Charge Of Uniform Density (80 NC/m3) Is Distributed Throughout A Hollow Cylindrical Region Formed By

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Understanding how electric charges distribute themselves within different geometrical regions is fundamental in electromagnetism. When charges are uniformly spread over a specific volume, they generate electric fields that depend heavily on the shape and size of the region. One notable configuration involves a hollow cylindrical region filled with a uniform charge density, which has numerous applications in electrical engineering, physics experiments, and the design of electromagnetic devices. In this article, we explore the case where a charge density of 80 NC/m3 is distributed throughout a hollow cylindrical region, detailing the physical setup, the mathematical approach to calculating electric fields, and the implications of such a distribution.

Physical Context and Significance of a Hollow Cylindrical Charge Distribution

A hollow cylindrical charge distribution refers to a three-dimensional region shaped like a pipe or tube, where the volume within the cylinder is empty, but the shell or the surface of the cylinder is filled with a uniform charge density. This setup is common in various physical systems, including:

- Capacitors with cylindrical geometry
- Electromagnetic shielding
- Particle accelerators and beam pipes
- Electrostatic experiments involving cylindrical geometries

Understanding the electric field generated by such a distribution is crucial for designing devices that rely on precise control of electric fields, such as sensors, field-effect transistors, and electrostatic lenses.

Defining the Hollow Cylindrical Region and Charge Density

Geometry of the Hollow Cylinder

Let's define the geometry of the hollow cylindrical region:

- Inner radius: \(r_{inner} \)

- Outer radius: r_{outer}
- Height of the cylinder: h

The region is essentially a cylindrical shell with uniform charge density.

Charge Density Specification

The charge density, denoted by ρ, is given as:

$$
\rho = 80\, \text{NC/m}^3
$$

This indicates that each cubic meter of the hollow cylindrical shell contains 80 coulombs of charge, uniformly distributed throughout its volume.

Mathematical Approach to Electric Field Calculation

Calculating the electric field generated by a hollow cylindrical charge distribution involves applying Gauss's law, which relates the electric flux through a closed surface to the enclosed charge.

Applying Gauss’s Law

Gauss's law states:

$$
\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}
$$

where:

- $\vec{E}$ is the electric field
- $d\vec{A}$ is the differential area element
- Q_{enc} is the total charge enclosed within the Gaussian surface
- ε_0 is the permittivity of free space ($8.854 \times 10^{-12} \text{F/m}$)

The symmetry of the problem simplifies the calculations, assuming the electric field is directed radially outward and depends only on the radial distance from the axis.

Choosing the Gaussian Surface

Depending on the position relative to the cylinder, different Gaussian surfaces are used:

1. Outside the cylinder ($r > r_{outer}$)
2. Within the shell ($r_{inner} < r < r_{outer}$)
3. Inside the hollow region ($r < r_{inner}$)

Each case requires calculating the enclosed charge Q_{enc} based on the geometry.

Electric Field Outside the Hollow Cylinder ($r > r_{outer}$)

For points outside the entire shell, the Gaussian surface is a cylinder coaxial with the charged shell, with radius r.

- Enclosed charge:

$$
Q_{enc} = \rho \times \text{Volume of the shell}
$$

$$
Q_{enc} = \rho \times \pi (r_{outer}^2 - r_{inner}^2) h
$$

- Electric field magnitude:

$$
E(r) = \frac{Q_{enc}}{2\pi r h \varepsilon_0}
$$

$$
E(r) = \frac{\rho \pi (r_{outer}^2 - r_{inner}^2) h}{2\pi r h \varepsilon_0} = \frac{\rho (r_{outer}^2 - r_{inner}^2)}{2 r \varepsilon_0}
$$

This expression indicates that beyond the shell, the field behaves as if all the charge were concentrated along the axis.

Electric Field Inside the Hollow Region ($r < r_{inner}$)

Within the hollow part of the cylinder—where there's no charge—the electric field is:

$$
E(r) = 0
$$

since no charge is enclosed within the Gaussian surface.

Electric Field Within the Shell ($r_{inner} < r < r_{outer}$)

In this region, the Gaussian surface encloses only a portion of the shell's charge:

- Enclosed charge:

$$
Q_{enc} = \rho \times \text{Volume of the cylindrical segment within radius } r
$$

$$
Q_{enc} = \rho \times \pi (r^2 - r_{inner}^2) h
$$

- Electric field magnitude:

$$
E(r) = \frac{Q_{enc}}{2\pi r h \varepsilon_0} = \frac{\rho \pi (r^2 - r_{inner}^2) h}{2\pi r h \varepsilon_0} = \frac{\rho (r^2 - r_{inner}^2)}{2 r \varepsilon_0}
$$

This shows the electric field increases linearly with r within the shell region.

