Confirm That The Integral Test Can Be Applied To The Series. Then Use The Integral Test To Determine

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Understanding the convergence or divergence of infinite series is a fundamental topic in calculus, especially in the study of series. One of the most powerful tools available for this purpose is the Integral Test. Before applying the Integral Test, it is crucial to confirm that the series in question meets the necessary conditions for the test's validity. This article provides a comprehensive guide to confirming the applicability of the Integral Test, explaining the criteria involved, and demonstrates how to use the test to analyze the convergence of a series with detailed examples.

What Is the Integral Test?

The Integral Test is a method used to determine whether an infinite series converges or diverges by comparing it to a related improper integral. Specifically, it applies to series whose terms are positive, decreasing, and continuous functions over a certain interval.

Formal Statement of the Integral Test:

Suppose $f(n) = a_n$ is a continuous, positive, decreasing function for $x \geq N$, where N is a positive integer. Then, the series

$$
\sum_{n=N}^{\infty} a_n
$$

and the improper integral

$$
\int_{N}^{\infty} f(x) \, dx
$$

either both converge or both diverge.

Conditions for Applying the Integral Test

Before applying the Integral Test, verify that the series satisfies the following key conditions:

1. Positivity of the Terms

- All terms a_n in the series must be positive ($a_n > 0$) for sufficiently large n.
- This ensures the integral comparison is meaningful because the integral of a positive function provides an accurate basis for convergence analysis.

2. Monotonic Decrease of the Function

- The function $f(x) = a_x$ must be decreasing for all $x \geq N$.
- That is, $f'(x) \leq 0$ for $x \geq N$.
- Monotonicity guarantees that the integral comparison is valid because the function does not oscillate.

3. Continuity of the Function

- The function $f(x)$ should be continuous on the interval $[N, \infty)$.
- Continuity ensures that the integral is well-defined and can be evaluated or estimated accurately.

4. Proper Behavior at Infinity

- The improper integral $\int_{N}^{\infty} f(x) \, dx$ should be properly defined.
- For convergence, the improper integral must approach a finite value as the upper limit tends to infinity.

Steps to Confirm the Applicability of the Integral Test

To systematically confirm whether the Integral Test can be applied, follow these steps:

1. **Identify the general term a_n:** Extract the expression for the series term.

2. **Define the corresponding function $f(x)$:** Replace n with x to get a continuous function $f(x)$.

3. **Verify positivity:** Confirm $f(x) > 0$ for all $x \geq N$.

4. **Check for monotonicity:** Determine whether $f(x)$ is decreasing for $x \geq N$. You can do this by computing $f'(x)$.

5. **Ensure continuity:** Confirm $f(x)$ is continuous on $[N, \infty)$.

6. **Identify N:** Choose an appropriate N such that all conditions hold for $x \geq N$.

Once these conditions are satisfied, the Integral Test can be confidently applied to analyze the series.

Applying the Integral Test: Step-by-Step Example

Let's apply this process to a concrete example to illustrate how to confirm the conditions and then determine the series' convergence.

Example Series:

$$
\sum_{n=2}^{\infty} \frac{1}{n^p}
$$

where p is a real number.

Step 1: Identify a_n

$$
a_n = \frac{1}{n^p}
$$

Step 2: Define $f(x)$

$$
f(x) = \frac{1}{x^p}
$$

Note: $f(x)$ is defined for $x \geq 1$ (or $x \geq 2$ to match the series).

Step 3: Verify positivity

- For $x > 0$, $f(x) = \frac{1}{x^p} > 0$ as long as $x \neq 0$.
- Since $x \geq 2$, the function is positive.

Step 4: Check for monotonicity

- Compute the derivative:

$$
f'(x) = -p x^{-p-1}
$$

- For $x \geq 2$:

- If $p > 0$, then $f'(x) < 0$, indicating the function is decreasing.
- If $p \leq 0$, the function may not be decreasing; further analysis needed.

- Conclusion: For $p > 0$, $f(x)$ is decreasing on $[2, \infty)$.

Step 5: Confirm continuity

- $f(x)$ is continuous for $x > 0$.

Step 6: Choose N

- Since the conditions hold for $x \geq 2$, set $N=2$.

Using the Integral Test to Determine Convergence

Having confirmed the conditions, now evaluate the improper integral:

$$
\int_{2}^{\infty} \frac{1}{x^{p}} \, dx
$$

Evaluation:

- For $p \neq 1$:

$$
\int_{2}^{\infty} x^{-p} \, dx = \left[\frac{x^{-p+1}}{-p+1} \right]_2^{\infty}
$$

- Case 1: $p > 1$

- $-p + 1 < 0$, so as $x \to \infty$, $x^{-p+1} \to 0$.

