Consider The Following.f(x, Y, Z) = Xe3yz, P(1, 0, 3), U = < 2/3 , -1/3, 2/3>Find The Gradient

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Introduction: Understanding the Problem and Its Significance

In the realm of multivariable calculus, the concept of the gradient plays a fundamental role in understanding how functions change in space. The gradient vector points in the direction of the steepest ascent of a function and provides critical insights into the behavior of multivariable functions, especially in optimization problems, physics, and engineering applications.

The problem at hand involves a specific multivariable function:

$$f(x, y, z) = x e^{3 y z}$$

along with a particular point $P(1, 0, 3)$ and a direction vector $U = \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle$. The task is to find the gradient of the function f at point P, which involves calculating the partial derivatives, and then understanding how the gradient interacts with the given point and direction.

This problem is not just an exercise in differentiation; it also illustrates how gradients are used to analyze the behavior of functions in three-dimensional space, which is particularly relevant in fields such as physics (e.g., potential fields), computer graphics, and optimization algorithms.

Understanding the Function $f(x, y, z)$

Before diving into the gradient calculation, it's essential to understand the structure of the function:

$$f(x, y, z) = x e^{3 y z}$$

This is a product of a linear term x and an exponential function $e^{3 y z}$. The exponential component depends on the product $y z$, making the function sensitive to changes along those variables.

Key properties of the function:

- It is smooth and differentiable everywhere in $\mathbb{R}^3$.
- The exponential term $e^{3 y z}$ amplifies or diminishes the value depending on the signs and magnitudes of y and z.
- At the point $P(1, 0, 3)$, the function simplifies because $y=0$, leading to $f(1, 0, 3) = 1 \times e^{0} = 1$.

Calculating the Gradient of $f(x, y, z)$

The gradient vector ∇f of a scalar function $f(x, y, z)$ is composed of its partial derivatives with respect to each variable:

$$
\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)
$$

Calculating each component involves applying rules of differentiation, particularly the chain rule for the exponential term.

Partial Derivative $\frac{\partial f}{\partial x}$

Since $f(x, y, z) = x e^{3 y z}$, treating y and z as constants:

$$
\frac{\partial f}{\partial x} = e^{3 y z}
$$

This is straightforward because the exponential component is independent of x.

Partial Derivative $\frac{\partial f}{\partial y}$

Here, x is constant, and we differentiate with respect to y:

$$
\frac{\partial f}{\partial y} = x \frac{\partial}{\partial y} e^{3 y z}
$$

Applying the chain rule:

$$
\frac{\partial}{\partial y} e^{3 y z} = e^{3 y z} \times 3 z
$$

Thus,

$$
\frac{\partial f}{\partial y} = x \times e^{3 y z} \times 3 z
$$

Partial Derivative $\frac{\partial f}{\partial z}$

Similarly, differentiating with respect to z:

$$
\frac{\partial f}{\partial z} = x \times \frac{\partial}{\partial z} e^{3 y z}
$$

Applying the chain rule again:

$$
\frac{\partial}{\partial z} e^{3 y z} = e^{3 y z} \times 3 y
$$

Therefore,

$$
\frac{\partial f}{\partial z} = x \times e^{3 y z} \times 3 y
$$

Evaluating the Gradient at Point $P(1, 0, 3)$

Now that we have the general expressions, substitute the point $P(1, 0, 3)$ into each component.

Step 1: Calculate $e^{3 y z}$ at P:

$$
e^{3 \times 0 \times 3} = e^{0} = 1
$$

Step 2: Compute each partial derivative:

- $\frac{\partial f}{\partial x}$:

$$
e^{3 y z} = 1
$$

$$
\left. \frac{\partial f}{\partial x} \right|_{P} = 1
$$

- $\frac{\partial f}{\partial y}$:

$$
x = 1, \quad z=3, \quad e^{3 y z} = 1
$$

$$
\left. \frac{\partial f}{\partial y} \right|_{P} = 1 \times 1 \times 3 \times 3 = 9
$$

- $\frac{\partial f}{\partial z}$:

$$
x=1, \quad y=0, \quad e^{3 y z} = 1
$$

$$
\left. \frac{\partial f}{\partial z} \right|_{P} = 1 \times 1 \times 3 \times 0 = 0
$$

Thus, the gradient vector at $P(1, 0, 3)$ is:

$$
\nabla f(1, 0, 3) = \left(1, 9, 0 \right)
$$

\]

Understanding the Gradient Vector and Its Applications

The gradient vector $\nabla f(1, 0, 3) = \left(1, 9, 0 \right)$ points in the direction of the steepest increase of the function f at point P. Its magnitude and direction provide valuable information for various applications.