Implications of the Charge Distribution

The uniform charge density within a hollow cylindrical shell produces a predictable electric field pattern that can be summarized as:

- Zero field inside the hollow region ($r < r_{inner}$)
- Increasing electric field within the shell ($r_{inner} < r < r_{outer}$)
- Decreasing field outside the shell ($r > r_{outer}$) following inverse proportionality to r

This behavior is key in understanding the electrostatics of cylindrical conductors and insulators, as well as in designing devices that rely on precise electric field control.

Applications and Practical Examples

Understanding the distribution of a uniform charge density in a hollow cylindrical region has wide-ranging applications:

- Designing cylindrical capacitors: Calculating the electric field between concentric cylinders helps

optimize capacitor performance.
- Electrostatic shielding: Cylindrical shells can shield sensitive equipment from external electric fields.
- Particle beam guidance: Hollow cylinders with specific charge distributions guide charged particles in accelerators.
- Sensor development: Electric field patterns generated by such distributions are used in sensors and detectors.

Conclusion

Distributing a charge of 80 NC/m3 throughout a hollow cylindrical region results in a well-defined electric field pattern that depends on the geometry and position relative to the cylinder. Through the application of Gauss's law, the electric field can be derived for various regions: inside the hollow, within the shell, and outside the shell. Such analyses are fundamental in the field of electromagnetism, informing the design of electrostatic devices and systems. Understanding these principles enables engineers and physicists to manipulate electric fields efficiently in various technological applications.

By mastering the concepts discussed in this article, one gains deeper insight into the behavior of electric fields in cylindrical geometries, a cornerstone in the study of electrostatics and electromagnetic theory.

Frequently Asked Questions

What is the significance of the uniform charge density (80 NC/m^3) in a hollow cylindrical region?

The uniform charge density indicates that the charge is evenly spread throughout the volume of the hollow cylindrical region, affecting the electric field distribution both inside and outside the cylinder based on Coulomb's law and Gauss's law.

How does the distribution of a uniform charge density in a hollow cylinder influence its electric field?

The electric field inside the hollow region depends on the enclosed charge; for a uniform charge density, the field is zero within the hollow space and varies with distance outside the cylinder, following principles derived from Gauss's law.

What are the key steps to calculate the electric field at a point outside or inside a hollow cylinder with uniform charge density?

The main steps involve determining the total enclosed charge using the volume charge density, applying Gauss's law with a suitable Gaussian surface (such as a coaxial cylinder), and then calculating the electric field based on the symmetry and enclosed charge.

How is the total charge distributed in a hollow cylindrical region with a given uniform charge density of 80 NC/m³?

The total charge is found by multiplying the charge density (80 NC/m³) by the volume of the hollow cylindrical shell, which is calculated from its inner and outer radii and height, resulting in a total charge that is uniformly spread throughout the shell's volume.

In what practical applications might understanding the charge distribution in a hollow cylinder with uniform density be important?

This understanding is crucial in designing capacitors, electromagnetic shielding, particle accelerators, and other devices where precise control of electric fields and charge distributions within cylindrical geometries is necessary.

Additional Resources

Charge Of Uniform Density (80 NC/m³) Is Distributed Throughout A Hollow Cylindrical Region Formed By

Understanding how a charge of uniform density (80 NC/m³) is distributed throughout a hollow cylindrical region provides insight into electrostatics, field calculations, and the behavior of charges in symmetrical environments. Such problems are fundamental in physics and engineering, especially in designing devices like capacitors, insulators, and shielding systems. This article offers a comprehensive guide to analyzing the distribution of charge within a hollow cylindrical geometry, breaking down the concepts, calculations, and principles involved.

Introduction to Charge Distribution in Cylindrical Geometries

When dealing with electrostatics, the distribution of charge is crucial in determining the electric field and potential at various points in space. A hollow cylindrical region with a uniform volume charge density is a classic problem that illustrates the application of Gauss's law, symmetry considerations, and cylindrical coordinate systems.

In our case, the charge density (ρ) is given as 80 NC/m³, meaning that in every cubic meter within the hollow cylindrical volume, there is an accumulated charge of 80 Coulombs.

Defining the Geometry and Charge Distribution

1. Description of the Hollow Cylindrical Region

- Inner radius (a): The radius of the hollow space.
- Outer radius (b): The outer boundary of the cylindrical shell.
- Height (h): The length of the cylinder along its axis.

- Charge distribution: Uniform, meaning the charge density is constant throughout the volume of the shell.

2. Why Consider a Hollow Cylinder?

Hollow cylinders are significant in various applications:

- Capacitors: where the charge resides on the surfaces or within the volume.
- Shielding: to contain or block electric fields.
- Waveguides: where electromagnetic waves propagate within cylindrical geometries.

Fundamental Principles and Tools for Analysis

1. Gauss's Law

Gauss's law relates the electric flux through a closed surface to the enclosed charge:

$$
\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{enc}}{\varepsilon_0}
$$

where:

- $\mathbf{E}$ is the electric field,
- $d\mathbf{A}$ is the differential area vector,
- Q_{enc} is the enclosed charge,
- ε_0 is the permittivity of free space ($8.854 \times 10^{-12} \, C^2/(N \cdot m^2)$).

