# Calculate The Probability That The Speed Of Oxygen Molecule Lies Between 99.5 And 100.5 Meter/sec At

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Understanding the behavior of oxygen molecules at the microscopic level is a fundamental aspect of statistical mechanics and thermodynamics. One common question that arises in this context is how to determine the probability that a molecule's speed falls within a specific range. For instance, calculating the probability that the speed of an oxygen molecule lies between 99.5 and 100.5 meters per second provides insights into molecular dynamics, diffusion processes, and kinetic theory. This article offers a comprehensive guide to calculating this probability, integrating theoretical principles, mathematical formulas, and practical examples.

## Understanding Molecular Speed Distribution: The Maxwell-Boltzmann Framework

Before delving into probability calculations, it is essential to understand how molecular speeds are distributed in a gas. The Maxwell-Boltzmann distribution describes the statistical distribution of speeds among molecules in an ideal gas at thermal equilibrium.

### Key Concepts of Maxwell-Boltzmann Distribution

- **Speed Distribution Function:** Provides the probability density of molecules having a specific speed v.
- **Mathematical Expression:** The distribution function for molecular speeds in three dimensions is given by:

$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{m v^2}{2 k T}}$$

- **Variables:**
  - v: Speed of the molecule
  - m: Mass of an oxygen molecule
  - k: Boltzmann constant

    - T: Absolute temperature in Kelvin

### Why Use the Maxwell-Boltzmann Distribution?

This distribution allows us to calculate the probability that a molecule's speed is within any specified range by integrating the distribution function over that range.

## Calculating the Probability for a Specific Speed Range

To find the probability that an oxygen molecule's speed lies between 99.5 and 100.5 m/sec, follow these steps:

### Step 1: Identify the Parameters

- **Molecular mass of oxygen ($O_2$):** Approximately 32 atomic mass units (amu).
- **Conversion to kg:** 1 amu ≈ $1.6605 \times 10^{-27}$ kg, so:
    - $m = 32 \times 1.6605 \times 10^{-27} \approx 5.3136 \times 10^{-26}$ kg.

- **Temperature (T):** Assume a typical temperature, e.g., 300 K, or specify based on your context.
- **Constants:** Boltzmann constant, $k = 1.3806 \times 10^{-23}$ J/K.

### Step 2: Write the Probability Density Function (PDF)

The Maxwell-Boltzmann speed distribution function is:

```
f(v) = 4π (m / 2πkT)^{3/2} v^2 e^{- (m v^2) / (2 k T)}
```

## Step 3: Calculate the Normalization Constant

Define:

$$A = 4\pi (m / 2\pi kT)^{3/2}$$

Calculate A:

1. Compute $(m / 2\pi kT)$:

$$m / (2\pi kT) = 5.3136 \times 10^{-26} / (2\pi \times 1.3806 \times 10^{-23} \times 300)$$

2. Raise the result to the power of 3/2:

$$A = 4\pi \times [ (m / 2\pi kT)^{3/2} ]$$

## Step 4: Integrate the Distribution Function Over the Range

The probability that v is between $v_1$ and $v_2$ is given by:

$$P(v_1 \leq v \leq v_2) = \int_{v_1}^{v_2} f(v)\, dv$$

Because the integral involves an exponential function multiplied by $v^2$, it is often solved using substitution or lookup tables.

## Step 5: Use the Cumulative Distribution Function (CDF)

The integration can be simplified by employing the known form of the Maxwell speed distribution's CDF, which involves the error function (erf):

$$F(v) = \mathrm{erf}( v / v_{mp} ) - (2/\sqrt{\pi}) (v / v_{mp}) e^{ - (v / v_{mp})^2 }$$

where $v_{mp}$ is the most probable speed:

$$v_{mp} = \sqrt{2 k T / m}$$

Thus, the probability between $v_1$ and $v_2$ is:

$$P = F(v_2) - F(v_1)$$

# Applying the Calculation: A Practical Example

Suppose a sample of oxygen molecules at 300 K, and we want to know the probability that their speeds are between 99.5 and 100.5 m/sec.

