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When working with analog signals in the realm of signal processing, one of the core concepts is sampling—converting a continuous-time signal into a discrete-time signal for digital analysis or processing. Understanding how to properly sample an analogue signal like s(t) = 6 Cos(40t) + 4 Sin(80t) is fundamental to ensuring accurate signal reconstruction and avoiding issues such as aliasing. In this article, we will explore the principles of sampling for this specific signal, analyze the implications of the sampling rate, and discuss the key concepts involved in digital signal processing.

Understanding the Analogue Signal s(t)

Before diving into the sampling process, it is important to understand the nature of the given analogue signal.

Signal Composition and Frequency Components

The signal s(t) = 6 Cos(40t) + 4 Sin(80t) comprises two sinusoidal components:

- 6 Cos(40t): A cosine wave with amplitude 6 and angular frequency 40 rad/sec.
- 4 Sin(80t): A sine wave with amplitude 4 and angular frequency 80 rad/sec.

The overall signal is a sum of these two sinusoidal signals, each with distinct frequencies. These frequencies determine how the signal behaves over time and influence the sampling rate required for accurate digitization.

Frequency Analysis and Bandwidth

The key to proper sampling lies in understanding the highest frequency component within s(t). The frequencies are:

- $f_1 = 40 / (2\pi) \approx 6.37$ Hz
- $f_2 = 80 / (2\pi) \approx 12.74$ Hz

These frequencies are derived from the angular frequencies by dividing by 2π. The maximum frequency component in s(t) is approximately 12.74 Hz, which is essential for determining the necessary sampling rate.

Sampling Theorem and Its Application

The Nyquist-Shannon Sampling Theorem is a foundational principle in signal processing, stating that a band-limited signal can be perfectly reconstructed from its samples if it is sampled at a rate greater than twice the highest frequency component.

Nyquist Rate for s(t)

Given the highest frequency component of approximately 12.74 Hz, the Nyquist rate (minimum sampling frequency, A) should satisfy:

- $A > 2 \times 12.74$ Hz ≈ 25.48 Hz

Sampling at a rate just above 25.48 Hz ensures that the sampled data contains enough information to reconstruct the original signal without aliasing.

Choosing an Appropriate Sampling Rate

In practice, to avoid aliasing and to facilitate easier filtering, engineers often select a sampling rate significantly higher than the Nyquist rate. For example:

- A common choice might be 50 Hz, 100 Hz, or even higher, depending on the application's fidelity requirements.

The specific choice depends on factors such as the signal's bandwidth, noise considerations, and the capabilities of the analog-to-digital converter (ADC).

Implications of Sampling Rate on Signal Reconstruction

The sampling rate impacts how accurately the original analogue signal can be reconstructed from its discrete samples.

Aliasing Phenomenon

Aliasing occurs when the sampling rate is too low, causing higher frequency components to appear as lower frequencies in the sampled data. This distortion can lead to:

- Misinterpretation of frequencies
- Distortion in the reconstructed signal

To prevent aliasing:

- Apply an anti-aliasing filter before sampling
- Ensure the sampling rate exceeds twice the maximum frequency component

Reconstruction of the Signal

Once sampled appropriately, the original continuous-time signal can be reconstructed using interpolation techniques such as:

- Sinc interpolation
- Reconstruction filters (ideal low-pass filters)

These methods rely on the Nyquist-Shannon sampling theorem ensuring perfect reconstruction provided the sampling criteria are met.

Practical Considerations in Sampling s(t)

While theoretical calculations provide a basis, real-world scenarios require attention to several practical factors.

Choosing the Sampling Rate

Based on the analysis:

- The minimum sampling rate should be greater than 25.48 Hz, typically rounded up to 50 Hz for safety margin.
- Higher sampling rates improve accuracy but increase data size and processing power requirements.

Anti-Aliasing Filters

Before sampling:

- Implement an analog low-pass filter to limit the bandwidth of the signal to below half of the sampling rate (Nyquist frequency).
- This prevents frequency components above the Nyquist frequency from folding back into the baseband.

Sampling Hardware and Implementation

Selecting appropriate hardware involves:

- High-quality ADCs with sufficient sampling speed
- Proper synchronization and calibration to minimize jitter and distortion

Conclusion: Ensuring Accurate Digital Representation of the Analogue Signal

Sampling an analogue signal such as s(t) = 6 Cos(40t) + 4 Sin(80t) requires careful consideration of the signal's frequency components, the Nyquist rate, and practical filtering techniques. By understanding the highest frequency component—approximately 12.74 Hz in this case—engineers can choose an appropriate sampling rate that ensures faithful digital representation and precise reconstruction. Proper implementation of anti-aliasing filters and selection of high-quality hardware further guarantee the integrity of the sampled data.