- Therefore,

$$
\int_{2}^{\infty} \frac{1}{x^{p}} \, dx = \frac{2^{-p+1}}{p-1}
$$

which is finite.

- Case 2: $p \leq 1$

- When $p = 1$:

$$
\int_{2}^{\infty} \frac{1}{x} \, dx = \lim_{t \to \infty} \ln t - \ln 2 = \infty
$$

diverges to infinity.

- When $p < 1$:

$$
\int_{2}^{\infty} x^{-p} \, dx = \lim_{t \to \infty} \frac{t^{-p+1}}{-p+1}
$$

Since $-p + 1 > 0$, as $t \to \infty$, $t^{-p+1} \to \infty$, so the integral diverges.

Summary of Results:

p Value	Series Convergence	Explanation

| \(p > 1\) | Converges | The integral converges to a finite value. |
| \(p \leq 1\) | Diverges | The integral diverges to infinity. |

Conclusion: Series Convergence Based on the Integral Test

- The series \(\sum_{n=2}^{\infty} \frac{1}{n^p}\) converges if and only if \(p > 1\).
- It diverges for \(p \leq 1\).

This example clearly demonstrates how to confirm the applicability of the Integral Test and then use it to determine the convergence of a series.

Additional Tips for Applying the Integral Test

- Compare with known p-series: Many series with terms involving powers of \(n\) are directly comparable to p-series, which have well-understood convergence properties.
- Use substitution for complex integrals: When integrals involve more complicated functions, substitution or partial fractions can simplify evaluation.
- Be cautious with non-monotonic functions: The Integral Test requires the function to be decreasing; if not, consider other tests like the Comparison or Limit Comparison Test.

Summary

Applying the Integral Test effectively hinges on verifying key conditions: positivity

Frequently Asked Questions

What is the integral test and when can it be applied to a series?

The integral test states that if a function f(n) is positive, continuous, and decreasing for n ≥ N, then the infinite series Σ a_n and the improper integral ∫ f(x) dx either both converge or both diverge. It can be applied when these conditions are met for the terms of the series.

How do I verify if the integral test is applicable to a given series?

To verify applicability, check if the terms a_n are positive, continuous, and decreasing for $n \geq$ some N. If so, you can use the integral test to analyze the series' convergence.

Can the integral test be used for series with alternating signs?

No, the integral test is generally only applicable to series with positive terms that are decreasing. For alternating series, other tests like the Alternating Series Test are more appropriate.

What steps are involved in applying the integral test to determine if a series converges?

First, identify a suitable function $f(x)$ such that $a_n = f(n)$. Verify that $f(x)$ is positive, continuous, and decreasing for $x \geq N$. Then, evaluate the improper integral $\int_N^\infty f(x)\, dx$. If the integral converges, the series converges; if it diverges, the series diverges.

Can you give an example of applying the integral test to a series?

Yes. For example, consider the series $\Sigma\ 1/n^p$ with $p > 1$. We set $f(x) = 1/x^p$. Since $f(x)$ is positive, continuous, and decreasing for $x \geq 1$, we evaluate $\int_1^\infty 1/x^p\, dx$. The integral converges if $p > 1$, confirming the series converges for $p > 1$.

How does the integral test help in determining the convergence or divergence of a series?

The integral test relates the convergence of a series to the convergence of an associated improper integral. If the integral converges, the series converges; if the integral diverges, the series diverges, providing a straightforward way to analyze series behavior.

Are there limitations or cases where the integral test cannot be applied?

Yes. The integral test cannot be applied if the terms are not positive or not decreasing. Also, if the function $f(x)$ is not integrable over the interval or not continuous, the test is invalid.

Once the integral test indicates convergence, how can I determine the sum of the series?

The integral test only determines whether the series converges or diverges; it does not give the sum. To find the sum, other methods such as partial sums, telescoping, or known series formulas are needed.

Additional Resources

Confirm That The Integral Test Can Be Applied To The Series. Then Use The Integral Test To Determine

When analyzing the convergence or divergence of infinite series, mathematicians often turn to various tests to make their determination. One particularly powerful and elegant method is the Integral Test. Before applying the test, it is crucial to confirm that the conditions for its use are satisfied. This involves verifying the properties of the function associated with the series. Once confirmed, the Integral Test can then be employed to evaluate the series' behavior. This guide provides a comprehensive breakdown of how to confirm that the Integral Test can be applied, and demonstrates its use through a detailed example.

