

Chegg A. Solve The Linear Systems By Applying Gaussian Elimination To The Augmented Matrix $\begin{bmatrix} 1 & 1 & 2 & 1 \end{bmatrix}$.

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When tackling systems of linear equations, one of the most effective methods is Gaussian elimination. This systematic procedure transforms a system into a form that makes it easy to find solutions. In particular, applying Gaussian elimination to an augmented matrix allows for a clear, step-by-step approach to solving linear systems. In this article, we will explore how to use Gaussian elimination on the augmented matrix $\begin{bmatrix} 1 & 1 & 2 & 1 \end{bmatrix}$, and how Chegg can assist students in mastering this technique.

Understanding the Linear System and Its Augmented Matrix

Before delving into the elimination process, it's vital to understand the structure of the linear system and how it is represented as an augmented matrix.

What is an Augmented Matrix?

An augmented matrix is a compact way to represent a system of linear equations. It combines the coefficients of the variables with the constants from the right-hand side of the equations into a single matrix. For example, for a system:

```
\[
\begin{cases}
a_{11}x + a_{12}y + a_{13}z = b_1 \\
a_{21}x + a_{22}y + a_{23}z = b_2 \\
a_{31}x + a_{32}y + a_{33}z = b_3
\end{cases}
\]
```

the augmented matrix is written as:

```
\[
\left[ \begin{array}{ccc|c}
a_{11} & a_{12} & a_{13} & b_1 \\
a_{21} & a_{22} & a_{23} & b_2 \\
a_{31} & a_{32} & a_{33} & b_3
\end{array} \right]
\]
```

\]

Given Matrix: $[1 \ 1 \ 2 \ | \ 1]$

The matrix $[1 \ 1 \ 2 \ | \ 1]$ can represent a single equation with three variables:

$$\begin{bmatrix} x + y + 2z = 1 \end{bmatrix}$$

In the context of solving a system, this matrix may be part of a larger augmented matrix or a simplified case to demonstrate Gaussian elimination.

Applying Gaussian Elimination to the Matrix $[1 \ 1 \ 2 \ | \ 1]$

Gaussian elimination involves performing row operations to convert the matrix into row-echelon form, and then back-substituting to find solutions.

Step 1: Write the Initial Augmented Matrix

The initial matrix is:

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \end{array} \right]$$

Since this matrix contains only one equation, the goal is to express the variables in terms of each other or find specific solutions if possible.

Step 2: Identify the Variables and Goals

Our goal is to solve for x , y , and z . Because there's only one equation, the system may have infinitely many solutions or none, depending on further equations. For demonstration purposes, suppose we are working with a larger system that includes this equation alongside others, or we want to analyze the solution set.

Step 3: Expand to a System of Equations

Assuming this is part of a larger system, the equation:

\]

$$x + y + 2z = 1$$

is one of the equations. For a complete solution, consider a system such as:

$$\begin{cases} x + y + 2z = 1 \\ \text{(additional equations)} \end{cases}$$

In the absence of others, this single equation defines a plane in three-dimensional space, representing infinitely many solutions.

Using Gaussian Elimination in Multi-Equation Systems

While the single equation $[1 \ 1 \ 2 \ | \ 1]$ demonstrates the basic idea, Gaussian elimination truly shines when solving larger systems.

Example: Solving a 3x3 System

Suppose we have the following system:

$$\begin{cases} x + y + 2z = 1 \\ 2x + 3y + 3z = 4 \\ x + 2y + 3z = 3 \end{cases}$$

The augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & 3 & 3 & 4 \\ 1 & 2 & 3 & 3 \end{array} \right]$$

Step 1: Make the leading coefficient of the first row 1 (already achieved).

Step 2: Eliminate the x -terms from rows 2 and 3.

- Subtract 2 times row 1 from row 2:

```
\[
R_2 \rightarrow R_2 - 2 R_1
\]
```

```
\[
(2 - 2 \times 1) = 0, \quad (3 - 2 \times 1) = 1, \quad (3 - 2 \times 2) =
-1, \quad (4 - 2 \times 1) = 2
\]
```

- Subtract row 1 from row 3:

```
\[
R_3 \rightarrow R_3 - R_1
\]
```

```
\[
(1 - 1) = 0, \quad (2 - 1) = 1, \quad (3 - 2) = 1, \quad (3 - 1) = 2
\]
```

Updated matrix:

```
\[
\left[\begin{array}{ccc|c}
1 & 1 & 2 & 1 \\
0 & 1 & -1 & 2 \\
0 & 1 & 1 & 2
\end{array}\right]
\]
```

Step 3: Eliminate the (y) -term from row 3.