This process highlights the importance of the Nyquist-Shannon Sampling Theorem and practical signal processing techniques in modern electronics, communications, and digital systems. Whether for audio processing, telecommunications, or instrumentation, properly sampling and reconstructing signals is fundamental to achieving high fidelity and robust performance in digital systems.

By grasping these principles and applying them diligently, professionals can optimize their signal processing workflows, minimize errors, and ensure the reliability of their digital representations of analog signals.

Frequently Asked Questions

What is the frequency of the cosine component in the signal s(t) = 6 Cos(40t) + 4 Sin(80t)?

The frequency of the cosine component is 40 radians per second, which corresponds to approximately 6.37 Hz.

What is the frequency of the sine component in the signal s(t)?

The sine component has a frequency of 80 radians per second, approximately 12.73 Hz.

What is the Nyquist sampling rate required to accurately sample the signal s(t)?

The Nyquist rate must be at least twice the highest frequency component, so approximately 2 80 rad/sec / (2π) ≈ 25.46 Hz.

If the signal s(t) is sampled at a rate of 50 Hz, is it sufficient to avoid aliasing?

Yes, sampling at 50 Hz exceeds the Nyquist rate (~25.46 Hz), so it should prevent aliasing for this signal.

What is the significance of the frequencies 40t and 80t in the signal s(t)?

They represent the angular frequencies of the cosine and sine components, determining their oscillation rates over time.

How can one reconstruct the original continuous-time signal s(t) from its samples if sampled at rate A? (assuming A is above Nyquist rate)

Reconstruction can be achieved using ideal low-pass filtering (sinc interpolation) of the sampled data to recover the original signal accurately.

What potential issue arises if the sampling rate A is below the Nyquist rate for s(t)?

Sampling below the Nyquist rate causes aliasing, which distorts the signal and makes perfect reconstruction impossible.

What is the effect of increasing the sampling rate beyond the Nyquist rate for this signal?

Sampling at a higher rate than necessary improves the accuracy of digital representation and reduces potential reconstruction errors.

Additional Resources

Consider The Analogue Signal S(t): s(t) = 6 Cos(40t) + 4 Sin(80t) as a Case Study in Signal Sampling and Analysis

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Introduction

In the realm of signal processing, understanding how continuous-time signals are sampled, reconstructed, and analyzed is fundamental. The given analogue signal:

$$s(t) = 6 \cos(40t) + 4 \sin(80t)$$

serves as an excellent example to explore these concepts. This signal comprises two harmonic components at different frequencies, each with specific amplitudes and phases, making it an ideal candidate to examine the principles of sampling theory, frequency content, and potential issues such as aliasing.

This article aims to provide a comprehensive, detailed analysis of the signal $s(t)$, focusing on its properties, the implications of sampling it at a particular rate A, and the broader significance within the domain of digital signal processing (DSP). We will systematically delve into the mathematical characteristics of the signal, sampling criteria, the Nyquist theorem, and practical considerations for real-world implementations.

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1. Dissecting the Signal $s(t)$: Mathematical and Spectral Perspectives

1.1. Understanding the Composition

The signal:

$$s(t) = 6 \cos(40t) + 4 \sin(80t)$$

is a superposition of two sinusoidal functions. Each component can be analyzed individually:

- First component: $6 \cos(40t)$
- Second component: $4 \sin(80t)$

The frequencies of these components are derived from their arguments:

- For $\cos(40t)$, the angular frequency is $\omega_1 = 40$ rad/sec.
- For $\sin(80t)$, the angular frequency is $\omega_2 = 80$ rad/sec.

In the context of signal processing, the angular frequency ω relates to the regular frequency f (in Hz) via:

$$f = \frac{\omega}{2\pi}$$

Thus:

- $f_1 = \frac{40}{2\pi} \approx 6.366 \text{ Hz}$
- $f_2 = \frac{80}{2\pi} \approx 12.732 \text{ Hz}$

The signal contains two discrete frequency components at approximately 6.37 Hz and 12.73 Hz, with their respective amplitudes.

1.2. Spectral Analysis

The spectrum of $s(t)$ reveals the frequency domain characteristics:

- Spectral peaks at: $\pm 6.37 \text{ Hz}$ and $\pm 12.73 \text{ Hz}$
- Amplitudes: 6 and 4 for their respective components
- Phases: For simplicity, the phase shifts are zero; the cosine and sine functions are in their standard forms.

This spectral composition indicates that the signal is band-limited, with energy concentrated at these two frequencies. Such understanding is crucial when considering sampling, as the sampling theorem hinges upon the highest frequency component.

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2. The Sampling Process: How and Why?

2.1. Sampling Fundamentals

Sampling involves converting a continuous-time signal $s(t)$ into a discrete-time sequence $s[n] = s(nT)$, where T is the sampling period, and $f_s = \frac{1}{T}$ is the sampling frequency.

