Claire Completed The Steps To Solve The System Of Equations. 4 X Minus 2 Y = 4. 3 X + 3 Y = 21. Step

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When working through a system of equations, it's essential to follow a structured approach to find the values of the variables involved. Claire's methodical process to solve the given system demonstrates key algebraic strategies such as substitution, elimination, and simplification. This article provides an in-depth look at each step Claire took, explaining the rationale and methods used to arrive at the solution. Whether you're a student learning this process for the first time or someone looking to reinforce your understanding, this guide will clarify each stage of solving the system of equations:

- 4X - 2Y = 4
- 3X + 3Y = 21

Let's analyze how Claire approached and successfully solved this system.

Understanding the System of Equations

Equations Overview

The system consists of two linear equations:

1. 4X - 2Y = 4
2. 3X + 3Y = 21

These equations involve two variables, X and Y. The goal is to find the values of X and Y that satisfy both equations simultaneously.

Graphical Interpretation

Understanding these equations graphically can be helpful:
- Each equation represents a straight line on the coordinate plane.
- The solution to the system is the point(s) where these lines intersect.
- Since the equations are linear, there is typically one unique solution unless the lines are parallel or coincident.

Step 1: Simplify Each Equation (If Necessary)

Review and Simplify the Equations

Before solving, check if equations can be simplified for easier manipulation.

- For the first equation, $4X - 2Y = 4$, notice that all terms are divisible by 2:

$4X \div 2 = 2X$

$-2Y \div 2 = -Y$

$4 \div 2 = 2$

So, simplified, the first equation becomes:

$2X - Y = 2$

- The second equation, $3X + 3Y = 21$, can be simplified by dividing all terms by 3:

$3X \div 3 = X$

$3Y \div 3 = Y$

$21 \div 3 = 7$

Resulting in:

$X + Y = 7$

Advantages of Simplification

- Simplified equations are easier to work with.
- Reduces the risk of calculation errors.
- Clarifies the relationships between variables.

Step 2: Choose a Method to Solve the System

Common Methods

There are primarily three methods to solve systems of linear equations:

1. Substitution Method
2. Elimination Method
3. Graphical Method

Claire's Approach

In this case, Claire chose the substitution method because one of the equations was easily solved for a variable, making substitution straightforward.

Step 3: Solve One Equation for a Variable

Express Y in Terms of X from the Simplified Equation

From the simplified second equation:

$X + Y = 7$

Solve for Y:

$Y = 7 - X$

This expression allows Claire to substitute Y in the first equation.

Step 4: Substitute and Solve for X

Substitution Process

Recall the simplified first equation:

$2X - Y = 2$

Substitute $Y = 7 - X$ into this:

$2X - (7 - X) = 2$

Perform the Substitution

Distribute the negative sign:

$2X - 7 + X = 2$

Combine like terms:

$(2X + X) - 7 = 2$

$3X - 7 = 2$

Isolate X

Add 7 to both sides:

$3X = 9$

Divide both sides by 3:

$X = 3$

Claire successfully finds the value of X: $X = 3$.

Step 5: Find Y Using the Value of X

Substitute X Back into the Expression

Recall Y = 7 - X. Now, plug in X = 3:

$Y = 7 - 3$

$Y = 4$

Claire determines the value of Y: Y = 4.

Step 6: Verify the Solution

Substitute the Values into Both Equations

To ensure accuracy, substitute X = 3 and Y = 4 into the original equations:

- First original equation: $4X - 2Y = 4$

$4(3) - 2(4) = 12 - 8 = 4$ ✓

- Second original equation: $3X + 3Y = 21$

$3(3) + 3(4) = 9 + 12 = 21$ ✓

Both equations are satisfied, confirming the solution is correct.

Concluding the Solution

Final Answer

The solution to the system of equations is:

$X = 3, Y = 4$

Summary of the Steps Taken

- Simplified equations for easier manipulation.
- Chose the substitution method based on the form of the equations.
- Solved one equation for a variable.
- Substituted into the other to find the value of the remaining variable.
- Verified the solution by substituting back into original equations.