## Step 1: Calculate Most Probable Speed ($v_{mp}$)

Using:

$$v_{mp} = \sqrt{2 k T / m}$$

Plugging in the values:

$$v_{mp} = \sqrt{(2 \times 1.3806\times10^{-23} \times 300) / (5.3136\times10^{-26})}$$

Calculating numerator:

$$2 \times 1.3806\times10^{-23} \times 300 \approx 8.2836\times10^{-21}$$

Dividing:

$$8.2836\times10^{-21} / 5.3136\times10^{-26} \approx 1.559\times10^{5}$$

Taking the square root:

$$v_{mp} \approx \sqrt{1.559\times10^{5}} \approx 394.8\, \text{m/sec}$$

This indicates that the most probable speed for oxygen molecules at 300 K is approximately 395 m/sec.

## Step 2: Compute the Argument for the Error Function

Calculate:

$$x = v / v_{mp}$$

For $v_1$ = 99.5 m/sec:

$$x_1 = 99.5 / 394.8 \approx 0.252$$

For $v_2$ = 100.5 m/sec:

$$x_2 = 100.5 / 394.8 \approx 0.254$$

## Step 3: Evaluate the Error Function and Exponential Terms

Using standard error function tables or computational tools:

- $\mathrm{erf}(0.252) \approx 0.278$
- $\mathrm{erf}(0.254) \approx 0.2785$

Calculate the exponential terms:

$$e^{-x^2} \text{ for } x=0.252: e^{-(0.252)^2} \approx e^{-0.0635} \approx 0.9385$$
$$e^{-x^2} \text{ for } x=0.254: e^{-0.0645} \approx 0.9375$$

Now, compute F(v):

$$F(v) = \mathrm{erf}(x) - (2/\sqrt{\pi}) x\, e^{-x^2}$$

Calculate the coefficient:

$$(2/\sqrt{\pi}) \approx 1.128$$

For $v_1$:

$$F(99.5) \approx 0.278 - 1.128 \times 0.252 \times 0.9385 \approx 0.278 - 0.267 \approx 0.011$$

For $v_2$:

```
F(100.5) ≈ 0.2785 - 1.128 × 0.254 × 0.9375 ≈ 0.2785 - 0.268 ≈ 0.0115
```

### Step 4: Find the Probability

Subtract:

```
P ≈ 0.0115 - 0.011 = 0.0005
```

Thus, the probability that an oxygen molecule at 300 K has a speed between 99.5 and 100.5 m/sec

## Frequently Asked Questions

### What is the significance of calculating the probability that oxygen molecules have speeds between 99.5 and 100.5 m/sec?

It helps in understanding the distribution of molecular speeds in a gas, which is essential for studying kinetic theory and thermodynamic properties of oxygen at a given temperature.

### How do you model the distribution of oxygen molecule speeds for probability calculations?

The Maxwell-Boltzmann distribution models the speeds of molecules in a gas, allowing us to calculate the probability of finding molecules within a specific speed range.

### What steps are involved in calculating the probability that an oxygen molecule's speed lies between 99.5 and 100.5 m/sec?

First, determine the Maxwell-Boltzmann distribution parameters for oxygen at the given conditions, then integrate the probability density function between 99.5 and 100.5 m/sec to find the probability.

### Is it necessary to know the temperature when calculating this probability?

Yes, because the Maxwell-Boltzmann distribution depends on temperature, which influences the average and most probable speeds of molecules.

## Can this probability be approximated using standard normal distribution techniques?

Yes, for speeds close to the mean speed, the Maxwell-Boltzmann distribution can be approximated by a normal distribution, simplifying the probability calculation.

## What is the typical mean speed of an oxygen molecule at room temperature?

Approximately 470 m/sec, but the specific speed range of 99.5 to 100.5 m/sec is much lower, representing the tail of the distribution.