The core purpose of sampling is to enable digital processing of analog signals, which requires digitization through analog-to-digital conversion (ADC). The challenge lies in choosing an appropriate sampling rate to preserve the original signal's information without distortion.

2.2. The Critical Role of the Sampling Rate (A)

In the problem statement, the signal is sampled at a rate A. Although the exact value of A isn't specified here, the analysis revolves around understanding the implications of various sampling rates relative to the signal's frequency content.

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3. Sampling Theorem and Its Application to $s(t)$

3.1. Nyquist-Shannon Sampling Theorem

The cornerstone of sampling theory is the Nyquist-Shannon sampling theorem, which states:

> To perfectly reconstruct a band-limited signal, it must be sampled at a rate f_s greater than twice its highest frequency component:

$$ f_s > 2f_{max} $$

where f_{max} is the maximum frequency present in the signal.

Applying this to $s(t)$:

- Highest frequency component: $f_2 \approx 12.73 \text{ Hz}$
- Therefore, the minimum sampling rate:

$$ f_{min} = 2 \times 12.73 \approx 25.46 \text{ Hz} $$

Any sampling rate A below this threshold risks aliasing, which distorts the reconstructed signal.

3.2. Implications for Sampling Rate A

- If $A > 25.46 \text{ Hz}$: The sampling rate is sufficient to preserve the original signal perfectly.
- If $A = 25.46 \text{ Hz}$: The sampling is at the Nyquist limit. Reconstruction is theoretically possible but practically fragile.
- If $A < 25.46 \text{ Hz}$: Aliasing occurs, leading to overlapping spectral components and distortion.

These considerations emphasize the importance of selecting an appropriate sampling rate, especially in systems where accuracy is paramount.

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4. Aliasing: The Pitfall of Insufficient Sampling

4.1. Understanding Aliasing

Aliasing occurs when higher frequency components in the original signal are indistinguishable from lower frequency components after sampling. This happens if the sampling rate does not satisfy the Nyquist criterion.

Mathematically, the aliasing frequency f_a for a frequency component f when sampled at f_s is:

$$f_a = |f - n f_s| \quad \text{for some integer } n$$

This means a component at a frequency f above half the sampling rate appears as a lower frequency f_a in the sampled data, potentially altering the interpreted signal.

4.2. Visualizing Aliasing in $s(t)$

Suppose the sampling rate A is below 25.46 Hz, say 20 Hz:

- $f_{max} = 12.73 \text{ Hz}$
- Nyquist frequency: 10 Hz

The component at 12.73 Hz exceeds the Nyquist frequency, leading to aliasing:

$$f_{alias} = |f - n f_s| = |12.73 - 1 \times 20| = 7.27 \text{ Hz}$$

This aliasing causes the higher frequency component to appear at 7.27 Hz, misrepresenting the original signal and potentially leading to incorrect analysis or reconstruction.

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5. Practical Considerations in Sampling and Signal Reconstruction

5.1. Choosing an Appropriate Sampling Rate

In real-world applications, engineers often select a sampling rate significantly higher than the minimum Nyquist rate to account for:

- Filter roll-off (non-ideal filters)
- Hardware limitations
- Noise and distortion

For $s(t)$, a safe choice might be:

$$f_s \approx 2 \times f_{max} \times \text{margin factor}$$

For example, choosing $f_s \approx 50 \text{ Hz}$ ensures ample margin and minimal aliasing risk.

5.2. Reconstruction of the Signal

Assuming an appropriate sampling rate, the original $s(t)$ can be reconstructed using ideal low-pass filtering techniques (sinc interpolation). The reconstructed signal:

$$\hat{s}(t) = \sum_{n=-\infty}^{\infty} s(nT) \cdot \text{sinc}\left(\frac{t - nT}{T}\right)$$

where $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.

This process requires precise filtering to eliminate spectral images introduced during sampling.

5.3. Real-World Limitations

In practice, ideal filters do not exist, and hardware imperfections introduce errors. Nonetheless, understanding the underlying theory guides the design of systems that minimize distortion.

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6. Broader Implications and Applications

6.1. Signal Processing in Communications

The principles discussed are fundamental in designing communication systems, where signals often comprise multiple frequency components. Adequate sampling ensures data fidelity, efficient bandwidth utilization, and minimal error rates.

6.2. Audio and Speech Processing

In audio engineering, sampling rates like 44.1 kHz (standard for CDs) are chosen to capture the audible spectrum (roughly 20 Hz to 20 kHz). The concepts apply similarly to the $s(t)$ example but scaled for higher frequencies.

6.3. Medical Imaging and Remote Sensing

Techniques like MRI and radar rely heavily on proper sampling to reconstruct images and signals accurately, emphasizing the universal importance of sampling theory.

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7. Summary and Conclusions

The analysis of the analogue signal \(s(t

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