Understanding the Integral Test

The Integral Test offers a way to determine whether an infinite series converges or diverges by relating it to an improper integral. Specifically, for a series of the form:

$$ \sum_{n=1}^\infty a_n $$

where each term a_n is positive, decreasing, and continuous when viewed as a function $f(x)$ (with $a_n = f(n)$), the test states:

- The series converges if and only if the improper integral

$$ \int_{1}^\infty f(x) \, dx $$

converges.

- The series diverges if and only if this integral diverges.

Thus, the integral test provides a bridge between series and improper integrals, simplifying the analysis of convergence.

Step 1: Confirm the Conditions for the Integral Test

Before applying the Integral Test, it is essential to verify that the series' terms meet its criteria:

1. Positivity of Terms
- The series must have all positive terms: $a_n > 0$ for all $n \geq 1$.

2. Monotonic Decrease
- The sequence a_n must be decreasing: $a_{n+1} \leq a_n$ for all n.

3. Continuity of the Corresponding Function
- The function $f(x)$ associated with the series, where $a_n = f(n)$, should be continuous on the interval $[1, \infty)$.

Step 2: Confirm That the Function Meets the Conditions

Suppose you are given a specific series, such as:

$$ \sum_{n=1}^\infty \frac{1}{n^p} $$

where $p > 0$. To apply the Integral Test:

- Positivity: Since $\frac{1}{n^p} > 0$ for all n, the positivity condition is satisfied.
- Monotonic Decrease: For $p > 0$, $\frac{1}{n^p}$ is decreasing because as n increases, the denominator increases, decreasing the term.
- Continuity: The function $f(x) = \frac{1}{x^p}$ is continuous for $x \geq 1$.

If these conditions are satisfied, then the Integral Test can be confidently applied.

Step 3: Applying the Integral Test

Once the conditions are confirmed:

1. Set up the integral
Express the series in terms of $f(x)$, e.g., $f(x) = \frac{1}{x^p}$.

2. Compute the improper integral
Evaluate:

$$ \int_{1}^\infty f(x) \, dx $$

- Determine whether this integral converges (has a finite value) or diverges (approaches infinity).

3. Draw conclusions
- If the integral converges, then the series converges.
- If the integral diverges, then the series diverges.

Example: Applying the Integral Test to a Specific Series

Let's walk through a detailed example to illustrate this process.

Series to analyze:

$$ \sum_{n=1}^\infty \frac{1}{n^2} $$

Step 1: Confirm Conditions

- $a_n = \frac{1}{n^2}$

- Positivity: $\frac{1}{n^2} > 0$ for all $n \geq 1$.

- Decreasing: Since n^2 increases with n, $\frac{1}{n^2}$ decreases.

- Continuity of $f(x) = \frac{1}{x^2}$: Continuous for $x \geq 1$.

All conditions are satisfied, so the Integral Test applies.

Step 2: Set up the integral

Evaluate:

$$\int_{1}^\infty \frac{1}{x^2} \, dx$$

Step 3: Compute the integral

Calculate:

$$\int_{1}^{\infty} x^{-2} \, dx = \left[-\frac{1}{x} \right]_1^{\infty}$$

As $x \to \infty$:

$$-\frac{1}{x} \to 0$$

At $x=1$:

$$-\frac{1}{1} = -1$$

Thus,

$$\int_{1}^\infty \frac{1}{x^2} dx = 0 - (-1) = 1$$

Step 4: Conclusion

Since the improper integral converges to 1, the series converges by the Integral Test.

Additional Tips for Applying the Integral Test

- Check the function's behavior at infinity: Confirm that the integral's limits are from 1 to infinity, or adjust accordingly.
- Use substitution or known integral formulas: For more complex functions, substitution, partial fractions, or standard integral results can be helpful.
- Compare with known p-series: Series of the form $\sum 1/n^p$ are well-understood; the integral test confirms the well-known convergence criteria.

Summary

Applying the Integral Test involves a systematic process:

1. Verify the conditions:
- All terms are positive.
- The sequence a_n is decreasing.
- The associated function $f(x)$ is continuous on $[1, \infty)$.

2. Set up and evaluate the improper integral:
- Compute $\int_{1}^\infty f(x) \, dx$.

3. Interpret the integral's convergence:
- If the integral converges, the series converges.
- If the integral diverges, the series diverges.

This method provides a reliable and often straightforward way to analyze the behavior of many series, especially those resembling p-series or integrable functions with known properties.

Final Remarks

Confirming that the integral test can be applied to a series is a crucial step in the analysis process. By carefully checking the conditions and evaluating the corresponding improper integral, mathematicians can determine the convergence or divergence of infinite series with confidence. The integral test not only offers theoretical insight but also practical computational tools, making it a vital part of the calculus toolkit for series analysis.

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