Applications include:

- Optimization: In finding local maxima or minima, the gradient helps identify the direction to move toward increasing or decreasing the function.
- Directional Derivatives: The rate of change of f in an arbitrary direction U can be calculated using the dot product of ∇f and U.
- Physics: The gradient of potential functions indicates force fields and energy gradients.

Calculating the Directional Derivative in the Direction U

Given the direction vector:

$$
U = \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle
$$

To find the rate of change of f at P in the direction U:

$$
D_U f = \nabla f \cdot U
$$

First, verify that U is a unit vector:

$$
|U| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = \sqrt{\frac{4}{9} + \frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{9}{9}} = 1
$$

Since U is a unit vector, the directional derivative simplifies to the dot product:

$$
D_U f = (1, 9, 0) \cdot \left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right) = 1 \times \frac{2}{3} + 9 \times \left(-\frac{1}{3}\right) + 0 \times \frac{2}{3}
$$

Calculating:

$$
\frac{2}{3} - 3 + 0 = \frac{2}{3} - 3 = - \frac{7}{3}
$$

Interpretation:

The negative value indicates that in the direction U, the function f decreases at a rate of $\frac{7}{3}$.

Conclusion: Summary and Insights

Calculating the gradient of a multivariable function is a foundational skill in calculus, and understanding its implications is essential across various scientific fields. In this problem

Frequently Asked Questions

How do you find the gradient of the function $f(x, y, z) = x e^{3 y z}$ at the point P(1, 0, 3)?

To find the gradient, compute the partial derivatives of f with respect to x, y, and z, then evaluate at the point P(1, 0, 3).

What are the steps to calculate the partial derivatives of $f(x, y, z) = x e^{3 y z}$?

First, differentiate f with respect to each variable: $\partial f/\partial x = e^{3 y z}$, $\partial f/\partial y = x e^{3 y z} 3z$, and $\partial f/\partial z = x e^{3 y z} 3y$.

How do you evaluate the gradient components at the point P(1, 0, 3)?

Substitute x=1, y=0, z=3 into the partial derivatives: $\partial f/\partial x = e^{0} = 1$, $\partial f/\partial y = 1 e^{0} 33 = 9$, and $\partial f/\partial z = 1 e^{0} 30 = 0$.

What is the resulting gradient vector of f(x, y, z) at point P(1, 0, 3)?

The gradient vector is <1, 9, 0>.

What is the significance of the gradient vector in multivariable calculus?

The gradient vector points in the direction of the steepest ascent of the function and its magnitude indicates the rate of increase.

How does the given vector U = <2/3, -1/3, 2/3> relate to the gradient of f at P?

U is a direction vector; to find the directional derivative of f in direction U at P, take the dot product of the gradient and U.

How do you compute the directional derivative of f at P in the direction of

u?

Calculate the dot product of the gradient vector <1, 9, 0> and u = <2/3, -1/3, 2/3>: (1)(2/3) + (9)(-1/3) + (0)(2/3) = 2/3 - 3 + 0 = -7/3.

What is the value of the directional derivative of f at P in the direction of u?

The directional derivative is -7/3, indicating the rate of change of f in that direction at P.

Additional Resources

Consider The Following: f(x, y, z) = x e^{3 y z}, P(1, 0, 3), u = < 2/3, -1/3, 2/3 > — Find The Gradient

In the realm of multivariable calculus, understanding how functions change in space is critical. The gradient vector offers a powerful tool for capturing the directional rate of change of a multivariable function at a specific point. Today, we explore a concrete example involving the function $f(x, y, z) = x e^{3 y z}$, with a particular focus on computing its gradient at a specific point and understanding the significance of this vector. This exploration not only sharpens our calculus skills but also deepens our appreciation for how functions behave in three-dimensional space.

Introduction to the Gradient and Its Importance

Before diving into the problem, it's essential to grasp what the gradient represents. In simple terms, the gradient of a function $f(x, y, z)$, denoted as ∇f, is a vector composed of the partial derivatives of the function with respect to each variable:

$$
\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle
$$

This vector points in the direction of the greatest rate of increase of the function at a given point, and its magnitude indicates how steep that increase is. Conversely, the negative of the gradient points toward the direction of greatest decrease.

Calculating the gradient at a specific point allows us to understand how the function behaves locally—information crucial for optimization problems, fluid flow analysis, and various applications in physics and engineering.

The Function and the Given Data

Our function of interest is:

$$
f(x, y, z) = x e^{3 y z}
$$

The point at which we need to compute the gradient is:

$$
P(1, 0, 3)
$$

Additionally, a vector $U = \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle$ is provided, which often indicates a direction for directional derivatives or other contextual interpretations. However, for the scope of this article, the primary focus is on computing the gradient at point P.

