Consider The DE (1+ye ^xy)dx+(2y+xe ^xy)dy=0, Then The DE Is ,F_X =, Hence (x,y)= And G (y)= _____

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Introduction

Differential equations (DEs) are fundamental in mathematical modeling, describing phenomena across physics, engineering, biology, and economics. They serve as essential tools to understand how quantities change concerning each other. Among the various types of differential equations, first-order differential equations are particularly significant due to their broad applications and relatively manageable solution methods.

In this article, we delve into a specific first-order differential equation:

$$
(1 + y e^{xy})\, dx + (2 y + x e^{xy})\, dy = 0
$$

This particular equation encapsulates interesting features, including potential for exactness, integrating factors, and substitution strategies. Our goal is to analyze this DE thoroughly, determine the functions F_x and F_y, identify the solution family (x, y), and find the auxiliary function $G(y)$.

Understanding the structure and solution process for such equations not only enhances mathematical proficiency but also broadens the toolkit for tackling real-world problems involving differential relationships.

Understanding the Differential Equation Structure

The given differential equation:

\[

$$
(1 + y e^{xy})\, dx + (2 y + x e^{xy})\, dy = 0
$$

is expressed in differential form:

$$
M(x, y)\, dx + N(x, y)\, dy = 0
$$

where

$$
M(x, y) = 1 + y e^{xy}
$$
$$
N(x, y) = 2 y + x e^{xy}
$$

Our first step is to analyze whether this DE is exact, which is often the quickest route to a solution.

Checking for Exactness

A differential equation of the form $M\, dx + N\, dy = 0$ is exact if

$$
\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}
$$

Let's compute these derivatives.

Partial derivatives:

$$
\frac{\partial M}{\partial y} = \frac{\partial}{\partial y} \left(1 + y e^{xy} \right) = 0 + \frac{\partial}{\partial y} \left(y e^{xy} \right)
$$

Applying the product rule:

$$
\frac{\partial}{\partial y} \left(y e^{xy} \right) = e^{xy} + y \frac{\partial}{\partial y} (e^{xy})
$$

Since e^{xy} depends on y:

$$
\frac{\partial}{\partial y} (e^{xy}) = e^{xy} \cdot x
$$

Therefore,

$$
\frac{\partial M}{\partial y} = e^{xy} + y \cdot e^{xy} \cdot x = e^{xy} (1 + xy)
$$

Similarly, compute $\frac{\partial N}{\partial x}$:

$$
\frac{\partial N}{\partial x} = \frac{\partial}{\partial x} \left(2 y + x e^{xy} \right) = 0 + \frac{\partial}{\partial x} \left(x e^{xy} \right)
$$

Applying product rule:

$$
e^{xy} + x \cdot \frac{\partial}{\partial x} (e^{xy})
$$

Since

$$
\frac{\partial}{\partial x} (e^{xy}) = e^{xy} \cdot y
$$

we get:

$$
\frac{\partial N}{\partial x} = e^{xy} + x \cdot y e^{xy} = e^{xy} (1 + xy)
$$

Conclusion on Exactness:

$$
\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = e^{xy} (1 + xy)
$$

Thus, the differential equation is exact.

Finding the Potential Function \(F(x, y)\)

Since the DE is exact, there exists a potential function \(F(x, y)\) such that:

\[
\frac{\partial F}{\partial x} = M(x, y) = 1 + y e^{xy}
\]
\[
\frac{\partial F}{\partial y} = N(x, y) = 2 y + x e^{xy}
\]

Our goal is to find \(F(x, y)\), then the general solution will be \(F(x, y) = C \).

Integrate \(M(x, y)\) with respect to \(x\):

\[
F(x, y) = \int M(x, y)\, dx + h(y)
\]

\[
F(x, y) = \int \left(1 + y e^{xy} \right) dx + h(y)
\]

Compute \(\int 1\, dx\):

\[
\int 1\, dx = x
\]

Compute \(\int y e^{xy}\, dx\):

Note that \(y \) is treated as a constant during integration with respect to \(x \). So:

\[

\int y e^{xy}\, dx
\]

Let’s set:

\[
u = xy \Rightarrow du = y\, dx \Rightarrow dx = \frac{du}{y}
\]

Then:

\[
\int y e^{xy}\, dx = y \int e^{u} \cdot \frac{du}{y} = \int e^{u}\, du = e^{u} + C = e^{xy}
\]

Therefore,

\[
F(x, y) = x + e^{xy} + h(y)
\]

Determine \(h(y)\) by differentiating \(F(x, y)\) with respect to \(y \)

\[
\frac{\partial F}{\partial y} = \frac{\partial}{\partial y} \left(x + e^{xy} + h(y) \right) = 0 + \frac{\partial}{\partial y} (e^{xy}) + h'(y)
\]

Calculate \(\frac{\partial}{\partial y} (e^{xy})\):

\[
e^{xy} \cdot x
\]

Thus,

\[
\frac{\partial F}{\partial y} = x e^{xy} + h'(y)
\]

But earlier, we have:

\[
\frac{\partial F}{\partial y} = N(x, y) = 2 y + x e^{xy}

\]

Set these equal:

\[
x e^{xy} + h'(y) = 2 y + x e^{xy}
\]

Subtract \(x e^{xy}\) from both sides:

\[
h'(y) = 2 y
\]

Integrate with respect to \(y \):

\[
h(y) = \int 2 y\, dy = y^{2} + C
\]

We can ignore the constant \(C \) in the potential function, since it will be absorbed into the general solution.