- Subtract row 2 from row 3:

```
\[
R_3 \rightarrow R_3 - R_2
\]
```

```
\[
(1 - (-1)) = 2, \quad (2 - 2) = 0
\]
```

Updated matrix:

```
\[
\left[\begin{array}{ccc|c}
1 & 1 & 2 & 1 \\
0 & 1 & -1 & 2 \\
0 & 0 & 2 & 0
\end{array}\right]
\]
```

\]

Step 4: Back-substitution to find variables.

- From the third row:

\[

$$2z = 0 \rightarrow z = 0$$

\]

- From the second row:

\[

$$y - z = 2 \rightarrow y - 0 = 2 \rightarrow y = 2$$

\]

- From the first row:

\[

$$x + y + 2z = 1 \rightarrow x + 2 + 0 = 1 \rightarrow x = -1$$

\]

Solution:

\[

$$x = -1, \quad y = 2, \quad z = 0$$

\]

This example illustrates the power of Gaussian elimination in solving systems efficiently.

The Role of Chegg in Learning Gaussian Elimination

Chegg offers valuable resources for students learning about Gaussian elimination and solving linear systems.

Chegg Study and Step-by-Step Solutions

Chegg's platform provides detailed, step-by-step solutions to a wide range of math problems, including those involving Gaussian elimination. Students can input their matrices or systems and receive guided solutions that clarify each step.

Video Tutorials and Concept Explanations

Chegg also offers video tutorials that explain the principles behind Gaussian elimination, helping students visualize the process and understand why each step is performed.

Practice Problems and Quizzes

Practicing with Chegg's curated problems enables students to reinforce their understanding. These exercises often mirror real test questions, preparing learners for exams and assignments.

Benefits of Using Gaussian Elimination for Solving Linear Systems

Applying Gaussian elimination offers several advantages:

- **Systematic Approach:** Provides a clear, step-by-step method to solve complex systems.
- **Efficiency:** Reduces computational effort, especially with larger systems.
- **Foundation for Advanced Topics:** Serves as a basis for understanding matrix algebra, LU decomposition, and other advanced methods.
- **Accuracy:** Minimizes errors compared to ad hoc solving methods.

Conclusion

Solving linear systems by applying Gaussian elimination to the augmented matrix $[1 \ 1 \ 2 \ | \ 1]$ exemplifies a fundamental technique in linear algebra. Whether working with a single equation or multiple interconnected systems, Gaussian elimination provides a structured way to find solutions efficiently. For students seeking to master this technique, resources like Chegg are invaluable, offering comprehensive tutorials, step-by-step solutions, and practice problems. By understanding and practicing Gaussian elimination, learners develop critical problem-solving skills that are essential in mathematics, engineering, computer science, and many other disciplines.

Remember, the key to mastering Gaussian elimination is understanding

Frequently Asked Questions

What is Gaussian elimination and how is it used to solve linear systems?

Gaussian elimination is a systematic method for solving systems of linear equations by converting the augmented matrix into row echelon form and then back-substituting to find the solutions.

How do you apply Gaussian elimination to the augmented matrix $[1 \ 1 \ 2 \ | \ 1]$?

Since the matrix has only one row, the solution is directly derived by solving the equation $1x + 1y = 1$, which simplifies to $y = 1 - x$. Additional equations are needed for a complete system, but with this single row, you can express one variable in terms of the other.

What are the steps involved in solving the linear system represented by the augmented matrix $[1 \ 1 \ 2 \ | \ 1]$?

The steps include: 1) Write the augmented matrix. 2) Use row operations to reach row echelon form. 3) Solve for variables using back-substitution. In this case, with a single equation, you directly solve for one variable.

Can Gaussian elimination be applied to a single-row augmented matrix like $[1 \ 1 \ 2 \ | \ 1]$? Why or why not?

Yes, Gaussian elimination can be applied, but with a single equation, it simplifies to solving that equation directly. The method is more useful for larger systems with multiple equations.

What does the augmented matrix $[1 \ 1 \ 2 \ | \ 1]$ represent in terms of a system of equations?

It represents the equation $x + y = 1$ with an additional constant 2, which indicates that the system may be incomplete or needs clarification, but typically it corresponds to the linear equation $x + y = 1$.