## Why is the probability of finding an oxygen molecule with speed exactly 100 m/sec very small?

Because molecular speeds follow a continuous distribution, the probability of an exact value is zero; instead, we calculate the probability within a range.

## How does the mass of the oxygen molecule affect the probability calculation?

The molecular mass influences the shape and scale of the Maxwell-Boltzmann distribution, affecting the probability of molecules having certain speeds.

## Can the probability that the molecule's speed lies within a narrow range be used to estimate reaction rates?

Yes, since reaction rates depend on molecular speeds, knowing the probability within specific speed ranges helps estimate how many molecules have sufficient energy to react.

## What assumptions are made when calculating the probability of oxygen molecules having speeds between 99.5 and 100.5 m/sec?

Assumptions include molecules obeying the Maxwell-Boltzmann distribution, the gas being ideal, and the temperature remaining constant during the calculation.

# Additional Resources

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Understanding the probabilistic behavior of molecules in gases is fundamental to statistical mechanics and thermodynamics. When analyzing the kinetic properties of oxygen molecules, a common question involves calculating the likelihood that a molecule's speed falls within a specific range. In this detailed review, we explore the theoretical framework, mathematical formulations, and practical approaches to determine the probability that an oxygen molecule's speed lies between 99.5 m/sec and 100.5 m/sec.

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# Introduction to Molecular Speed Distributions

At the microscopic level, the molecules of a gas such as oxygen ($O_2$) are in constant, random motion. Their speeds are not uniform but follow a probability distribution governed by the principles of statistical mechanics. The Maxwell-Boltzmann distribution describes the statistical distribution of molecular speeds in an ideal gas, providing the foundation for calculating probabilities of molecules having certain speeds.

Key points:

- Molecular speeds are inherently stochastic, characterized by a probability density function.
- The Maxwell-Boltzmann distribution gives the probability density function (PDF) of molecular speeds.
- The distribution depends on temperature, molecular mass, and fundamental constants.

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# The Maxwell-Boltzmann Speed Distribution

The Maxwell-Boltzmann distribution for the speed $v$ of molecules in an ideal gas is expressed as:

$$
f(v) = \left( \frac{m}{2 \pi k T} \right)^{3/2} 4 \pi v^2 \exp \left( - \frac{m v^2}{2 k T} \right)
$$

where:

- $v$ = molecular speed
- $m$ = mass of a molecule
- $k$ = Boltzmann's constant ($1.380649 \times 10^{-23} \, \text{J/K}$)
- $T$ = absolute temperature in Kelvin
- $f(v)$ = probability density function for speed $v$

This function is normalized such that:

$$
\int_0^{\infty} f(v) \, dv = 1
$$

The distribution is characterized by a peak at a most probable speed, and the shape depends heavily on temperature.

---

# Parameters Specific to Oxygen Molecules

To accurately calculate the probability for oxygen molecules, specific parameters are needed:

1. Molecular Mass of $O_2$:

Oxygen has an atomic mass of approximately 16 u; thus, for $O_2$:

$$
m_{O_2} = 2 \times 16 \, \text{u} = 32 \, \text{u}
$$

Since 1 atomic mass unit (u) = $1.660539 \times 10^{-27}$ kg,

$$
m_{O_2} = 32 \times 1.660539 \times 10^{-27} \approx 5.3137 \times 10^{-26} \, \text{kg}
$$

2. Temperature:

Assume room temperature (commonly 298 K) unless specified otherwise.

3. Constants:

- Boltzmann's constant: $1.380649 \times 10^{-23} \, \text{J/K}$

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# Calculating the Probability for Speeds Between 99.5 and 100.5 m/sec

The probability that an oxygen molecule's speed lies within a narrow range (here, 99.5 to 100.5 m/sec) involves integrating the probability density function over that interval:

$$
P = \int_{99.5}^{100.5} f(v) \, dv
$$

Given the narrow range, an approximation using the probability density function at the midpoint (100 m/sec) can be employed:

\[

P \approx f(100) \times (100.5 - 99.5) = f(100) \times 1
\]

or more precisely, for small intervals, the probability is approximately the value of the PDF at the midpoint times the width.