Final Form of the Potential Function

\[
F(x, y) = x + e^{xy} + y^{2}
\]

The general solution to the differential equation is:

\[
F(x, y) = \text{constant}
\]
or

\[
x + e^{xy} + y^{2} = C
\]

Expressing the Solution and Function \(G(y)\)

The implicit solution is:

$$
x + e^{xy} + y^{2} = C
$$

which relates x and y. To explicitly express x as a function of y, rearrangement might be complicated; however, the implicit form suffices for many applications.

In some contexts, it's helpful to define functions based on this relation:

- The solution family (x, y) is defined by the implicit relation:

$$
(x, y) \quad \text{such that} \quad x + e^{xy} + y^{2} = C
$$

- The function $G(y)$ can be viewed as a particular solution when solving for x:

$$
G(y) = \text{a function satisfying} \quad x = G(y)
$$

Given the implicit relationship, one could, for specific C, write:

$$
x = C - y^{2} - e^{xy}
$$

which generally requires numerical methods for explicit solutions.

Summary of Key Results

- Exactness: The DE is exact because

$$
\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = e^{xy} (1 + xy)
$$

- Potential Function $F(x, y)$:

$$
F(x, y) = x + e^{xy
$$

Frequently Asked Questions

What is the given differential equation in the problem?

The differential equation is $(1 + y e^{xy}) dx + (2y + x e^{xy}) dy = 0$.

How do we determine if the differential equation is exact?

By checking if the partial derivative of M with respect to y equals the partial derivative of N with respect to x, where $M = 1 + y e^{xy}$ and $N = 2y + x e^{xy}$.

What is the value of F_x for the given differential equation?

F_x is the partial derivative of the potential function $F(x,y)$ with respect to x, which can be found by integrating $M = 1 + y e^{xy}$ with respect to x.

How do we find the function G(y) in the solution?

G(y) is determined by integrating the partial derivative of F with respect to y, which is $N = 2y + x e^{xy}$, after integrating M with respect to x.

What is the general solution form for the differential equation?

The general solution is $F(x,y) = C$, where $F(x,y)$ is a potential function satisfying $F_x = M$ and $F_y = N$.

What are the key steps to solve this differential equation?

First, verify if it's exact; second, find F_x by integrating M w.r.t. x; third, determine G(y) by integrating the corresponding partial derivative; and finally, write the general solution as $F(x,y) = \text{constant}$.

Additional Resources

Consider The DE $(1 + y e^{xy}) dx + (2y + x e^{xy}) dy = 0$: A Step-by-Step Guide to Solving the Differential Equation

Understanding and solving differential equations is a fundamental aspect of advanced mathematics, engineering, and physics. In particular, the differential equation $(1 + y e^{xy}) dx + (2y + x e^{xy}) dy = 0$ presents an interesting case that involves exponential functions and variable coefficients. This guide aims to provide a comprehensive, step-by-step approach to analyzing, solving, and interpreting this differential equation, including identifying potential integrating factors, exactness, and solutions.

Introduction to the Differential Equation

The differential equation under consideration is:

$$
(1 + y e^{xy})\, dx + (2y + x e^{xy})\, dy = 0
$$

This is a first-order differential equation, and it appears to involve nonlinear terms due to the exponential e^{xy}, as well as polynomial components. The goal is to determine whether this differential equation is exact or can be made exact via an integrating factor, and then to find its general solution.

Step 1: Recognizing the Structure

Before attempting to solve, analyze the structure:

- The equation is in the form $M(x, y) dx + N(x, y) dy = 0$, where:

$$
M(x, y) = 1 + y e^{xy}
$$
$$
N(x, y) = 2y + x e^{xy}
$$

- To determine if the differential equation is exact, check the partial derivatives:

$$
\frac{\partial M}{\partial y} \quad \text{and} \quad \frac{\partial N}{\partial x}
$$

Step 2: Checking for Exactness

Computing Partial Derivatives:

1. $\frac{\partial M}{\partial y}$:

$$
\frac{\partial}{\partial y} (1 + y e^{xy}) = e^{xy} + y \frac{\partial}{\partial y} e^{xy}
$$

