtonian gravity with real-world applications. This calculation not only demonstrates the principles governing the gravitational force but also illustrates how the force varies with changes in distance from the Earth's surface. Understanding such variations is essential in fields ranging from aerospace engineering to satellite deployment, where precise calculations of gravitational forces are crucial for mission success. In this article, we will explore the underlying physics, derive the necessary formulas, and perform detailed calculations to determine the gravitational force acting on a 1 kg mass located 3.2106 meters above Earth's surface.

Understanding Gravitational Force

The Newtonian Law of Universal Gravitation

The foundation of calculating gravitational force is Newton's Law of Universal Gravitation, which states that every point mass attracts every other point mass in the universe with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.

Mathematically, the law is expressed as:

$$F = G \frac{m_1 m_2}{r^2}$$

Where:

- F is the magnitude of the gravitational force between the two masses,
- G is the gravitational constant ($6.67430 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$),
- m_1 and m_2 are the masses involved,
- r is the distance between the centers of the two masses.

The Earth and the Test Mass

In our scenario:

- m_1 is the Earth's mass (M_e),
- m_2 is the test mass, which is 1 kg,
- r is the distance from Earth's center to the mass.

Since the mass is only 3.2106 meters above Earth's surface, the primary concern is how this slight elevation affects the gravitational force compared to the force at the surface.

Key Data and Constants

Before proceeding with calculations, it is essential to establish the known data and constants:

- Earth's mass, $(M_e): (5.972 \times 10^{24} \text{ kg})$
- Earth's mean radius, $(R_e): (6,371,000 \text{ m})$ (approximately 6,371 km)
- Gravitational constant, $(G): (6.67430 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)$
- Mass of the object, $(m): 1 \text{ kg}$
- Height above Earth's surface, $(h): 3.2106 \text{ m}$

Calculating the Distance from Earth's Center

At Earth's Surface

The distance from Earth's center to the mass at the surface is simply Earth's radius:

$$r_{\text{surface}} = R_e = 6,371,000 \text{ m}$$

At 3.2106 M Above Surface

The distance from Earth's center to the mass at the given height is:

$$r_{\text{height}} = R_e + h$$

$$r_{\text{height}} = 6,371,000 \text{ m} + 3.2106 \text{ m} \approx 6,371,003.2106 \text{ m}$$

Given the negligible difference in height compared to Earth's radius, the change in gravitational force will be minimal but still calculable for accuracy.

Calculating the Gravitational Force at the Height

Force at Earth's Surface

Using Newton's law:

$$F_{\text{surface}} = G \frac{M_e \times m}{R_e^2}$$

Plugging in the known values:

$$F_{\text{surface}} = (6.67430 \times 10^{-11}) \times \frac{(5.972 \times 10^{24}) \times 1}{(6,371,000)^2}$$

Calculating denominator:

$$(6,371,000)^2 = 4.058 \times 10^{13} \text{ m}^2$$

Calculating numerator:

$$6.67430 \times 10^{-11} \times 5.972 \times 10^{24} \approx 3.986 \times 10^{14}$$

Therefore:

$$F_{\text{surface}} \approx \frac{3.986 \times 10^{14}}{4.058 \times 10^{13}} \approx 9.81 \text{ N}$$

This aligns with the standard gravitational acceleration ($g \approx 9.81 \text{ m/s}^2$), confirming the force on 1 kg at Earth's surface.

Force at 3.2106 M Above Surface

Using the same formula:

$$F_{\text{height}} = G \frac{M_e \times m}{r_{\text{height}}^2}$$

Calculate the denominator:

$$r_{\text{height}}^2 = (6,371,003.2106)^2 \approx 4.058 \times 10^{13} \text{ m}^2$$

Since the difference is minimal, the force can be estimated as:

$$F_{\text{height}} \approx 9.81 \times \left(\frac{R_e}{r_{\text{height}}} \right)^2$$

Calculating the ratio:

$$\frac{R_e}{r_{\text{height}}} \approx \frac{6,371,000}{6,371,003.2106} \approx 0.999999498$$

Squaring this:

$$(0.999999498)^2 \approx 0.999999 \text{ (approximately)}$$

Thus, the gravitational force at the height:

$$F_{\text{height}} \approx 9.81 \times 0.999999 \approx 9.80999 \text{ N}$$

Alternatively, performing the precise calculation:

$$F_{\text{height}} = 3.986 \times 10^{14} / 4.058 \times 10^{13} \approx 9.80999 \text{ N}$$

Difference in Force:

$$\Delta F = F_{\text{surface}} - F_{\text{height}} \approx 9.81 - 9.80999 \approx 0.00001 \text{ N}$$

This negligible difference demonstrates that at only a few meters above Earth's surface, the gravitational force remains essentially unchanged.

Implications of the Calculations

Understanding Gravity's Dependence on Distance

The calculation confirms that gravitational force decreases with the square of the distance from Earth's center. Even a few meters' difference yields an imperceptibly small change in force. This illustrates why Earth's gravity can be considered constant for most everyday applications, but becomes significantly different at larger altitudes.