2. Symmetry Considerations

- The problem exhibits cylindrical symmetry, implying the electric field depends only on the radial distance r from the axis.
- The field is directed radially outward if the charge is positive.
- The axial and angular components can be ignored or simplified based on symmetry.

3. Coordinate System

- Use cylindrical coordinates (r, θ, z) for simplicity, with the axis along the height of the cylinder.

Step-by-Step Analysis of the Charge Distribution

1. Calculating Total Charge in the Hollow Cylinder

Given the uniform volume charge density $\rho = 80\, NC/m^3$:

$$
Q_{total} = \rho \times V
$$

\]

where the volume \(V\) of the hollow cylindrical shell is:

\[
V = \pi h (b^2 - a^2)
\]

Thus,

\[
Q_{total} = 80 \times \pi h (b^2 - a^2)
\]

This total charge is spread uniformly throughout the volume, influencing the electric field at various regions.

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2. Electric Field Inside and Outside the Cylinder

The distribution's symmetry allows us to use Gauss's law to find the electric field at different points:

- Inside the hollow (for \(r < a\)): No enclosed charge, so the electric field is zero.
- Within the shell (for \(a < r < b\)): The enclosed charge is proportional to the volume within radius \(r\).
- Outside the shell (for \(r > b\)): The entire charge is enclosed, and the electric field behaves as if all charge were concentrated at the center.

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3. Deriving the Electric Field at a Radial Distance \(r\)

For \(a < r < b\):

The enclosed charge \(Q_{enc}\) within radius \(r\):

\[
Q_{enc}(r) = \rho \times V_{r}
\]

where

\[
V_{r} = \pi h (r^2 - a^2)
\]

Hence,

\[
Q_{enc}(r) = 80 \times \pi h (r^2 - a^2)

\]

Applying Gauss's law:

\[
E(r) \times 2\pi r h = \frac{Q_{enc}(r)}{\varepsilon_0}
\]

So,

\[
E(r) = \frac{Q_{enc}(r)}{2 \pi r h \varepsilon_0} = \frac{80 \times \pi h (r^2 - a^2)}{2 \pi r h \varepsilon_0}
\]

Simplify:

\[
E(r) = \frac{80 (r^2 - a^2)}{2 r \varepsilon_0}
\]

or,

\[
E(r) = \frac{40 (r^2 - a^2)}{r \varepsilon_0}
\]

This gives the electric field at any point within the shell.

---

4. Electric Field for \(r > b\):

The entire charge \(Q_{total}\) is enclosed:

\[
Q_{total} = 80 \pi h (b^2 - a^2)
\]

Applying Gauss's law:

\[
E(r) \times 2 \pi r h = \frac{Q_{total}}{\varepsilon_0}
\]

Thus,

\[
E(r) = \frac{80 \pi h (b^2 - a^2)}{2 \pi r h \varepsilon_0} = \frac{40 (b^2 - a^2)}{r \varepsilon_0}
\]

Visualizing the Charge Distribution and Electric Field

1. Charge Density Map

- The charge density is homogeneous throughout the volume.
- The total charge depends on the dimensions of the hollow structure.

2. Electric Field Profile

- Zero inside the hollow core ($r < a$)
- Increasing within the shell ($a < r < b$), depending on r
- Decreasing as $1/r$ outside the shell ($r > b$)

Practical Applications and Implications

1. Capacitor Design

- Understanding the charge distribution helps in optimizing the dielectric properties and electrode placement.
- Hollow cylindrical capacitors are common in high-voltage applications.

2. Shielding and Containment

- The distribution informs how electric fields behave around cylindrical conductors or insulators.

3. Material and Structural Considerations

- The uniform charge distribution affects mechanical stresses and potential breakdown points.

Summary and Key Takeaways

- A charge of uniform density ($80\ NC/m^3$) distributed within a hollow cylindrical shell leads to calculable electric fields based on symmetry and Gauss's law.
- The total charge is proportional to the volume of the shell, calculated as $\rho \pi h (b^2 - a^2)$.
- Electric field behavior varies across regions: zero inside the hollow core, increasing within the shell, and diminishing outside as $1/r$.
- Analyzing such systems aids in the design of electrostatic devices, understanding field interactions, and developing shielding mechanisms.

Final Remarks

Grasping the principles behind the charge of uniform density ($80\ NC/m^3$) distributed throughout a hollow cylindrical region is foundational in electrostatics. The symmetry simplifies complex calculations, enabling engineers and physicists to predict electric field behavior accurately. Whether designing capacitors, insulators, or electromagnetic shields, these concepts serve as essential tools in the scientific toolkit, illustrating the profound interplay between geometry, charge distribution, and electric fields in our physical world.

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