Step-by-Step Calculation of the Gradient

1. Computing Partial Derivatives

To find the gradient ∇f, we need to compute the partial derivatives with respect to each variable.

a. Partial derivative with respect to x:

Since $f(x, y, z) = x e^{3 y z}$, treating y and z as constants:

$$
\frac{\partial f}{\partial x} = e^{3 y z}
$$

b. Partial derivative with respect to y:

Here, x is treated as a constant, and we differentiate:

$$
f(x, y, z) = x e^{3 y z}
$$

Using the chain rule:

$$
\frac{\partial f}{\partial y} = x \times \frac{\partial}{\partial y} e^{3 y z} = x \times e^{3 y z} \times 3 z = 3 x z e^{3 y z}
$$

c. Partial derivative with respect to z:

Similarly, differentiating with respect to z:

$$
\frac{\partial f}{\partial z} = x \times e^{3 y z} \times 3 y = 3 x y e^{3 y z}
$$

---

2. Evaluating the Partial Derivatives at $P(1, 0, 3)$

Now, substitute $x=1$, $y=0$, and $z=3$ into the derivatives.

a. $\frac{\partial f}{\partial x}$:

$$
e^{3 \times 0 \times 3} = e^{0} = 1
$$

b. $\frac{\partial f}{\partial y}$:

$$
3 \times 1 \times 3 \times e^{0} = 3 \times 3 \times 1 = 9
$$

c. $\frac{\partial f}{\partial z}$:

$$
3 \times 1 \times 0 \times e^{0} = 0
$$

---

3. Assembling the Gradient Vector

The gradient vector at point $P(1, 0, 3)$ is:

$$
\nabla f (1, 0, 3) = \left\langle 1, 9, 0 \right\rangle
$$

This vector indicates the direction and rate of the steepest ascent of the function at P. Notably, the z-component is zero, suggesting no change in the function's steepness along the z-axis at this point.

Significance of the Gradient

The calculated gradient $\left\langle 1, 9, 0 \right\rangle$ provides several insights:

- Direction of fastest increase: The vector points roughly in a direction where x increases slightly, y increases significantly, and z remains unchanged.
- Magnitude: The magnitude of the gradient:

$$
|\nabla f| = \sqrt{1^2 + 9^2 + 0^2} = \sqrt{1 + 81} = \sqrt{82} \approx 9.055
$$

indicates a relatively steep increase in the function's value in that direction.

The Role of the Directional Vector U

Given the vector $U = \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle$, a natural next step in many applications is to compute the directional derivative of f at P in the direction of U.

Note: For U to be a valid direction vector, it should be a unit vector. Let's verify whether U is normalized.

$$
|U| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = \sqrt{\frac{4}{9} + \frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{9}{9}} = 1
$$

Since the magnitude is 1, U is a unit vector.

Computing the Directional Derivative in U

The directional derivative $D_U f$ at P is given by:

$$
D_u f = \nabla f \cdot u
$$

Calculating the dot product:

$$
D_u f = (1)(\frac{2}{3}) + (9)(-\frac{1}{3}) + (0)(\frac{2}{3}) = \frac{2}{3} - 3 + 0 = \frac{2}{3} - 3 = -\frac{7}{3}
$$

This negative value indicates that, in the direction of u, the function decreases at the point P.

Practical Implications and Applications

Understanding the gradient and directional derivatives in three-dimensional functions like $f(x, y, z) = x e^{3 y z}$ has numerous real-world applications:

- Optimization: Finding local maxima or minima of functions, which is essential in engineering design, economics, and machine learning.
- Physical sciences: Describing how physical quantities such as temperature, pressure, or concentration change in space.
- Navigation and path planning: Determining optimal paths along which a quantity increases or decreases most rapidly.

The example demonstrates how a seemingly complex function can be analyzed systematically to reveal its local behavior.

Summary

In this comprehensive exploration, we've:

- Defined the gradient and its significance.
- Calculated the partial derivatives of $f(x, y, z) = x e^{3 y z}$.
- Evaluated the derivatives at a specific point $P(1, 0, 3)$.
- Assembled the gradient vector $\left\langle 1, 9, 0 \right\rangle$.
- Interpreted the gradient's meaning in the context of the function's behavior.
- Verified the direction vector u and computed the directional derivative.

Through this process, we've seen how calculus tools enable us to understand and predict the behavior of multivariable functions in space, providing critical insights for science, engineering, and beyond.

Final Thoughts

The ability to compute and interpret the gradient is fundamental for anyone working with multivariate functions. Whether optimizing a design, analyzing physical phenomena, or navigating complex data landscapes, the gradient provides a compass pointing toward the most significant changes. As you've seen in this example, methodical calculation and interpretation turn abstract mathematical concepts into powerful tools for real-world problem-solving.

Consider The Following F X Y Z Xe3yz P 1 0 3 U Lt 2 3 1 3 2 3 Gt Find The Gradient

Consider The Following F X Y Z Xe3yz P 1 0 3 U Lt 2 3 1 3 2 3 Gt Find The Gradient

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