Practical Significance

- Satellite Orbit Calculations: For satellites in low Earth orbit (~200 km altitude), the change in gravitational force is more significant and must be precisely calculated.
- Engineering and Design: Structures or objects at different heights require understanding of the slight variations in gravity for sensitive measurements.
- Educational Demonstration: The calculation provides a clear example of how gravitational force varies with distance, reinforcing Newtonian physics principles.

Conclusion

Calculating the gravitational force on a 1 kg mass positioned 3.2106 meters above Earth's surface reveals a force of approximately 9.81 N, virtually identical to the force at Earth's surface. The tiny difference, on the order of (10^{-5}) N, underscores the fact that Earth's gravity remains effectively constant within a few meters from the surface. However, as the height increases to thousands or millions of meters, the force diminishes significantly, following the inverse-square law. This exploration exemplifies the profound relationship between distance and gravitational attraction, a cornerstone of classical physics. Understanding these principles is crucial for scientific endeavors that involve precise gravitational measurements, space exploration, and the development of technologies operating in varying gravitational fields.

Frequently Asked Questions

How do you calculate the force of gravity on a 1 kg mass located 3.2106 meters above Earth's surface?

The force of gravity can be calculated using Newton's law of universal gravitation: $F = G (m_1 m_2) / r^2$, where G is the gravitational constant, m_1 is Earth's mass, m_2 is the mass (1 kg), and r is the distance from Earth's center (Earth's radius plus 3.2106 m).

What is the value of Earth's radius used in calculating the gravitational force at 3.2106 meters above the surface?

Earth's average radius is approximately 6,371 kilometers, or 6,371,000 meters. For calculations, this value is added to the height above the surface to get the total distance from Earth's center.

Does the small height of 3.2106 meters significantly affect the gravitational force on a 1 kg mass?

No, at such a small height relative to Earth's radius, the change in gravitational force is negligible, but precise calculations can account for this slight difference.

What is the approximate gravitational force on a 1 kg mass just 3.2106 meters above Earth's surface?

Using the formula, the gravity at Earth's surface is about 9.81 N. At 3.2106 meters above, the force is approximately 9.81 N ($\text{Earth's radius} / (\text{Earth's radius} + 3.2106))^2$, which is very close to 9.81 N.

Why does the gravitational force decrease slightly as the altitude increases?

Because gravitational force is inversely proportional to the square of the distance from Earth's center, increasing the height reduces the force slightly.

Can you provide a step-by-step calculation for the gravity on 1 kg at 3.2106 meters above Earth's surface?

Yes. First, note Earth's radius $R \approx 6,371,000$ m. The distance $r = R + 3.2106$ m. Then, $F = G (M_{\text{earth}} 1 \text{ kg}) / r^2$. Using $G \approx 6.674 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$ and $M_{\text{earth}} \approx 5.972 \times 10^{24} \text{ kg}$, plug in the values to compute the force, which will be approximately 9.81 N minus a tiny amount due to the increased distance.

Additional Resources

Calculate The Force Of Gravity On The 1-Kilogram Mass If It Were 3.2106 M Above Earth's Surface

Understanding the force of gravity acting on objects near Earth's surface is fundamental to physics, encapsulating concepts from Newton's law of universal gravitation to practical applications in engineering and everyday life. In this detailed exploration, we will analyze the gravitational force exerted on a 1-kg mass located precisely 3.2106 meters above Earth's surface. This scenario, seemingly straightforward, opens a window into the intricacies of gravitational calculations, the significance of altitude, and the broader implications in scientific understanding.

Fundamentals of Gravitational Force

Before delving into the specific calculation, it's essential to revisit the foundational principles governing gravitational force.

Newton's Law of Universal Gravitation

Sir Isaac Newton formulated the law that describes the attractive force between two masses:

$$F = G \frac{m_1 m_2}{r^2}$$

Where:

- F = magnitude of the gravitational force between the two masses (in Newtons, N),
- G = gravitational constant ($6.67430 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$),
- m_1 and m_2 = masses involved (in kilograms, kg),
- r = the distance between the centers of the two masses (in meters, m).

In this context, m_1 would be Earth's mass, and m_2 is the 1-kg mass.

Earth's Mass and Radius

- Earth's approximate mass: $M_e \approx 5.972 \times 10^{24} \text{ kg}$,
- Earth's mean radius: $R_e \approx 6,371 \text{ km} = 6.371 \times 10^6 \text{ m}$.

The calculation hinges on the distance from Earth's center to the object, which is the Earth's radius plus the altitude above the surface.

Determining the Distance from Earth's Center

Given the problem states the object is 3.2106 meters above Earth's surface, the total distance from Earth's center (r) is:

$$r = R_e + h$$

Where:

- $R_e = 6.371 \times 10^6 \text{ m}$,
- $h = 3.2106 \text{ m}$.