How do you interpret the solution of the system from the augmented matrix $[1 \ 1 \ 2 \ | \ 1]$?

The solution is expressed as $y = 1 - x$, where x can be any real number, making this a system with infinitely many solutions along the line $y = 1 - x$.

What are common pitfalls when applying Gaussian elimination to small matrices like $[1 \ 1 \ 2 \ | \ 1]$?

Common pitfalls include misinterpreting the matrix as a complete system when it may not be, forgetting to perform row operations correctly, or attempting to apply elimination when only a single equation is present, which may not fully determine all variables.

How can Chegg A assist students in solving linear systems using Gaussian elimination?

Chegg A provides step-by-step solutions, explanations, and practice problems to help students understand and correctly apply Gaussian elimination to various systems of equations, including simple matrices like $[1 \ 1 \ 2 \ | \ 1]$.

Additional Resources

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Introduction: The Significance of Solving Linear Systems

Linear systems of equations are foundational to numerous scientific, engineering, and mathematical applications. They underpin everything from modeling physical phenomena to optimizing complex systems. As such, methods for solving these systems efficiently and accurately have become critical tools for students, researchers, and professionals alike. Among these methods, Gaussian elimination stands out due to its systematic approach and general applicability.

This article delves into a specific example of solving a linear system using Gaussian elimination, focusing on the augmented matrix represented by the entries "1 1 2 1." We will explore the theoretical underpinnings of Gaussian elimination, interpret the specific matrix, and provide a step-by-step analysis of how to solve the system. Furthermore, the discussion will include insights into the method's practical applications, advantages, and limitations.

Understanding the Augmented Matrix: Deciphering "1 1 2 1"

Before diving into the solution, it's crucial to understand what the matrix "1 1 2 1" signifies. Typically, in the context of solving linear systems, an augmented matrix combines the coefficients of the variables with the constants from the equations.

Interpreting the Matrix

Given the sequence "1 1 2 1," and considering standard notation, it most likely represents a single linear equation with two variables and a constant term, possibly formatted as:

```
\[
\begin{bmatrix}
1 & 1 & | & 2
\end{bmatrix}
\]
```

or, in equation form:

```
\[
x + y = 2
\]
```

However, the mention of "1 1 2 1" suggests perhaps a more complex interpretation, such as a 2x2 system, where the augmented matrix might be written as:

```
\[
\begin{bmatrix}
1 & 1 & | & 2 \\
\text{(second row)}
\end{bmatrix}
\]
```

or possibly a single row with four entries: 1, 1, 2, 1, which could represent a matrix with two rows, each with two coefficients and a constant.

Assumption for Clarity: Given the context, the most probable scenario is that the system involves two equations with two variables, and the augmented matrix is:

```
\[
\begin{bmatrix}
1 & 1 & | & 2 \\
\text{second row data}
\end{bmatrix}
\]
```

For illustration, let's assume the complete augmented matrix is:

```
\[
\begin{bmatrix}
1 & 1 & | & 2 \\
0 & 1 & | & 1
\end{bmatrix}
\]
```

which corresponds to the system:

```
\[
\begin{cases}
x + y = 2 \\
\quad \quad y = 1
\end{cases}
\]
```

The goal is to use Gaussian elimination to solve such a system systematically.

Overview of Gaussian Elimination: The Theoretical Framework

What Is Gaussian Elimination?

Gaussian elimination is a method for solving systems of linear equations by converting the system's augmented matrix into an upper triangular form, from which the solutions can be obtained through back-substitution. This process involves a series of elementary row operations that preserve the solution set.

Elementary Row Operations

The three types of elementary row operations used in Gaussian elimination are:

1. Swapping two rows.
2. Multiplying a row by a non-zero scalar.
3. Adding or subtracting a multiple of one row to/from another row.

These operations facilitate the elimination of variables systematically.

The Process Outline

1. Forward Elimination: Transform the augmented matrix into an upper triangular form. This involves zeroing out elements below the main diagonal.
2. Back Substitution: Starting from the last row, solve for variables and substitute back into the previous equations to find all unknowns.

This method is universally applicable to linear systems with unique solutions, infinite solutions, or no solutions.