Step 1: Compute the Most Probable Speed

The most probable speed \( v_{mp} \) is given by:

\[
v_{mp} = \sqrt{\frac{2 k T}{m}}
\]

Plugging in the values:

\[
v_{mp} = \sqrt{\frac{2 \times 1.380649 \times 10^{-23} \times 298}{5.3137 \times 10^{-26}}}
\]

Calculations:

\[
\text{Numerator} = 2 \times 1.380649 \times 10^{-23} \times 298 \approx 8.227 \times 10^{-21}
\]

\[
v_{mp} = \sqrt{\frac{8.227 \times 10^{-21}}{5.3137 \times 10^{-26}}} \approx \sqrt{154,811} \approx 393.5 \, \text{m/sec}
\]

This indicates that at room temperature, the most probable speed for oxygen molecules is approximately 393.5 m/sec, which is substantially higher than 100 m/sec. Therefore, the probability that a molecule has a speed between 99.5 and 100.5 m/sec is expected to be very small, as the Maxwell-Boltzmann distribution peaks at around 393.5 m/sec.

Step 2: Express the PDF at \( v = 100 \, \text{m/sec} \)

The probability density function simplifies to:

\[
f(v) = 4 \pi \left( \frac{m}{2 \pi k T} \right)^{3/2} v^2 \exp \left( - \frac{m v^2}{2 k T} \right)
\]

Calculate the pre-factor:

\[
A = 4 \pi \left( \frac{m}{2 \pi k T} \right)^{3/2}
\]

Step-by-step:

1. Compute $\frac{m}{2 \pi k T}$:

$$
\frac{5.3137 \times 10^{-26}}{2 \pi \times 1.380649 \times 10^{-23} \times 298}
$$

Denominator:

$$
2 \pi \times 1.380649 \times 10^{-23} \times 298 \approx 2 \times 3.1416 \times 1.380649 \times 10^{-23} \times 298 \approx 2.589 \times 10^{-20}
$$

Numerator:

$$
5.3137 \times 10^{-26}
$$

So,

$$
\frac{m}{2 \pi k T} \approx \frac{5.3137 \times 10^{-26}}{2.589 \times 10^{-20}} \approx 2.052 \times 10^{-6}
$$

2. Raise to the $3/2$ power:

$$
(2.052 \times 10^{-6})^{3/2} = (2.052 \times 10^{-6})^{1.5} \approx 2.952 \times 10^{-9}
$$

3. Multiply by $4 \pi$:

$$
A = 4 \pi \times 2.952 \times 10^{-9} \approx 4 \times 3.1416 \times 2.952 \times 10^{-9} \approx 3.711 \times 10^{-8}
$$

4. Now, compute $v^2$ and the exponential term:

$$
v = 100\, \text{m/sec}
$$

$$
v^2 = 10,000
$$

Exponent:

$$
- \frac{m v^2}{2 k T} = - \frac{5.3137 \times 10^{-26} \times 10,000}{2 \times 1.380649 \times 10^{-23} \times 298}
$$

Calculations:

Numerator:

$$
5.3137 \times 10^{-26} \times 10,000 = 5.3137 \times 10^{-22}
$$

Denominator:

$$
2 \times 1.380649 \times 10^{-23} \times 298 \approx 8.227 \times 10^{-21}
$$

Exponential argument:

$$
- \frac{5.3137 \times 10^{-22}}{8.227 \times 10^{-21}} \approx -0.0646
$$

Exponent:

$$
\exp(-0.0646) \approx 0.9375
$$

Putting it all together:

$$
f(100) \approx 3.711 \times 10^{-8} \times 10,000 \times 0.9375 \approx 3.711 \times 10^{-8} \times 9,375 \approx 3.481 \times 10^{-4}
$$

Step 3: Approximate Probability

Since the interval width is 1 m/sec:

$$
P \approx f(100) \times 1 \approx 3.481 \times 10^{-4}
$$

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