Thus,

$$r = 6.371 \times 10^6 \text{ m} + 3.2106 \text{ m} \approx 6.371000032106 \times 10^6 \text{ m}$$

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The addition of approximately 3.21 meters is minuscule relative to Earth's radius, but for precise calculations, especially in scientific contexts, such detail is important.

Calculating the Gravitational Force

Using Newton's law:

$$F = G \frac{M_e m}{r^2}$$

Where:

- $M_e = 5.972 \times 10^{24} \text{ kg}$,
- $m = 1 \text{ kg}$,
- $r \approx 6.371000032106 \times 10^6 \text{ m}$,
- $G = 6.67430 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$.

Substituting the values:

$$F = (6.67430 \times 10^{-11}) \times \frac{(5.972 \times 10^{24}) \times 1}{(6.371000032106 \times 10^6)^2}$$

Calculating the denominator:

$$r^2 \approx (6.371000032106 \times 10^6)^2$$

Since the additional 3.21 meters is negligible at this scale, the square of (r) is:

$$r^2 \approx (6.371 \times 10^6)^2 + 2 \times 6.371 \times 10^6 \times 3.2106 + (3.2106)^2$$

But because (3.2106) meters is tiny compared to (6.371×10^6) meters, we can approximate:

$$r^2 \approx (6.371 \times 10^6)^2 + \text{negligible terms}$$

Calculating $((6.371 \times 10^6)^2)$:

$$\begin{aligned} & (6.371 \times 10^6)^2 = 6.371^2 \times 10^{12} \approx 40.58 \times 10^{12} = 4.058 \times 10^{13} \end{aligned}$$

Thus,

$$F \approx 6.67430 \times 10^{-11} \times \frac{5.972 \times 10^{24}}{4.058 \times 10^{13}}$$

Simplify numerator:

$$6.67430 \times 10^{-11} \times 5.972 \times 10^{24} = (6.67430 \times 5.972) \times 10^{(-11 + 24)} = 39.878 \times 10^{13}$$

Now, divide by the denominator:

$$F \approx \frac{39.878 \times 10^{13}}{4.058 \times 10^{13}} = \frac{39.878}{4.058} \approx 9.829 \, \text{N}$$

Therefore, the gravitational force acting on a 1-kg mass approximately 9.83 Newtons at 3.2106 meters above Earth's surface.

Comparison with Standard Gravity and Effect of Altitude

A key insight from the calculation is that at Earth's surface, the gravitational acceleration (g) is approximately:

$$g = \frac{GM_e}{R_e^2} \approx 9.81 \, \text{m/s}^2$$

And the force on a 1-kg mass at the surface:

$$F_{\text{surface}} = m g \approx 1 \times 9.81 = 9.81 \, \text{N}$$

Our calculated force at 3.2106 meters above the surface ($\sim 9.83 \, \text{N}$) is marginally higher than at the surface, which seems counterintuitive because gravity diminishes with altitude.

Why does the force slightly increase?

- The calculation above is based on the total distance from Earth's center, which is essentially unchanged at such a small altitude.
- The minute increase in (r) (from $(6.371 \times 10^6, \text{m})$ to $(6.3710000032106 \times 10^6, \text{m})$) causes a negligible change in (r^2) and thus in the force.
- However, because the force is inversely proportional to (r^2) , increasing (r) slightly decreases the force, so the force at 3.21 meters above the surface should be slightly less than at the surface.

Reconciliation:

The approximate calculation indicates a force of about 9.83 N, slightly higher than Earth's surface gravity. This discrepancy results from the approximations used in the calculations, especially ignoring the tiny change in denominator when just adding 3.21 meters.

In reality, the gravitational acceleration at the Earth's surface is about 9.81 m/s^2 , and at 3.21 meters above the surface, it can be approximated as:

$$g_h = g_0 \left(1 - \frac{2h}{R_e} \right)$$

for small (h) , using a linear approximation, giving:

$$g_h \approx 9.81 \times \left(1 - \frac{2 \times 3.21}{6.371 \times 10^6} \right) \approx 9.81 \times \left(1 - 1.006 \times 10^{-6} \right) \approx 9.81 - 0.0099 \text{ N}$$

which confirms that the force at 3.21 meters is approximately:

$$F \approx 9.81 \text{ N} - 0.0099 \text{ N} \approx 9.80 \text{ N}$$

indicating a negligible decrease compared to the force at Earth's surface.

Broader Implications and Practical Applications

Understanding the minute differences in gravitational force at small altitudes is critical in various scientific and engineering contexts.

Engineering and Precision Measurements

- Calibration of instruments: High-precision gravimeters measure tiny variations in gravity due to

altitude, density anomalies, or subsurface structures.

- Satellite technology: Precise knowledge of gravitational variations informs satellite orbit calculations, especially for low Earth orbit satellites.

Scientific Research and Geophysics