Applying Gaussian Elimination to the System Represented by "1 1 2 1"

Step 1: Setting up the System

Assuming the augmented matrix for a 2x2 system is:

```
\[
\begin{bmatrix}
1 & 1 & | & 2 \\
0 & 1 & | & 1
\end{bmatrix}
\]
```

which corresponds to the equations:

```
\[
\begin{cases}
x + y = 2 \\
\quad \quad y = 1
\end{cases}
\]
```

Step 2: Forward Elimination

In this setup, the matrix is already in an upper triangular form because the second row has a zero in the first column, and the second row's leading coefficient is 1.

Step 3: Back Substitution

- From the second row: $y = 1$.
- Substitute $y = 1$ into the first equation:

```
\[
x + 1 = 2 \Rightarrow x = 1
\]
```

Thus, the solution to the system is:

```
\[
x = 1, \quad y = 1
\]
```

Alternative Scenario: A Different Matrix

Suppose the original augmented matrix is:

```
\[
\begin{bmatrix}
1 & 1 & | & 2 \\
1 & 2 & | & 3
\end{bmatrix}
\]
```

corresponding to:

$$\begin{cases} x + y = 2 \\ x + 2y = 3 \end{cases}$$

Applying Gaussian elimination:

- Use the first row to eliminate x from the second row:

Subtract row 1 from row 2:

$$(1 - 1)x + (2 - 1)y = 3 - 2 \rightarrow 0x + 1y = 1$$

which simplifies to:

$$y = 1$$

- Back-substitute into the first equation:

$$x + 1 = 2 \rightarrow x = 1$$

The solution remains $(x = 1, y = 1)$, confirming the method's robustness.

The Significance of Gaussian Elimination in Practical Applications

Computational Efficiency

Gaussian elimination is favored for its straightforward, algorithmic approach, making it suitable for computer implementation. It scales well with systems of larger size, especially when optimized with partial pivoting to reduce numerical errors.

Broader Applications

- Engineering: Solving circuit equations, structural analysis.
- Computer Graphics: Transformations and rendering calculations.
- Economics: Input-output models.
- Data Science: Least squares solutions in regression analysis.

Limitations and Challenges

Despite its strengths, Gaussian elimination has limitations:

- Numerical instability: When pivot elements are very small, errors can amplify.
- Singular systems: Systems with no solutions or infinitely many solutions require modifications like partial pivoting or alternative methods.
- Computational cost: For very large systems, more advanced methods like LU decomposition or iterative methods are preferred.

Critical Insights and Analysis

The Method's Elegance and Systematic Approach

Gaussian elimination's step-by-step procedure offers clarity and transparency in solving linear systems. Its systematic nature makes it accessible for educational purposes and implementable in computational algorithms.

The Role of Pivoting

In practice, partial pivoting—reordering equations to position the largest pivot element—improves numerical stability. This is especially critical when dealing with real-world data where coefficients may vary widely in magnitude.

Limitations in Large-Scale Problems

While effective for small to medium-sized systems, Gaussian elimination's computational cost grows cubically with the number of variables, prompting the development of more efficient algorithms for large matrices, such as iterative methods or sparse matrix techniques.

Theoretical Significance

From a theoretical standpoint, Gaussian elimination connects to matrix factorization techniques like LU decomposition, which enhance computational efficiency and stability. It also provides insights into matrix invertibility and rank.

Conclusion: The Enduring Relevance of Gaussian Elimination

The example of applying Gaussian elimination to the system represented by the matrix "1 1 2 1" underscores the method's simplicity, versatility, and importance in solving systems of linear equations. Whether for academic purposes, engineering calculations, or computational algorithms, Gaussian elimination remains a fundamental technique.

Its straightforward approach allows users to convert complex systems into manageable forms, leading to solutions that are critical across diverse

fields. As data and problem sizes grow, understanding the principles behind Gaussian elimination and its optimized variants continues to be essential for scientists, engineers, and mathematicians.

In essence, mastering Gaussian elimination not only equips practitioners with a powerful problem-solving tool but also deepens their understanding of linear algebra's core concepts, fostering analytical thinking and computational proficiency.

References

- Lay, David C. Linear Algebra and Its Applications. Addison-Wesley, 2012.
- Strang, Gilbert. Introduction to Linear Algebra. Wellesley-Cambridge Press, 2016.
- Anton, Howard, and Chris Rorres. Elementary Linear Algebra. Wiley, 2013.
- Chegg Study Resources and Tutorials on Gaussian Elimination.

Note: The specific input "1 1 2 1" was interpreted within a typical context of a linear system. For precise solutions, additional details about the system's equations or matrix form would be necessary.

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