Recall that:

$$
\frac{\partial}{\partial y} e^{xy} = x e^{xy}
$$

Thus:

$$
\frac{\partial M}{\partial y} = e^{xy} + y \cdot x e^{xy} = e^{xy} (1 + xy)
$$

2. $\frac{\partial N}{\partial x}$:

$$
\frac{\partial}{\partial x} (2y + x e^{xy}) = 0 + e^{xy} + x \frac{\partial}{\partial x} e^{xy}
$$

Again,

$$
\frac{\partial}{\partial x} e^{xy} = y e^{xy}
$$

So:

$$
\frac{\partial N}{\partial x} = e^{xy} + x y e^{xy} = e^{xy} (1 + xy)
$$

Observing the results:

$$
\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = e^{xy} (1 + xy)
$$

Conclusion: The partial derivatives are equal, which implies that the differential equation is exact.

Step 3: Finding the Potential Function $F(x, y)$

Since the differential equation is exact, there exists a function $F(x, y)$ such that:

$$
\frac{\partial F}{\partial x} = M(x, y) = 1 + y e^{xy}
$$
$$
\frac{\partial F}{\partial y} = N(x, y) = 2y + x e^{xy}
$$

Integrate $M(x, y)$ with respect to x:

$$
F(x, y) = \int (1 + y e^{xy}) dx + h(y)
$$

Note that $h(y)$ is an arbitrary function of y because the integration is with respect to x.

Step 4: Computing $F(x, y)$

Integrate term-by-term:

1. $\int 1\, dx = x$

2. $\int y e^{xy} dx$:

Let’s focus on this integral:

$$
I = y \int e^{xy} dx
$$

Since y is treated as a constant during integration with respect to x, and:

$$
\int e^{xy} dx
$$

Let $u = xy \Rightarrow du/dx = y$, so:

$$
dx = du / y
$$

Thus:

\[

$$
I = y \int e^{u} \frac{du}{y} = \int e^{u} du = e^{u} + C = e^{xy}
$$

Therefore:

$$
F(x, y) = x + e^{xy} + h(y)
$$

Step 5: Determine $h(y)$ by Differentiation

Differentiate $F(x, y)$ with respect to y:

$$
\frac{\partial F}{\partial y} = \frac{\partial}{\partial y} \left(x + e^{xy} + h(y) \right) = 0 + x e^{xy} + h'(y)
$$

Recall that:

$$
\frac{\partial F}{\partial y} = N(x, y) = 2y + x e^{xy}
$$

Set equal:

$$
x e^{xy} + h'(y) = 2y + x e^{xy}
$$

Subtract $x e^{xy}$ from both sides:

$$
h'(y) = 2y
$$

Integrate:

$$
h(y) = \int 2y\, dy = y^2 + C
$$

We can ignore the constant C because it will be absorbed in the constant of integration of the solution.

Step 6: Write the General Solution

The potential function $F(x, y)$ is:

$$
F(x, y) = x + e^{xy} + y^2
$$

The solution to the differential equation is given by setting $F(x, y) = \text{constant}$:

$$
x + e^{xy} + y^2 = C
$$

where C is an arbitrary constant.

Summary:

- The differential equation $(1 + y e^{xy}) dx + (2y + x e^{xy}) dy = 0$ is exact.
- The potential function $F(x, y)$ is:

$$
F(x, y) = x + e^{xy} + y^2
$$

- The general solution is:

$$
x + e^{xy} + y^2 = C
$$

Additional Insights: Deriving F_x, F_y, and the Solution Set

Partial Derivative with respect to x:

$$
F_x = \frac{\partial}{\partial x} (x + e^{xy} + y^2) = 1 + y e^{xy}
$$

Partial Derivative with respect to y:

$$
F_y = \frac{\partial}{\partial y} (x + e^{xy} + y^2) = x e^{xy} + 2 y
$$

which matches the original components M and N, reinforcing the correctness of the solution.

Practical Applications and Further Analysis

This type of differential equation appears in various scientific contexts, such as:

- Population dynamics, where exponential terms model growth or decay.
- Thermodynamics, involving exponential relationships between variables.
- Electrical engineering, in the analysis of certain circuits with exponential voltage or current relations.

Understanding how to identify exact equations and find their potential functions simplifies solving complex differential equations, especially those involving exponential and polynomial terms.

Conclusion

In this comprehensive analysis, we've:

- Recognized the structure and identified the differential equation as exact.
- Computed the potential function \(F(x, y)\).
- Derived the general solution \(x + e^{xy} + y^2 = C\).
- Verified the solution through partial derivatives.

Mastering such techniques enhances problem-solving skills in differential equations, enabling the handling of more intricate systems with confidence. Whether you're a student, researcher, or professional, understanding the process of solving exact differential equations is invaluable in mathematical modeling and applied sciences.

Consider The De 1 Ye Xy Dx 2y Xe Xy Dy 0 Then The De Is F X Hence X Y And G Y

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