Additional Insights and Tips

- **Always check for simplification:** Simplifying equations reduces complexity and minimizes errors.
- **Choose the most straightforward method:** If one equation is easily solved for a variable, substitution is often preferable.
- **Verify your solutions:** Substituting the found values into original equations ensures correctness.
- **Practice different methods:** Understanding substitution, elimination, and graphical solutions enhances problem-solving flexibility.

Final Thoughts

Solving systems of equations is a fundamental skill in algebra that builds logical thinking and problem-solving capabilities. Claire's step-by-step approach exemplifies best practices—simplify where possible, select an effective method, carefully perform calculations, and verify results. Mastery of these techniques prepares you for more complex mathematical challenges and real-world applications involving multiple variables and equations.

Frequently Asked Questions

What is the first step Claire took to solve the system of equations 4X - 2Y

= 4 and 3X + 3Y = 21?

Claire likely chose to solve for one variable in one equation, such as solving for X or Y from one of the equations to substitute into the other.

How did Claire simplify the equations before solving for variables?

She probably simplified the equations by dividing or combining like terms to make the coefficients easier to work with.

What method did Claire use to solve the system: substitution, elimination, or graphing?

Based on the steps, Claire used the elimination method by aligning coefficients to cancel out one variable.

What is the value of X after Claire completed the steps?

After solving, Claire found that X = 3.

What is the value of Y after Claire completed the solution process?

She determined that Y = 1.

Why is it important to check the solution after solving the system?

Checking verifies that the solution satisfies both original equations, ensuring accuracy.

Can you explain the significance of the steps Claire completed in understanding systems of equations?

Yes, completing these steps demonstrates how to systematically find the point of intersection between two lines, which is fundamental in solving systems of equations.

What are common mistakes to avoid when solving systems of equations like Claire did?

Common mistakes include incorrect arithmetic, mixing up signs, or substituting incorrectly; careful step-by-step work and checking help prevent these errors.

Additional Resources

Comprehensive Review of Claire's Problem-Solving Approach to the System of Equations

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Introduction: Understanding the Significance of Solving Systems of Equations

Solving systems of equations is a fundamental skill in algebra that enables students and mathematicians to find common solutions that satisfy multiple conditions simultaneously. The ability to methodically approach such problems is essential for advanced mathematics, science, engineering, and real-world applications like economics and data analysis. When a student like Claire demonstrates mastery in solving a system of equations, it signifies a solid grasp of various algebraic techniques, logical reasoning, and problem-solving strategies.

The specific problem Claire tackled involves the system:

$$
\begin{cases}
4x - 2y = 4 \quad \text{(Equation 1)} \\
3x + 3y = 21 \quad \text{(Equation 2)}
\end{cases}
$$

This review delves into the step-by-step process Claire used, examines the methods applied, discusses alternative approaches, and explores the mathematical reasoning behind each step.

Step 1: Analyzing the Given System of Equations

Before solving, it's crucial to understand the structure and characteristics of the system.

Equation Forms and Coefficients

- Equation 1: $4x - 2y = 4$

- Equation 2: $3x + 3y = 21$

Notice the coefficients:

- For x: 4 in the first, 3 in the second.
- For y: -2 in the first, 3 in the second.

The coefficients indicate that the system can be approached via substitution, elimination, or graphing. Claire's choice of method reflects her understanding of the system's structure, which appears suitable for the elimination method due to the coefficients' manageable sizes and the potential for straightforward elimination.

Checking for Consistency and Dependence

Before proceeding, Claire might have checked if the system is consistent (has solutions) or inconsistent:

- Are the equations multiples of each other? No, because the ratios of coefficients differ.
- Do they represent parallel lines? Not necessarily; checking the slopes can clarify.

Calculating slopes:

- Equation 1 in slope-intercept form:

$$
4x - 2y = 4 \Rightarrow -2y = -4x + 4 \Rightarrow y = 2x - 2
$$

- Equation 2 in slope-intercept form:

$$
3x + 3y = 21 \Rightarrow 3y = -3x + 21 \Rightarrow y = -x + 7
$$

Since the slopes are 2 and -1, the lines are not parallel; they intersect at a unique point, indicating a single solution.

Step 2: Converting Equations to Slope-Intercept Form

Claire's next move was to rewrite both equations into the slope-intercept form $y = mx + b$, which simplifies the visualization and helps in applying substitution or elimination.

Equation 1 Conversion:

$$
4x - 2y = 4
$$

$$
-2y = -4x + 4
$$

$$
y = 2x - 2
$$

Equation 2 Conversion:

$$
3x + 3y = 21
$$

$$
3y = -3x + 21
$$

$$
y = -x + 7
$$

Outcome: Now, Claire has two explicit equations:

$$
\begin{cases}
y = 2x - 2 \\
y = -x + 7
\end{cases}
$$

This form makes it straightforward to find x and y by setting the right-hand sides equal, leading to the substitution method.

Step 3: Applying the Substitution Method

Claire's choice to use substitution is logical given the equations' forms. Here's a detailed breakdown of this method:

Equate the expressions for \(y \):

\[
2x - 2 = -x + 7
\]

Solve for \(x\):

- Add \(x\) to both sides:
\[
2x + x - 2 = 7
\]
\[
3x - 2 = 7
\]
- Add 2 to both sides:
\[
3x = 9
\]
- Divide both sides by 3:
\[
x = 3
\]

Result: \(x = 3 \)

Find \(y \) using one of the equations:

- Substitute \(x = 3\) into \(y = 2x - 2 \):
\[
y = 2(3) - 2 = 6 - 2 = 4
\]

Solution Pair: \((x, y) = (3, 4)\)

Claire’s application of substitution here is precise and efficient, leveraging the explicit forms of the equations for straightforward calculation.

Step 4: Verifying the Solution

Verification is a crucial step in problem-solving, ensuring that the solution satisfies both original equations.

Substitute $(3, 4)$ into Equation 1:

$$
4(3) - 2(4) = 12 - 8 = 4
$$

which matches the right side, 4.

Substitute $(3, 4)$ into Equation 2:

$$
3(3) + 3(4) = 9 + 12 = 21
$$

which matches the right side, 21.

Since both equations are satisfied, Claire confidently concludes that $(3, 4)$ is the solution.

Step 5: Exploring Alternative Solution Methods

While Claire's method was effective, it's beneficial to discuss alternative techniques to deepen understanding.

Elimination Method

- Multiply Equation 2 by 2 to align coefficients:

$$
2(3x + 3y) = 2 \times 21 \Rightarrow 6x + 6y = 42
$$

- Rewrite Equation 1:

$$
4x - 2y = 4
$$

- To eliminate y, manipulate the equations:
- Multiply Equation 1 by 3:

$$
12x - 6y = 12
$$

- Subtract the second from this:

$$
(12x - 6y) - (6x + 6y) = 12 - 42
$$

$$
12x - 6y - 6x - 6y = -30
$$

$$
6x - 12y = -30
$$

- But this approach might be more complex than necessary; alternatively, eliminating x or y directly by multiplying equations appropriately can be more straightforward.

- For example, multiply Equation 1 by 3:

$$
12x - 6y = 12
$$

- Multiply Equation 2 by 4:

$$
12x + 12y = 84
$$

- Subtract:

$$
(12x + 12y) - (12x - 6y) = 84 - 12
$$

$$
12x + 12y - 12x + 6y = 72
$$

$$
18y = 72
$$

$$
y = 4
$$

- Then substitute back into one of the original equations to find x.

Advantage of elimination: It’s systematic and often better suited for systems with coefficients that are easy to manipulate.

Graphical Method

Plotting the two equations:

- $y = 2x - 2$
- $y = -x + 7$

The intersection point visually confirms the solution $(3,4)$. This method is particularly useful for visual learners and when the system represents geometric entities.

Step 6: Analyzing the Solution's Uniqueness and Behavior

Given the distinct slopes, the system has a unique solution. Claire may have reflected on this:

- The lines are not parallel or coincident.
- The intersection point is a single solution, $(3, 4)$.

This understanding highlights the importance of analyzing the system's structure before solving, an essential skill in algebra.

Step 7: Reflection and Mathematical Reasoning

Claire's systematic approach demonstrates several important mathematical principles:

- Method selection: Choosing substitution based on explicit forms simplifies calculations.
- Verification: Always verifying solutions confirms correctness and builds confidence.
- Flexibility: Being aware of alternative methods (elimination, graphing) enhances problem-solving versatility.
- Understanding the system: Analyzing slopes and coefficients informs expectations about solutions.

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